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Minimizing the Waiting Time of Emergency Patients in the Operating Rooms

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Abstract: Operating Room (OR) departments are one of the costliest and most revenue-generating departments in the hospitals. Due to limited resources, increasing demand, and conflicting priorities, managing an OR department is a challenging task. Having an efficient OR planning and scheduling technique is one way to manage the OR departments. OR planning and scheduling problem deals with how to allocate limited resources to minimize cost and improve quality of care and service. The use of deterministic procedures such as surgical durations is not an effective tool since uncertainty is one of the factors that highly affects the planning and scheduling of the patients in the OR departments. This study focuses on the minimization of costs in ORs and improving the waiting time of patients by developing mathematical models.

Key words: scheduling, operating rooms, healthcare, operations research
JEL code: C61

1. Introduction

Operating rooms (ORs) are highly considered departments in hospitals for generating more than 40% of a hospital's total revenue and consuming almost 30% of resource costs (Davila, 2013; Fei et al., 2010). They have a major effect on hospitals' performance since they are directly connected with several hospital departments. Operating Room Planning and Scheduling (ORPS) problem deals with allocating limited resources of surgeons, ORs, and times to patients and sequencing the patients in the ORs. ORPS problem is an important problem since ORs are one of the highest revenue and cost centers in the hospitals, and it is a challenging problem due to conflicting priorities of patients, surgeons, and hospital management, and unexpected arrival of emergency patients and changes in the surgical durations. Using deterministic assumptions in surgical procedures, such as length of stay, patient flow, and duration of surgical operations (SOs), is not an effective solution in OR planning and scheduling since surgical procedures have some degree of uncertainty and variability; therefore, there is a need to use stochastic methodologies in OR planning and scheduling.

Hospitals may reserve some OR capacity for emergent surgeries to deal with the uncertain arrival of emergency patients. There are multiple options for reserving some OR capacity. The first option is having dedicated ORs. In this option, emergency patients are taken to the dedicated ORs for their surgeries if these

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dedicated ORs are available. If not, they may wait until a dedicated OR is available or transfer to a nearby hospital if the urgency level of emergency patients is too high. This option results in low utilization of the ORs since the dedicated ORs will be empty if there are no emergency patients. The second option is using elective ORs for emergency patients. ORs with regular surgeries are called elective ORs. In this option, hospitals do not have dedicated ORs reserved for emergency patients. Using this option, an emergency patient is taken to an available elective OR to have surgery. Elective ORs are available for emergency surgeries before an elective surgery starts or after it finishes. These times or moments, i.e., before or after the operation of elective surgeries, are called Break-In-Moments (BIMs), as shown in Figure 1. The last option is the combination of the first and second options. In this option, an emergency patient is taken to a dedicated OR to be operated on, otherwise, the emergency patient is operated on in an elective OR once it is available. So, it is highly important to spread the elective surgeries as evenly as possible in the elective ORs to minimize the waiting time of the emergency patients.

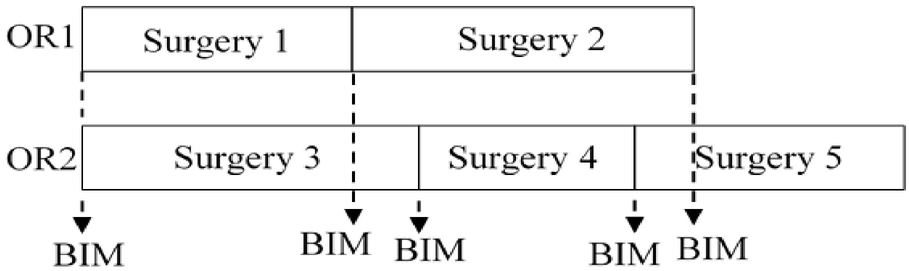

Figure 1 The BIM Problem

This paper uses the last option of dealing with emergency patients to maximize the utilization of the ORs and operate the emergency patients in the dedicated ORs if they have a high urgency level. For this purpose, two mathematical models, Mixed Integer Linear Programming (MILP), are developed. The first model is for scheduling elective patients and maximizing the number of BIMs. The second model is for scheduling emergency patients and rescheduling elective patients while maximizing the number of BIMs.

2. Literature Review

Erdem (2013) studied OR scheduling and rescheduling problem by developing mathematical models. In his study, the author considered minimizing the cost of ORs, hiring additional surgical teams, and PACU beds as downstream resources. However, the author did not include emergency patients' waiting time, the use of dedicated ORs for scheduling emergency patients, and patient prioritization in his study.

