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A Mathematical Model for Scheduling of Elective and Emergency Patients in the Operating Rooms

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Abstract: Facing numerous challenges has created the need for efficient use of limited resources in healthcare systems. In this regard, operating rooms (ORs) are of utmost importance as they generate a significant portion of the hospital's revenues. ORs managers need to take into consideration several conflicting priorities such as surgeons' schedules, patients' preferences and arrival time, uncertainty in surgery durations, and ORs availability. Uncertainties, limited resources, and increased patient demands make the ORs planning and scheduling one of the most complex tasks in healthcare systems. Healthcare providers are under pressure to decrease hospital costs, improve patients waiting time, and optimally allocate available resources. Accordingly, this study is about developing an efficient OR planning and scheduling system to minimize ORs costs and improve patients waiting time. In this respect, a mathematical model is developed to schedule patients in ORs. Two sources of schedule disruptions are considered in the model: (i) the arrival of emergency patients, and (ii) changes in the surgical durations. In the presented case study, a standard optimization algorithm, LINGO 18, is used to solve the developed model.

Key words: operating rooms, scheduling, optimization
JEL code: C61

1. Introduction

Healthcare is one of the most complex and fastest-growing industries in the developed world and the largest domestic industry in the U.S. (Lamiri et al., 2008; Agdestein, 2012). The national health expenditures in the U.S. increased from around \$1.3 trillion in 2000 to around \$3.3 trillion in 2016, which accounts for 17.9 percent of the gross domestic product [4]. It is important to mention that hospital care expenditures were around \$1.1 trillion in 2016, which is almost one-third of the total health expenditures.

Operating rooms (ORs) are considered the central engine of hospitals. ORs generate more than 40% of a hospital's total revenue and consume almost 30% of resource costs (Davila, 2013; Fei et al., 2010). Moreover, ORs have a major effect on hospitals' performance since they are directly connected with several hospital departments (Cardoen et al., 2010). Although ORs are financially crucial, they have an average utilization of only 68% (Abedini et al., 2017). Thus, improving the utilization of ORs may generate substantial revenue for hospitals. However, it should be emphasized that OR planning and scheduling (ORPS) is a complex task due to many

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conflicting factors with a high level of uncertainty. The uncertainty associated with length of stay, patient flow, and duration of surgical operations (SOs) demands effective stochastic methodologies in ORPS (Davila, 2013).

ORPS process consists of three stages: pre-operative stage, intra-operative stage, and post-operative stage, as shown in Figure 1. The cycle time from confirming a surgery decision to starting a surgical procedure is the pre-operative stage in which all the preparations to start a surgical procedure, such as collection of patients' information, physical examination, and medical tests, are performed. The required time in this stage ranges from minutes, for emergency patients, to days or months, for elective patients, with a surgery planned, in advance. The intra-operative stage begins with patients admitted to an OR. In this stage, surgeries and all other activities during a surgical procedure are executed. This stage ends with patients discharged from ORs and transferred to recovery areas such as Post Anesthesia Care Units (PACU) or Surgical Intensive Care Units (SICU) (Abedini et al., 2017; Batun, 2011).