Jeang and Chiang (2010) minimized idle time and overtime in ORs by modeling the OR scheduling problem as a nonlinear integer programming. The authors considered how to reduce inpatients' length of stay and waiting time for a surgery in their model. Their model allows for rescheduling if minor changes happen. However, their model does not include interval or turnover time between SOs, even though they admit that the turnover time should be about one hour.

Another big limitation of the model is that the operating time of SOs is a deterministic variable instead of a random variable with a probabilistic distribution.

Van der Lans et al. (2005) developed various operational off-line heuristics to study the BIM problem. Then, they tested these heuristics through a simulation study and compared five different methods in the BIM problem:

reserving dedicated rooms for emergency patients, reserving some OR capacity under a subset of ORs without BIM and with BIM, reserving some OR capacity under the whole ORs capacity without BIM and with BIM. The authors concluded that the best option is reserving some OR capacity under all ORs with BIM. They only considered operational off-line level in a BIM problem to minimize the waiting time for emergency patients. However, rescheduling of elective patients upon the arrival of emergency patients was not considered.

Van Essen et al. (2012) provided some heuristic and exact solution methods for the BIM problem. They developed an integer programming model to maximize the number of BIMs for the OR scheduling problem. They assumed that elective patients were already assigned in the ORs. Furthermore, they did not consider the rescheduling of elective patients due to disruptions in the schedule.

3. Methods

In this part of the paper, two MILP models are developed. The first MILP model schedules elective patients for their surgeries. A waiting list of elective patients with the type of surgery they request is assumed to be known. Models consider downstream resources as Post Anesthesia Care Unit (PACU) beds to transfer patients right after their surgeries finish in the ORs. There are two stages in the models. Patients are assigned to days and ORs in the first stage. Then, in the second stage, patients are sequenced in the ORs. The availability of the ORs, surgical teams, and PACU beds are considered in the SM. Overtime constraints for the ORs and PACU beds are included as well. There is an upper limit for the PACU beds due to the limited number of resources. Surgical durations are assumed to be a stochastic variable with known probability. Hence, different scheduling scenarios are generated for the surgical durations. Patient priority is considered in the models. If a patient needs surgery sooner than others, that patient will be scheduled earlier in the ORs. A waiting list of elective patients with the type of surgery they will have is assumed to be known in advance. The index and parameters of the first model are shown in Table 1. The decision variables of the first model are shown in Table 2.

Table 1 Indices and Parameters of the First Model

Indices	
i, i'	: Elective patient indices; $i, i' \in \{1, \dots, I\}$.
j	: SO type index; $j \in \{1, \dots, J\}$.
t, t'	: Time period indices; $t, t' \in \{1, \dots, T\}$.
h, k	: Auxiliary time period indices; $h, k \in T + 3$.
d, d'	: Day indices; $d \in \{1, \dots, D\}$.
m, m'	: OR indices; $m, m' \in \{1, \dots, N\}$.
w	: Scenario index; $w \in \{1, \dots, W\}$.
Parameters	
FC	: Fixed cost of opening an OR during planning cycle;
MAX_i	: Maximum operation hours for patient i;
COR	: Overtime utilization cost of an OR during planning cycle (cost/hour);
$CPACU$	: Unit expansion cost of PACU during planning cycle (cost/bed);
$BPACU$	: Current capacity of PACU in terms of beds;
$UPACU$	: Upper limit on the over-utilization of the PACU capacity in terms of beds;
CD	: Cost of deferring a patient to next planning cycle;
CC	: Cost of total completion time for all surgeries in each OR;
CR	: Penalty cost of repeating the completion times for surgeries;

(Table 1 to be continued)

(Table 1 continued)

OP_{jw}	: Operation time (hours) for surgery j under scenario w;
PC_j	: Length of stay (hours) at PACU for surgery type j;
RT	: Total number of regular working hours for ORs;
s_{ij}	: Equal to 1 if patient i requests surgery type j, 0 otherwise;
OP_{iw}	: Operation time (hours) time for patient i under scenario w;
PL_i	: Priority level of patient i;
TO	: Turnover time (hours);
P_w	: Probability of scenario w;
WT_i	: Waiting time (days) for patient i
HS_i	: Hospitalization cost of patient i (cost/day);
M	: A sufficient large number;

Table 2 Decision Variables of the First Model

Decision Variables	
DF_i	: 1 if patient i is deferred to next planning cycle, 0 otherwise;
C_i	: Surgery completion time for patient i;
$CMAX_{dm}$	: The last surgery completion time on day d in OR m;
WC_{id}	: Waiting cost of patient *i* on day *d*;
F_{md}	: Equal to 1 if OR m is open on day d, 0 otherwise;
OT_{mdw}	: Amount of overtime utilization of OR m on day d under scenario w;
$OPACU$	: Amount of additional capacity (beds) placed in PACU;
Y_{idtmw}	: 1 if patient *i* has a surgery on day d at time t in OR m under scenario w, 0 otherwise;
X_{idtm}	: 1 if surgery starts on day d at time t in OR m for patient i, 0 otherwise
G_{idtw}	: 1 if a patient i occupies a bed in PACU on day d at time t under scenario w, 0 otherwise;
$z_{ik}, RP_{ii'}$	: Auxiliary decision variables to calculate the BIMs.
$RP_{ii'}$	: Completion time repeats for patients i and i'.

The following calculation is used for converting the operation hours of surgeries to the operation hours of patients.

$$OP_{i,w} = \sum_{j\in J}(s_{ij} * OP_{j,w}), \quad \forall i \in I, w \in W$$

The first model is developed as follows:

$$Minimize\ \ Z = \begin{cases} \sum_{i\in I}\sum_{d\in D}\sum_{t\in T}\sum_{m\in N}(X_{idtm} * WC_{id}) \\ + \\ \sum_{i\in I}(CD * DF_i) \\ + \\ \sum_{i\in I}\sum_{i'\in I}(CR * RP_{ii'}) \\ + \\ \sum_{m\in N}\sum_{d\in D}(CC * CMAX_{dm}) + (FC * F_{md}) \\ + \\ \sum_{m\in N}\sum_{d\in D}\sum_{w\in W}(P_w * COR * OT_{mdw}) \\ + \\ \sum_{w\in W}(P_w * CPACU * OPACU) \end{cases}$$

Subject to

$$\sum_{d\in D}\sum_{t\in T}\sum_{m\in N} X_{idtm} + DF_i = 1, \qquad \forall i \in I \tag{1}$$

$$\sum_{i\in I} X_{idtm} \le 1, \qquad \forall d \in D, t \in T, m \in N \tag{2}$$

$$\sum_{d\in D}\sum_{t\in T}\sum_{m\in N}\left(t + MAX_i + \left((d-1) * T\right)\right) * X_{idtm} = C_i, \qquad \forall i \in I \tag{3}$$

$$\sum_{t\in T}(t+MAX_i)*X_{idtm}=ORC_{idm}, \qquad \forall i\in I,d\in D,m\in N \tag{4}$$

$$ORC_{idm}\le CMAX_{dm}, \qquad \forall i\in I,d\in D,m\in N \tag{5}$$

$$\sum_{i\in I}\sum_h X_{idhm}\le 1, \forall d\in D,t\in T,m\in N,w\in W,h=\max\left(1,t-OP_{i,w}+1\right)-TO,\dots,t \tag{6}$$

$$\sum_{i\in I} Y_{idtmw}\le N, \qquad \forall m\in N,d\in D,t\in T,w\in W \tag{7}$$

$$WC_{id}=HS_i*WT_i*d, \quad \forall i\in I,d\in D \tag{8}$$

$$Y_{idkmw}\ge s_{ij}*\sum_{m\in N}X_{idtm}, \forall m\in N,i\in I,j\in J,d\in D,w\in W,k=\mathrm{t},\dots,t+OP_{jw}-1 \tag{9}$$

$$\sum_{i\in I}\sum_{m\in N}s_{ij}*Y_{idtmw}\le ST_{jdt}, \ \forall j\in J,d\in D,t\in T,w\in W \tag{10}$$

$$\sum_{i\in I}\sum_t Y_{idtmw}\le RT, \qquad \forall m\in N,d\in D,t\in T,w\in W,t\in\{1,\dots,RT\} \tag{11}$$

$$\sum_{i\in I}\sum_t Y_{idtmw}=OT_{mdw}, \quad \forall m\in N,d\in D,t\in T,w\in W,t\in\{RT+1,\dots,RT+OT\} \tag{12}$$