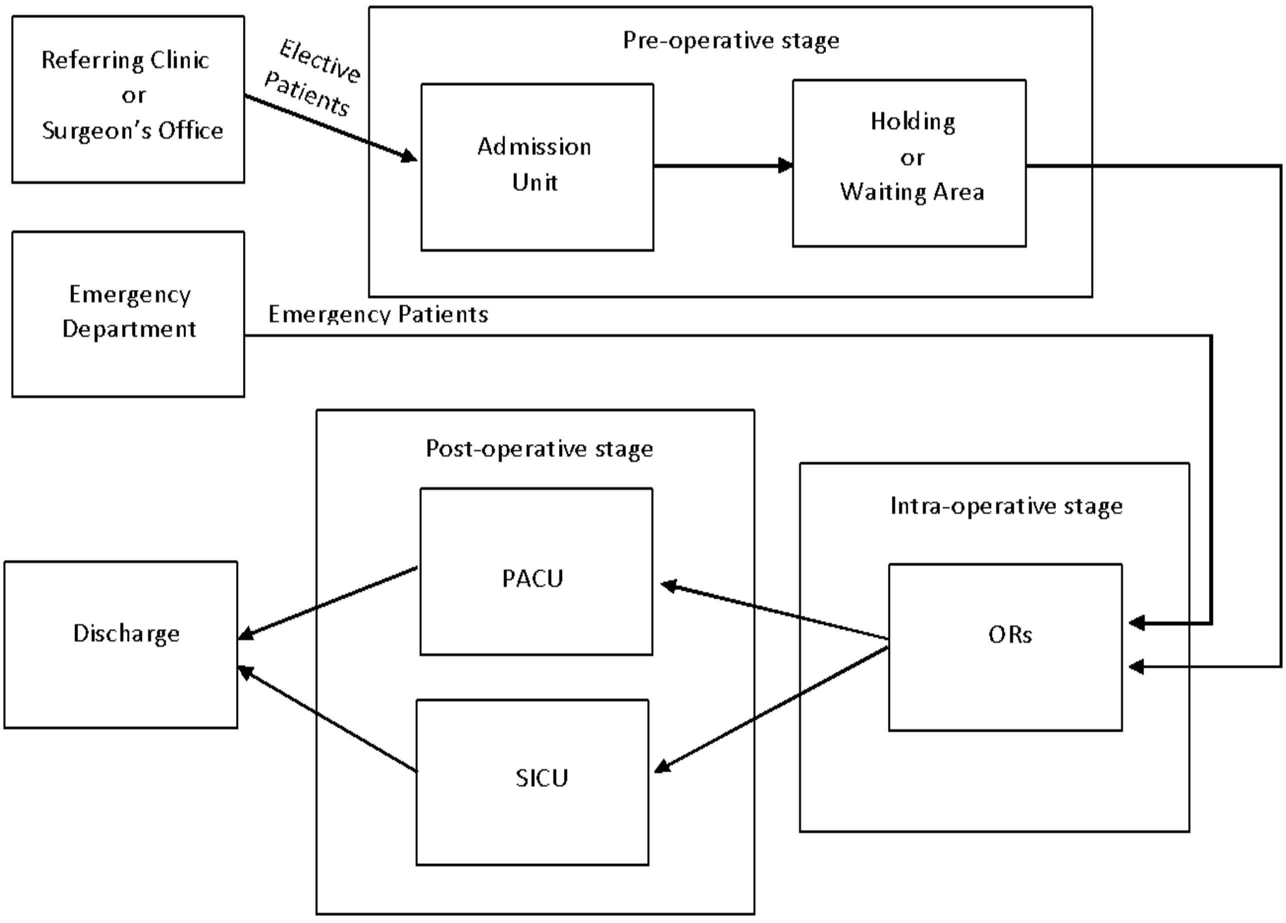

Figure 1 Flow of Surgical Patients

To deal with the unexpected arrival of emergency patients, hospitals normally reserve certain OR capacity for urgent surgeries. This can be done in multiple ways. Dedicating certain ORs to emergency patients is the first option. These ORs are called dedicated rooms. This option might result in low utilization of ORs. The second option is serving urgent surgeries in elective ORs. In this option, elective rooms are available for urgent surgeries before an elective surgery starts or after it is done. The times before or after the operation of elective surgeries are called Break-In-Moments (BIMs), as shown in Figure 2. In other words, urgent surgeries will have to wait until elective surgeries are completed. The third option is the combination of the above-mentioned options in which an emergency patient is taken to a dedicated room, otherwise, the emergency patient is taken to an elective room once it is available. Hence, BIMs are the times that an urgent operation would start immediately. To minimize total waiting time for emergency patients, these BIMs need to be spread as evenly as possible (Van Essen et al., 2012).

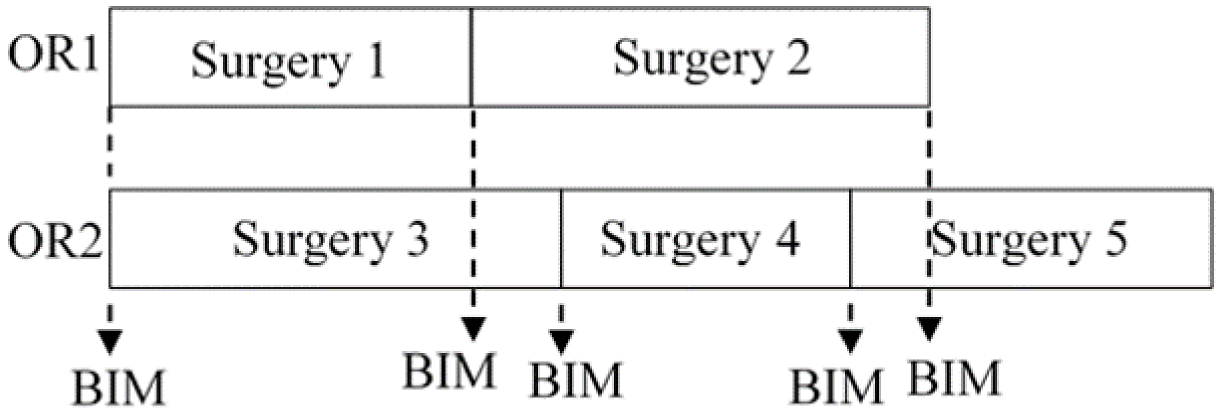

Figure 2 Break-In-Moments (BIMs)

The problem of minimizing total emergency patients' waiting time is investigated by a handful of researchers. However, these studies do not consider the rescheduling of elective patients due to unexpected disruptions. Accordingly, the objective of this research is to minimize total OR costs and improve total waiting time for elective and emergency patients in the operational offline/online scheduling stage. In doing so, a mathematical model is developed for scheduling and rescheduling elective and emergency patients. Two sources of disruptions are considered in the model: (i) the arrival of emergency patients, and (ii) changes in surgical durations.

The remainder of this study is organized as follows. Section 2 provides the background and literature review. The mathematical model is presented in section 3. A Numerical example is discussed in section 4. Finally, section 5 provides the conclusions and future research directions for the ORPS problem.

2. Background

Globerman et al. (2013) considered reducing patients' waiting time in ORs since the long waiting times are the main reason for patient dissatisfaction. Jebali and Ladet (2006) attempted to increase bed availability and patient satisfaction by minimizing OR overtime and patient waiting time. Denton et al. (2010) optimized overtime and patient waiting time for a single isolated OR while considering uncertainty in surgery durations, however, they did not consider reactive scheduling in case of any disruptions. Denton et al. (2006) developed a two-stage stochastic model using both open and block scheduling techniques to optimize patient and surgeon waiting time, OR idle time, and OR overtime costs. Tànfani and Testi (2009) studied Master Surgery Scheduling Problem by formulating a binary linear programming model and then using heuristics to solve the model assuming no emergency or outpatients. while considering the optimization of OR cost, overtime, patients' length of stay and waiting time, available OR equipment, number of surgeons, number of stays, and SICU beds.

Erdem (2013) developed several OR scheduling and rescheduling models considering scheduling elective patients to minimize the cost of ORs, hiring additional surgical teams, and some downstream resources, such as PACU beds. This work is one of the pioneer works in rescheduling elective patients due to the arrival of emergency patients. Surgical durations were assumed to be stochastic with a known probability. However, the author did not consider emergency patients' waiting time in the research.

Heydari and Soudi (2015) developed an operational off-line two-stage stochastic model for elective patient scheduling by considering the probability of the arrival of emergency patients. Their model minimizes the makespan, overtime, and the expected cost of disruption. Surgical durations are assumed to be deterministic. They did not consider the operational online scheduling level, which reschedules the elective patients at the time of emergency patients' arrival and schedules the arriving emergency patients.

Stuart and Kozan (2011) proposed an optimization model to sequence the elective and non-elective patients in a single OR. They assumed some changes in the surgical durations and unexpected arrival of non-elective patients. Fei et al. (2008) adopted a two-stage approach for OR scheduling assuming that surgical teams are available all the time.

3. OR Scheduling and Rescheduling Model

This section presents an OR scheduling and rescheduling optimization model. This rescheduling model, (RSM), considers rescheduling and resequencing elective patients disrupted by the arrival of emergency patients and changes in surgical durations. After completion of surgeries in ORs, patients are transferred to PACU for recovery. The bed capacity in the PACU is limited. Thus, if beds are unavailable in the PACU, patients have to wait in ORs. Moreover, surgical teams include surgeons, surgical assistants, nurses, anesthesiologists, etc. This study takes into account the availability of surgical teams as a whole.

It is assumed that all the cost terms are independent and they do not affect each other. This is a limiting assumption which needs to be addressed in the future works. In addition, this study assumes that all of the ORs are identical and surgeries can be performed in any available ORs. However, in real life, ORs are classified into groups based on the surgeries that can be performed in them. This work also assumes that all the required equipment for surgeries is available with no failure and no breakdowns during the surgeries.