$$\sum_{i\in I}\sum_t Y_{idtmw}=0, \ \forall m\in N,d\in D,w\in W,t\ge RT+OT+1 \tag{13}$$

$$\sum_{k\in K}\sum_{w\in W}Y_{idkmw}\le\sum_{j\in J}\sum_{w\in W}\left(s_{ij}*OP_{jw}\right)*\sum_{t\in T}X_{idtm}, \quad \forall d\in D,m\in N,i\in I \tag{14}$$

$$G_{idkw}\ge s_{ij}*\sum_{m\in N}X_{idtm}, \forall i\in I,j\in J,d\in D,t\in T,w\in W,k=\mathrm{t}+OP_{jw},\dots,t+OP_{jw}+PC_j-1 \tag{15}$$

$$\sum_{i\in I}G_{idtw}\le BPACU+OPACU, \quad \forall d\in D,t\in T,w\in W \tag{16}$$

$$OPACU\le UPACU \tag{17}$$

$$\sum_{k\in I}\sum_{w\in W}G_{idkw}\le W*PC_j, \quad \forall i\in I,j\in J,d\in D,s_{ij}=0,1 \tag{18}$$

$$X_{idtm}\le F_{md}, \qquad \forall i\in I,m\in N,t\in T,d\in D \tag{19}$$

$$X_{i'd't'm'}*PL_{i'}-X_{idtm}*PL_i\le M*(1-X_{idtm}), \forall i,i'\in I,m,m'\in N,t,t'\in T,d,d'\in D \tag{20}$$

$$\sum_{k\in K}k*z_{ik}\le C_i, \qquad \forall i\in I \tag{21}$$

$$\sum_{k\in K}z_{ik}\le 1, \qquad \forall i\in I \tag{22}$$

$$RP_{ii'}\ge z_{ik}+z_{i'k}-1, \qquad \forall i,i'\in I,k\in K \tag{23}$$

$$RP_{ii'}\ge 1-\sum_{k\in K}z_{ik}-\sum_{k\in K}z_{i'k}, \qquad \forall i,i'\in I \tag{24}$$

$$D_i,C_i,CMAX_{dm},WC_{idtm},F_{md},X_{idtm},Y_{idtmw},OT_{mdw},RP_{ii'}\ge 0, \forall i,i'\in I,m\in N,t\in T,d\in D,w\in W \tag{25}$$

$$F_{md},X_{idtm},Y_{idtmw},D_i,z_{ik} \ \ binary, \ \forall i\in I,m\in N,t\in T,d\in D,w\in W,k\in K \tag{26}$$

The objective function of the first model has six terms. The first term minimizes elective patients' waiting cost. The second term minimizes the cost of deferring elective patients to the next planning cycle. In the third term, the number of BIMs will be maximized. The fourth one minimizes the cost of completing the last surgeries in ORs. The fifth term minimizes the cost of opening ORs. The sixth one minimizes the overtime cost in ORs. And the last term minimizes the overtime cost in PACU. A short description of the constraints is presented in Table 3.

Table 3 Description of the Constraints of the First Model

Constraints #	Description
1	Guarantees that every patient will be either scheduled to have a surgery or deferred to the next planning period.
2	Ensures no more than 1 patient to start a surgery in any OR at the same time.
3	Calculates the completion time of surgeries.
4	Calculates the surgery completion time of patients for each day in each OR.
5	Shows the last surgery completion time for each day in each OR
6	Guarantees that once a patient starts a surgery, another surgery cannot be started in the same OR until the initial surgery is finished
7	Guarantees that the number of ongoing surgeries cannot be more than the number of ORs.
8	Calculates the waiting cost of patients

(Table 3 to be continued)

(Table 3 continued)

9	Provides the link between the start and continuation of the surgeries.
10	Guarantees that the existing number of surgical teams will be equal or more than the ongoing surgeries.
11	Determines the total utilization of the ORs in the planning cycle.
12	Calculates the amount of overtime utilized in ORs.
13	Ensures that surgeries cannot be performed outside of the planning period.
14	Ensures that decision variable $\boldsymbol{Y_{idtmw}}$ will be zero if a patient finishes his/her surgery.
15	Shows that patients will transfer and stay for a certain period of time in the PACU.
16	Shows that the current plus additional capacity (if needed) in PACU will be enough to satisfy the transferring patients from the ORs.
17	Determines the upper limit on the PACU capacity.
18	Makes sure that the decision variable of the PACU, $\boldsymbol{G_{idtw}}$, will be zero if the PACU is empty or there is no patient in it.
19	Guarantees that an OR will be closed if there are no ongoing surgeries in that OR.
20	Priority constraint for the patients
21-24	Calculate the number of BIMs