The key constraints applied in the developed model are as follows:

- Patients with a higher priority level (PL) must be scheduled earlier than other patients.
- Patients might be deferred to the next planning period.
- If there are available beds in the PACU, patients will be transferred to the PACU right after surgeries.
- There are a limited number of surgeons available, and they cannot be assigned to more than one surgery at a time.
- There must be enough time to perform the surgeries when they are scheduled.
- There must be a turnover (cleaning and preparation) time between surgeries.

When disruptions happen in the current schedule, it is modified by postponing, preponing, or canceling previously scheduled patients to tackle the disruptions. Although different scenarios are generated for the surgical durations, surgeries may last shorter or longer than expected. If surgical durations last shorter than expected, the ORs will be empty until the next planned surgery, and therefore, OR utilization will be decreased. If surgical durations take more time, patients' waiting time would increase. Having surgical durations longer than expected may also cause overtime in the ORs and canceled patients. Both scenarios, having durations shorter or longer than expected, require rescheduling and resequencing patients in the ORs.

The other source of disruption is emergency patient arrivals. As discussed earlier, in the RSM, emergency patients are operated on in elective ORs. The RSM answers the questions of what if surgical durations take shorter or longer and what if emergency patients arrive with an urgent need for ORs. When emergency patients arrive, the model allocates them to their closest BIMs to minimize their total waiting time. If more than one emergency patient arrives at the same time, the RSM schedules them based on their urgency levels. Urgency levels are characterized by waiting time of emergency patients. If there is any ongoing or completed surgery at the time of emergency patient arrivals, they are not rescheduled by the RSM because either their surgeries are completed and they are transferred to PACU or SICU or ongoing surgeries cannot be canceled or postponed. Surgical durations of emergency patients are assumed to be stochastic with known probability and they can have surgeries in any available OR. Table 1 shows the notation for index and parameters used in the RSM.

Table 1 Notation for Index and Parameters of the RSM

Indices	
i, i'	: Elective and emergency patient indices; $i, i' \in \{1, \dots, I\}$.
j	: SO type index; $j \in \{1, \dots, J\}$.
t, t'	: Time period indices; $t, t' \in \{1, \dots, T\}$.
h, k	: Auxiliary time period indices; $h, k \in T + 3$.
d, d'	: Day indices; $d \in \{1, \dots, D\}$.
m, m'	: OR indices; $m, m' \in \{1, \dots, N\}$.
w	: Scenario index; $w \in \{1, \dots, W\}$.
Parameters	
FC	: Fixed cost of opening an OR during planning cycle;
MAX_i	: Maximum operation hours for patient i;
COR	: Overtime utilization cost of an OR during planning cycle (cost/hour);
$CPACU$	: Unit expansion cost of PACU during planning cycle (cost/bed);
$BPACU$	: Current capacity of PACU in terms of beds;
$UPACU$	: Upper limit on the over-utilization of the PACU capacity in terms of beds;
CD	: Cost of deferring a patient to next planning cycle;
CC	: Cost of total completion time for all surgeries in each OR;
CR	: Penalty cost of repeating the completion times for surgeries;
OP_{jw}	: Operation time (hours) for surgery j under scenario w;
PC_j	: Length of stay (hours) at PACU for surgery type j;
RT	: Total number of regular working hours for ORs;
s_{ij}	: Equal to 1 if patient i requests surgery type j, 0 otherwise;
OP_{iw}	: Operation time (hours) time for patient i under scenario w;
PL_i	: Priority level of patient i;
TO	: Turnover time (hours);
P_w	: Probability of scenario w;
WT_i	: Waiting time (days) for patient i
HS_i	: Hospitalization cost of patient i (cost/day);
t_s	: Reference starting time for emergency patients (i.e., the time when the emergency patients arrive);
BIM_{mdt}	: Equal to 1 if there is a BIM in OR m on day d at time t;
FOS_{idtm}	: Equal to 1 if there is an ongoing or finished surgery for patient i on day d at time t in OR m when the emergency patients arrive;
ECh_i	: Waiting cost for emergency patient i;
M	: A sufficient large number;

Table 2 shows the notation for decision variables of the RSM.

Table 2 Notation for Decision Variables of the RSM

Decision Variables
DF_i : 1 if patient i is deferred to next planning cycle, 0 otherwise;
C_i : Surgery completion time for patient i;
$CMAX_{dm}$: The last surgery completion time on day d in OR m;
WC_{id} : Waiting cost of patient *i* on day *d*;
F_{md} : Equal to 1 if OR m is open on day d, 0 otherwise;
OT_{mdw} : Amount of overtime utilization of OR m on day d under scenario w;
$OPACU$: Amount of additional capacity (beds) placed in PACU;
Y_{idtmw} : 1 if patient *i* has a surgery on day d at time t in OR m under scenario w, 0 otherwise;
X_{idtm} : 1 if surgery starts on day d at time t in OR m for patient i, 0 otherwise
G_{idtw} : 1 if a patient i occupies a bed in PACU on day d at time t under scenario w, 0 otherwise;
$z_{ik}, RP_{ii'}$: Auxiliary decision variables to calculate the BIMs.
$RP_{ii'}$: Completion time repeats for patients i and i'.

The following calculation is used for converting operation hours of surgeries to operation hours of patients.

$$OP_{i,w} = \sum_{j \in J} (s_{ij} * OP_{j,w}), \quad \forall i \in I, w \in W$$

The RSM model is developed as follows:

$Minimize\ \ RSM =$

$$\left\{ \begin{array}{c} \sum_{i \in I} \sum_{d \in D} \sum_{t \in T} \sum_{m \in N} (X_{idtm} * WC_{id}) \\ + \\ \sum_{i \in I} (CD * DF_i) \\ + \\ \sum_{i \in I} \sum_{i' \in I} (CR * RP_{ii'}) \\ + \\ \sum_{m \in N} \sum_{d \in D} (CC * CMAX_{dm}) + (FC * F_{md}) \\ + \\ \sum_{m \in N} \sum_{d \in D} \sum_{w \in W} (P_w * COR * OT_{mdw}) \\ + \\ \sum_{w \in W} (P_w * CPACU * OPACU) + \sum_{i \geq 9} \sum_{d \in BIM_{mdt}} \sum_{t \geq t_s, t \in BIM_{mdt}} \sum_{m \in BIM_{mdt}} (ECh_i * (t - t_s) * X_{idtm}) \end{array} \right.$$

Subject to

$$\sum_{d \in D} \sum_{t \in T} \sum_{m \in N} X_{idtm} + D_i = 1, \ \forall i \in I \tag{1}$$

$$\sum_d \sum_t \sum_m X_{idtm} = 1, \ \forall i \in I, i \geq 9, t \geq t_s, d, t, m \in BIM_{mdt} \tag{2}$$

$$X_{idtm} = 1, \ i, d, t, m \in FOS_{idtm} \tag{3}$$

$$\sum_{i \in I} X_{idtm} \leq 1, \ \forall d \in D, t \in T, m \in N \tag{4}$$

$$\sum_{d \in D} \sum_{t \in T} \sum_{m \in N} \Big(t + MAX_i + \big((d-1) * T\big)\Big) * X_{idtm} = C_i, \ \forall i \in I \tag{5}$$

$$\sum_{t \in T} (t + MAX_i) * X_{idtm} = ORC_{idm}, \ \forall i \in I, d \in D, m \in N \tag{6}$$

$$ORC_{idm} \leq CMAX_{dm}, \forall i \in I, d \in D, m \in N \tag{7}$$

$$\sum_{i \in I} \sum_h X_{idhm} \leq 1, \forall d \in D, t \in T, m \in N, w \in W, h = \max\big(1, t - OP_{i,w} + 1\big) - TO, \ldots, t \tag{8}$$

$$\sum_{i\in I} Y_{idtmw} \leq N, \quad \forall m \in N, d \in D, t \in T, w \in W \tag{9}$$

$$WC_{id} = HS_i * WT_i * d, \ \forall i \in I, d \in D \tag{10}$$

$$Y_{idkmw} \geq s_{ij} * \sum_{m\in N} X_{idtm}, \forall m \in N, i \in I, j \in J, d \in D, w \in W, k = \mathrm{t}, \dots, t + OP_{jw} - 1, \tag{11}$$

$$\sum_{i\in I}\sum_{m\in N} s_{ij} * Y_{idtmw} \leq ST_{jdt}, \ \forall j \in J, d \in D, t \in T, w \in W \tag{12}$$

$$\sum_{i\in I}\sum_{t} Y_{idtmw} \leq RT, \ \forall m \in N, d \in D, t \in T, w \in W, t \in \{1, \dots, RT\} \tag{13}$$