Then, the first model is updated to cover the unexpectedly arriving emergency patients. The second model is developed to schedule emergency patients and reschedule elective patients. When emergency patients arrive, they may disrupt the current schedule. Thus, the second model needs to reschedule elective patients when scheduling emergency patients. Arriving emergency patients are categorized into three groups; high-urgency level emergency patients who require immediate surgery, medium-urgency level emergency patients who require surgery in two hours, and low-urgency level emergency patients who require surgery in six hours. The second model first checks the elective ORs to schedule the emergency patients. If there is no availability in the elective ORs, then the model schedules them in the dedicated ORs, based on their urgency levels. Otherwise, emergency patients are transferred to a nearby hospital. The following notations, shown in Table 4, are used in the second mode.

Table 4 Notations Used in the Second Model

I : set of total patients (elective and emergency);	J : set of surgical operations;
I_{PE} : set of emergency patients;	D : set of days;
T : set of time periods (hours);	W : set of scenarios;
N : set of operating rooms;	j : surgical operation type index;
N_{RD} : total number of dedicated rooms;	m, m' : operating room indices;
H, K : set of auxiliary time periods (hours), H, K = T + 3;	d, d' : day indices;
FC : fixed cost of opening an OR during planning cycle;	t, t' : time period indices;
$FCDR$: fixed cost of dedicated rooms per hour use.	t_s : arrival time for emergency patients;
MAX_i : maximum operation hours for patient i;	w : scenario index;
COR : overtime utilization cost of an OR during planning cycle (cost/hour);	h, k : auxiliary time period indices;
	P_w : probability of scenario w;
CD : cost of deferring a patient to next planning cycle;	WT_i : waiting time (days) for patient i;
CC : cost of total completion time for all surgeries in each OR;	i, i': elective patient indices;
$OP_{j,w}$: operation time (hours) for surgery j under scenario w;	HS_i : hospitalization cost of patient I (cost/day);
RT : total number of regular working hours for ORs;	ET_P : high-urgency level emergency patients
OT : total number of overtime hours for ORs;	UT_P : medium-urgency level emergency patients
s_{ij} : equal to 1 if patient i requests surgery type j, 0 otherwise;	NUT_P : low-urgency level emergency patients
$OP_{i,w}$: operation time (hours) time for patient i under scenario w;	PL_i : priority level of patient i;

(Table 4 to be continued)

(Table 4 continued)

WC_{id}: waiting cost of patient i on day d; CR : penalty cost of having the same completion time for surgeries; CTR_i : cost of transfer or loss of revenue for emergency patient i; FOS_{idtm} : Equal to 1 if there is a finished or ongoing surgery for patient i on day d at time t in OR m when an emergency patient arrives, 0 otherwise; $CPACU$: Unit expansion cost of PACU during planning cycle (cost/bed);	TO : turnover time (hours); ECh_i : waiting cost for emergency patient i; BIM_{mdt} : Equal to 1 if there is a BIM in OR m on day d at time t; $BPACU$: Current capacity of the PACU in terms of beds; $UPACU$: Upper limit on the over-utilization of the PACU capacity in terms of beds;

The following calculation is used for converting the operation hours of surgeries to the operation hours of patients.

$$OP_{i,w} = \sum_{j \in J} (s_{ij} * OP_{j,w}), \quad \forall i \in I, w \in W$$

The following equations are used for calculating the cost of transfers or loss of revenue for each level of emergency patients if they are transferred to nearby hospitals due to unavailable capacity.

$$CTR_i = 60000 * OP_{i,w} \quad , \quad \forall i \in ET_P, w \in W$$
$$CTR_i = 30000 * OP_{i,w} \quad , \quad \forall i \in UT_P, w \in W$$
$$CTR_i = 20000 * OP_{i,w} \quad , \quad \forall i \in NUT_P, w \in W$$

The following shows the decision variables of the second model.