$$\sum_{i\in I}\sum_{t} Y_{idtmw} = OT_{mdw}, \ \forall m \in N, d \in D, t \in T, w \in W, t \in \{RT+1, \dots, RT+OT\} \tag{14}$$

$$\sum_{i\in I}\sum_{t} Y_{idtmw} = 0, \ \forall m \in N, d \in D, w \in W, t \geq RT + OT + 1 \tag{15}$$

$$\sum_{k\in K}\sum_{w\in W} Y_{idkmw} \leq \sum_{j\in J}\sum_{w\in W}\left(s_{ij} * OP_{jw}\right) * \sum_{t\in T} X_{idtm}, \quad \forall d \in D, m \in N, i \in I \tag{16}$$

$$G_{idkw} \geq s_{ij} * \sum_{m\in N} X_{idtm}, \forall i \in I, j \in J, d \in D, t \in T, w \in W, k = \mathrm{t} + OP_{jw}, \dots, t + OP_{jw} + PC_j - 1 \tag{17}$$

$$\sum_{i\in I} G_{idtw} \leq BPACU + OPACU, \ \forall\, d \in D, t \in T, w \in W \tag{18}$$

$$OPACU \leq UPACU \tag{19}$$

$$\sum_{k\in I}\sum_{w\in W} G_{idkw} \leq W * PC_j, \forall\, i \in I, j \in J, d \in D, s_{ij} = 0{,}1 \tag{20}$$

$$X_{idtm} \leq F_{md}, \forall i \in I, m \in N, t \in T, d \in D \tag{21}$$

$$X_{i'd't'm'} * PL_{i'} - X_{idtm} * PL_i \leq M * (1 - X_{idtm}), \quad \forall i, i' \in I, m, m' \in N, t, t' \in T, d, d' \in D \tag{22}$$

$$\sum_{k\in K} k * z_{ik} \leq C_i, \forall i \in I \tag{23}$$

$$\sum_{k\in K} z_{ik} \leq 1, \ \forall i \in I \tag{24}$$

$$RP_{ii'} \geq z_{ik} + z_{i'k} - 1, \forall i, i' \in I, k \in K \tag{25}$$

$$RP_{ii'} \geq 1 - \sum_{k\in K} z_{ik} - \sum_{k\in K} z_{i'k}, \forall i, i' \in I \tag{26}$$

$$D_i, C_i, CMAX_{dm}, WC_{idtm}, F_{md}, X_{idtm}, Y_{idtmw}, OT_{mdw}, RP_{ii'} \geq 0, \ \forall i, i' \in I, m \in N, t \in T, d \in D, w \in W \tag{27}$$

$$F_{md}, X_{idtm}, Y_{idtmw}, D_i, z_{ik} \quad binary, \ \forall i \in I, m \in N, t \in T, d \in D, w \in W, k \in K \tag{28}$$

The objective function of the RSM model has eight terms. The first term minimizes the total waiting cost of elective patients. The second term minimizes the cost of deferring elective patients to the next planning cycle. The third term minimizes the penalty cost of surgery completion time repeats; thus, BIMs will be maximized. The fourth term minimizes the cost of completion of the last surgeries in ORs. The fifth term minimizes the cost of opening ORs. The sixth term minimizes the cost of overtime in ORs. The seventh term minimizes the cost of overtime in PACU. The last term minimizes the total waiting cost of emergency patients. A short description of the constraints is presented in Table 3.

Table 3 Description of the Constraints of the RSM

Constraint (1): This constraint guarantees that every patient will be either scheduled to have a surgery or deferred to next planning period.
Constraint (2): This constraint ensures that all of the emergency patients will be scheduled to have a surgery in the BIMs.
Constraint (3): This constraint guarantees that elective patients who already started or completed their surgeries cannot be rescheduled.
Constraint (4): This constraint ensures that we cannot have more than 1 patient to start a surgery in any OR at the same time.
Constraint (5): This constraint calculates the completion time of surgeries.
Constraint (6): This constraint calculates the surgery completion time of patients for each day in each OR.
Constraint (7): This constraint shows the last surgery completion time for each day in each OR.
Constraint (8): This constraint guarantees that once a patient starts a surgery, we have to wait till that surgery plus turnover time end to start another surgery.
Constraint (9): This constraint guarantees that the number of ongoing operations cannot be more than the number of ORs.
Constraint (10): This constraint calculates the waiting cost of patients.
Constraint (11): This constraint provides the link between the start and continuation of the SOs.
Constraint (12): This constraint guarantees that the existing number of surgical teams will be equal or more than the ongoing operations.
Constraint (13): This constraint determines the total utilization of the ORs in the planning cycle.
Constraint (14): This constraint calculates the amount of overtime utilized in ORs.
Constraint (15): This constraint ensures that we cannot have any ongoing operations outside of the planning period.
Constraint (16): This constraint ensures that decision variable Y_{idtmw} will be zero if a patient finishes his/her surgery.
Constraint (17): This constraint shows that patients will transfer and stay for a certain period of time in the PACU.
Constraint (18): This constraint shows that the current plus additional (if needed) capacity in PACU will be enough to satisfy the transferring patients from the ORs.
Constraint (19): This constraint determines the upper limit on the PACU capacity.
Constraint (20): This constraint makes sure that the decision variable of the PACU, G_{idtw}, will be zero if the PACU is empty or there is no patient in it.
Constraint (21): This constraint guarantees that an OR will be closed if there are no ongoing operations in that OR.
Constraint (22): This constraint is priority constraint.
Constraints (23)-(26): These constraints are the BIM constraints that calculate the BIMs.
Constraint (27): This constraint is the non-negativity constraint on all the decision variables.
Constraint (28): This constraint defines each $F_{md}, X_{idtm}, Y_{idtmw}, D_i,\ z_{ik}$ decision variable to be a binary variable.

4. A Numerical Example

Using the data in Erdem (2013), different scenarios are created for the duration of SOs. Table 4 shows the Duration of Operation (DOs) in hours for each scenario and each type of SOs. It is assumed that each scenario has a Corresponding Probability (CP) of 0.25. There are eight patients in the system waiting to have an SO with a Waiting Time (WT) of two days and a hospitalization (HS) cost of $300 daily for each patient. Turnover time is equal to one hour for each SO (Jeang & Chiang, 2010). Priority Level (PL) is 1 for each patient, except Patient 2, who has a higher PL, and equals 2.

It is assumed to be a one-day planning cycle, three available ORs, eight regular hours, and two overtime hours as shown in Table 5. The available number of surgical teams for SOs is shown in Table 6.

The cost of repeating or the penalty cost of having the same completion time for SOs is $5,000. The fixed cost of opening an OR is $2,500. The cost of completing the last SO in each OR and the cost of overtime are $1,000 per hour. The current capacity of the PACU is three beds and the length of stay is one hour for each SO. Bed expansion cost in PACU is $4,000 per bed and the upper limit is one bed. The cost of deferring a patient to the next planning cycle is $15,000.

Table 4 DOs in Hours for Each Scenario and Each Type of SOs

SO	DO (hours)				Patients	PL	WT (days)	HS ($/day)	Turnover (hr)
Cardio-Vascular	4	4	4	4	1	1	2	$300	1
Ear-Nose-Throat	2	2	2	2	2	2	2	$300	1
General Surgery	2	2	3	3	3	1	2	$300	1
Neurology	3	3	3	3	4	1	2	$300	1
OB/GYN	2	2	2	2	5	1	2	$300	1
Ophthalmology	2	3	2	3	6	1	2	$300	1
Orthopedics	4	4	4	4	7	1	2	$300	1
Hand	1	1	1	1	8	1	2	$300	1
Scenarios	1	2	3	4					
CP	0.25	0.25	0.25	0.25					

Table 5 The Planning Cycle

		Regular Time								Overtime	
		1	2	3	4	5	6	7	8	**1**	**2**
	ORs	08:00-09:00	09:00-10:00	10:00-11:00	11:00-12:00	12:00-13:00	13:00-14:00	14:00-15:00	15:00-16:00	**16:00-17:00**	**17:00-18:00**
Day 1	OR1										
	OR2										
	OR3										

Table 6 Available Number of Surgical Teams

SO	Day 1 Available Number of the Surgical Teams									
	1	2	3	4	5	6	7	8	9	10
Cardio-Vascular	1	1	1	1	1	1	1	1	1	1
Ear-Nose-Throat	1	1	1	1	1	1	1	1	1	1
General Surgery	1	1	1	1	1	1	1	1	1	1
Neurology	1	1	1	1	1	1	1	1	1	1
OB/GYN	1	1	1	1	1	1	1	1	1	1
Ophthalmology	1	1	1	1	1	1	1	1	1	1
Orthopedics	1	1	1	1	1	1	1	1	1	1
Hand	1	1	1	1	1	1	1	1	1	1