Decision Variables of the Second Model

D_i : equal to 1 if patient i is deferred to next planning cycle, 0 otherwise;

C_i : surgery completion time for patient i;

$CMAX_{dm}$: The last surgery completion time on day d in OR m;

OVT_{mdw}: Amount of overtime utilization of OR m on day d under scenario w;

$OPACU$: Amount of additional capacity (beds) placed in PACU;

F_{md}: equal to 1 if OR m is open on day d, 0 otherwise;

Y_{idtmw}: equal to 1 if patient i has a surgery on day d at time t in OR m under scenario w, 0 otherwise;

X_{idtm}: equal to 1 if surgery starts on day d at time t in OR m for patient i, 0 otherwise;

G_{idtw}: Equal to 1 if a patient i occupies a bed in PACU on day d at time t under scenario w, 0 otherwise;

TR_i: Equal to 1 if an emergency patient i is transferred to nearby hospital, 0 otherwise;

FDR_{mdt}: Equal to 1 if a dedicated room m is open on day d at time t, 0 otherwise;

z_{ik}, R_{ii^2}: auxiliary decision variables to calculate the BIMs.

The second model is developed as follows:

$$Minimize \sum_{i \in I} \sum_{d \in D} \sum_{t \in T} \sum_{m \in N} (X_{idtm} * WC_{id}) + \sum_{i \in I} (CD * D_i) + \sum_{i \in I} \sum_{i' \in I} (CR * RP_{ii'}) + \sum_{d \in D} \sum_{m \in N} CC * CMAX_{dm}$$

$$+ \sum_{m \in N} \sum_{d \in D} (FC * F_{md}) + \sum_{m \in N} \sum_{d \in D} \sum_{w \in W} (P_w * COR * OVT_{mdw}) + \sum_{w \in W} (P_w * CPACU * OPACU)$$

$$+ \sum_{i \in \{I - I_{PE} + 1, \ldots, I\}} \sum_{d \in BIM_{mdt}=1} \sum_{t \geq t_s, t \in BIM_{mdt}=1} \sum_{m \in BIM_{mdt}=1} (ECh_i * (t - t_s) * X_{idtm}) +$$

$$\sum_{i\in\{I-I_{PE}+1,\dots,I\}}(CTR*TR_i)\ +\sum_{m\in N}\sum_{d\in D}\sum_{t\in T}(FCDR*FDR_{mdt})$$

s.t.

$$\textstyle\sum_{d\in D}\sum_{t\in T}\sum_{m\in N}X_{idtm}+D_i=1,\quad \forall i\in I \tag{1}$$

$$\textstyle\sum_{d}\sum_{t}\sum_{m}X_{idtm}+TR_i=1,\ \forall i\in\{I-I_{PE}+1,\dots,I\},t\in\{t_s,\dots,T\},d,t,m\in BIM_{mdt} \tag{2}$$

$$X_{idtm}*\ (t-t_s)\le 0,\ \ \forall\ i\in ET_P,d\in D,t\in\{t_s,\dots,T\},m\in N \tag{3}$$

$$X_{idtm}*\ (t-t_s)\le 2,\ \forall\ i\in UT_P,d\in D,t\in\{t_s,\dots,T\},m\in N \tag{4}$$

$$X_{idtm}*\ (t-t_s)\le 6,\ \forall\ i\in NUT_P,d\in D,t\in\{t_s,\dots,T\},m\in N \tag{5}$$

$$Y_{idtmw}\le FDR_{mdt},\ \forall i\in\{I-I_{PE}+1,\dots,I\},m\in\{N-N_{RD}+1,\dots,N\},t\in T,d\in D \tag{6}$$

$$X_{idtm}=1,\ \ i,d,t,m\in FOS_{idtm} \tag{7}$$

$$\textstyle\sum_{i\in I}X_{idtm}\le 1,\forall d\in D,t\in T,m\in N \tag{8}$$

$$\textstyle\sum_{d\in D}\sum_{t\in T}\sum_{m\in N}\Big(t+MAX_i+\big((d-1)*T\big)\Big)*X_{idtm}=C_i,\ \forall i\in I \tag{9}$$

$$\textstyle\sum_{t\in T}(t+MAX_i)*X_{idtm}=ORC_{idm},\ \forall i\in I,d\in D,m\in N \tag{10}$$

$$ORC_{idm}\le CMAX_{dm},\ \forall i\in I,d\in D,m\in N \tag{11}$$

$$\textstyle\sum_{i\in I}\sum_{h}X_{idhm}\le 1,\ \forall d\in D,t\in T,m\in N,w\in W,h=\max\big(1,t-OP_{i,w}+1\big)-TO,\dots,t \tag{12}$$

$$\textstyle\sum_{i\in I}Y_{idtmw}\le N,\ \forall m\in N,d\in D,t\in T,w\in W \tag{13}$$

$$WC_{id}=HS_i*WT_i*d,\ \forall i\in I,d\in D \tag{14}$$

$$\textstyle Y_{idkmw}\ge s_{ij}*\sum_{m\in N}X_{idtm}\,,\ \forall m\in N,i\in I,j\in J,d\in D,t\in T,w\in W,k=\mathrm{t},\dots,t+OP_{jw}-1 \tag{15}$$

$$\textstyle\sum_{i\in I}\sum_{m\in N}s_{ij}*Y_{idtmw}\le ST_{jdt},\ \forall j\in J,d\in D,t\in T,w\in W \tag{16}$$

$$\textstyle\sum_{i\in I}\sum_{t}Y_{idtmw}\le RT,\ \forall m\in N,d\in D,t\in T,w\in W,t\in\{1,\dots,RT\} \tag{17}$$

$$\textstyle\sum_{i\in I}\sum_{t}Y_{idtmw}=OVT_{mdw},\ \forall m\in N,d\in D,t\in T,w\in W,\ \ t\in\{RT+1,\dots,RT+OT\} \tag{18}$$

$$\textstyle\sum_{i\in I}\sum_{t}Y_{idtmw}=0,\ \forall m\in N,d\in D,t\ge RT+OT+1,w\in W \tag{19}$$

$$\textstyle\sum_{k\in K}\sum_{w\in W}Y_{idkmw}\le\sum_{j\in J}\sum_{w\in W}\big(s_{ij}*OP_{jw}\big)*\sum_{t\in T}X_{idtm}\,,\ \forall d\in D,m\in N,i\in I \tag{20}$$

$$\textstyle G_{idkw}\ge s_{ij}*\sum_{m\in N}X_{idtm}\,,\ \forall i\in I,j\in J,d\in D,t\in T,w\in W,k=\mathrm{t}+OP_{jw},\dots,t+OP_{jw}+PC_j-1 \tag{21}$$

$$\textstyle\sum_{i\in I}G_{idtw}\le BPACU+OPACU,\ \forall\ d\in D,t\in T,w\in W \tag{22}$$

$$OPACU\le UPACU, \tag{23}$$

$$\textstyle\sum_{k\in I}\sum_{w\in W}G_{idkw}\le W*PC_j,\ \forall\ i\in I,j\in J,d\in D,s_{ij}=0{,}1 \tag{24}$$

$$X_{idtm}\le F_{md},\ \forall i\in I,m\in m\in\{1,\dots,N-N_{RD}\},t\in T,d\in D \tag{25}$$

$$X_{i'd't'm'}*PL_{i'}-X_{idtm}*PL_i\le M*(1-X_{idtm}),\ \forall i,i'\in I,m,m'\in N,t,t'\in T,d,d'\in D \tag{26}$$

$$\textstyle\sum_{k\in K}k*z_{ik}\le C_i,\ \forall i\in I \tag{27}$$

$$\textstyle\sum_{k\in K}z_{ik}\le 1,\ \forall i\in I \tag{28}$$

$$RP_{ii'}\ge z_{ik}+z_{i'k}-1,\ \forall i,i'\in I,k\in K \tag{29}$$

$$\textstyle RP_{ii'}\ge 1-\sum_{k\in K}z_{ik}-\sum_{k\in K}z_{i'k}\,,\ \forall i,i'\in I \tag{30}$$

$$D_i,C_i,CMAX_{dm},WC_{idtm},F_{md},X_{idtm},Y_{idtmw},OT_{mdw},RP_{ii'}\ge 0,\ \forall i,i'\in I,m\in N,t\in T,d\in D,w\in W \tag{31}$$

$$F_{md},X_{idtm},Y_{idtmw},FDR_{mdt},D_i,z_{ik}\ \ \ binary\,,\qquad \forall i\in I,m\in N,t\in T,d\in D,w\in W,k\in K \tag{32}$$

The objective function of the second model has ten different objectives with equal weights. The first one minimizes the total waiting cost of elective patients. The second one minimizes the cost of deferring elective patients to the next planning cycle. The third one minimizes the penalty cost of surgery completion time repeats; thus, BIMs will be maximized. The fourth one minimizes the cost of completion of the last surgeries in ORs. The fifth one minimizes the cost of opening ORs. The sixth one minimizes the cost of overtime in ORs. The seventh one minimizes the cost of overtime in PACU. The eighth one minimizes the total waiting cost of emergency patients. The ninth one minimizes the cost of transfer for emergency patients. The last one minimizes the usage cost of dedicated rooms.