To investigate how the RSM works, it is assumed that some emergency patients arrive and there are some changes in the surgical durations. In addition to data given in the first section regarding the number of patients and the type of surgeries they request, there are two emergency patients, Patients 9 and 10, arriving for an SO at time 3 (09:00-10:00). Waiting cost of emergency Patient 9 is $15,000/hr while the waiting cost of emergency Patient 10 is $5,000/hr. The other disruption is the surgical duration for Patient 2, which increases from two hours to three hours. Table 7 shows elective and arriving emergency patients with the type of surgery they request. Table 8 shows the scheduling and sequencing results of the RSM. It is seen from Table 8 that emergency Patient 9 is operated on after Patient 8 in OR1, while emergency patient 10 is operated on after Patient 2 in OR2. Patient 7 is deferred to the next planning cycle. As seen in Table 9, the cost of total waiting time for elective patients is $4,200 since one patient, Patient 7, is deferred to the next planning cycle. The cost of deferring is $15,000 because of Patient 7. Completion times for Patients 3 and 5 are the same, which makes the cost of BIM $5,000. Patient 1 finishes his/her surgery at time 10 in OR 1, Patient 3 finishes his/her surgery at time 11 in OR2, and Patient 5 finishes his/her surgery at time 11 in OR3. Based on these finish times, the cost of completion is $3,2000. All of the three ORs are used for scheduling, which makes the cost of ORs $7,500. Patient 1 uses one hour of overtime in OR1, Patient 3 uses one or two hours of overtime in OR2, and Patient 5 uses two hours of overtime in OR3, so the cost of overtime is $4,500. The cost of overtime in PACU is zero since having three beds in PACU will be enough for transferring patients from ORs to the recovery area.

Table 7 Elective and Emergency Patients With Surgeries They Request

SO	DO (hours)				Patients	
Cardio-Vascular	4	4	4	4	1	
Ear-Nose-Throat	3	3	3	3	2	
General Surgery	2	2	3	3	3	
Neurology	3	3	3	3	4	**10**
OB/GYN	2	2	2	2	5	**9**
Ophthalmology	2	3	2	3	6	
Orthopedics	4	4	4	4	7	
Hand	1	1	1	1	8	
CP	**0.25**	**0.25**	**0.25**	**0.25**		

Table 8 Scheduling and Sequencing Results of the RSM

		Regular Time								Overtime	
		1	2	3	4	5	6	7	8	**1**	**2**
	ORs	08:00-09:00	09:00-10:00	10:00-11:00	11:00-12:00	12:00-13:00	13:00-14:00	14:00-15:00	15:00-16:00	**16:00-17:00**	**17:00-18:00**
Day 1	**OR1**	Patient 8		**Patient 9**	**Patient 9**		Patient 1	Patient 1	Patient 1	Patient 1	
	OR2	Patient 2	Patient 2	Patient 2	**Patient 10**	**Patient 10**	**Patient 10**		Patient 3	Patient 3	*Patient 3*
	OR3	Patient 4	Patient 4	Patient 4		Patient 6	Patient 6	*Patient 6*		Patient 5	Patient 5

BIM BIM BIM BIM

Table 9 Cost Results of the RSM

Cost Functions	Cost ($)
Elective Waiting Time	4200
Emergency Waiting Time	5000
Deferring	15000
BIM	5000
Completion	32000
ORs	7500
OR Overtime	4500
PACU Overtime	0

5. Conclusion and Future Work

In this study, a Mixed Integer Linear Programming (MILP) model is developed to handle the OR scheduling and rescheduling problem. This MILP model minimizes the cost of recovery units since ORs and recovery departments are interconnected units. The overtime in ORs, opening ORs, and postponing patients to the next planning cycle are other cost factors minimized by the MILP model. When the number of BIMs is maximized and emergency patients arrive unexpectedly, the RSM model schedules and sequences emergency patients and reschedules elective patients considering all the cost and waiting factors. There are two disruption sources, namely having shorter or longer durations for SOs and the arrival of the emergency patients. Both are considered in this study. Future work includes considering another disruption source, such as no show-up by patients, for rescheduling the MILP model. When patients complete their SO in ORs, they are transferred to PACU to recover. However, some patients require a higher-than-normal level of care and they need to be transferred to Surgical Intensive Care Unit (SICU) from ORs. The current MILP model only considers patients transferring to PACU. The SICU is left for future study.

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