The second model has thirty-two constraints, and they are explained as follows; Constraint (1) guarantees that every elective patient will be either scheduled to have a surgery in elective ORs or deferred to next planning period. Constraint (2) ensures that all of the emergency patients will be scheduled to have a surgery in the BIMs or transferred to nearby hospitals. Constraint (3) shows that the waiting time limit for emergent patients is zero. Constraint (4) shows that the waiting time limit for urgent patients is 2 hours. Constraint (5) shows that the waiting time limit for non-urgent patients is 6 hours. Constraint (6) ensures that dedicated rooms are only used if there is no available capacity in elective rooms for emergency patients. Constraint (7) guarantees that elective patients who already started or completed their surgeries cannot be rescheduled. Constraint (8) ensures that we cannot have more than 1 patient to start a surgery in any OR at the same time. Constraint (9) calculates the completion time of surgeries. Constraint (10) calculates the surgery completion time of patients for each day in each OR. Constraint (11) shows the last surgery completion time for each day in each OR. Constraint (12) guarantees that once a patient starts a surgery, we have to wait till that surgery plus turnover time end to start another surgery. Constraint (13) guarantees that the number of ongoing operations cannot be more than the number of ORs. Constraint (14) calculates the waiting cost of patients. Constraints (15) provides the link between the start and continuation of the operations. Constraint (16) guarantees that the existing number of surgical teams will be equal or more than the ongoing operations. Constraint (17) determines the total utilization of the ORs in the planning cycle. Constraint (18) calculates the amount of overtime utilized in ORs. Constraint (19) ensures that we cannot have any ongoing operations outside of the planning period. Constraint (20) ensures that decision variable Y_{idtmw} will be zero if a patient finishes his/her surgery. Constraint (21) shows that patients will transfer and stay for a certain period of time in the PACU. Constraint (22) shows that the current capacity plus additional (if needed) capacity in PACU will be enough to satisfy the transferring patients from the ORs. Constraint (23) determines the upper limit on the PACU capacity. Constraint (24) makes sure that the decision variable of the PACU, G_{idtw}, will be zero if the PACU is empty or there is no patient in it. Constraint (25) guarantees that an OR will be closed if there are no ongoing operations in that OR. Constraint (26) is a priority constraint. Constraints (27)-(30) are the BIM constraints that calculate the BIMs. Constraint (31) is the non-negativity constraint on all the decision variables. Constraint (32) defines each $F_{md}, X_{idtm}, Y_{idtmw}, D_i, z_{ik}$ decision variable to be a binary variable.

3.1 Numerical Example

In this section of the study, the first and second models are solved to optimality. The data related to the type and duration of surgeries with Corresponding Probability (CP) are taken from Erdem (2013), as shown in Table 5.

Table 5 Type and Duration of Surgeries

Surgical Operation	Duration of operation (min)			Corresponding probability		
Cardio-Vascular (CV)	240			1		
Ear-Nose-Throat (ENT)	60	120		0.6	0.4	
General Surgery	60	120	180	0.2	0.5	0.3
Hand	60			1		
Neurology	240			1		
OB/GYN	120			1		
Ophthalmology	60	120		0.8	0.2	
Orthopedics	120			1		
Podiatry	120			1		
Urology	60	120		0.6	0.4	

The optimization software, LINGO 18, is unable to find a global optimal solution in a reasonable time, so this data is converted into a simplified version. The simplified data for the Surgical Operation (SO) with the Duration of Operation (DO) and the Corresponding Probability (CP) is shown in Table 6.

Table 6 Simplified Data

Surgical Operation (SO)	Duration of Operation (DO) (min)		Corresponding Probability (CP)	
Cardio-Vascular	240		1	
Ear-Nose-Throat	120		1	
General Surgery	120	180	0.5	0.5
Neurology	180		1	
OB/GYN	120		1	
Ophthalmology	120	180	0.5	0.5
Orthopedics	240		1	
Hand	60		1	

Using the data in Table 6, different scenarios are created for the duration of SOs. Table 7 shows the Duration of Operation (DOs) in hours for each scenario and each type of SOs. It is assumed that each scenario has a Corresponding Probability (CP) of 0.25. There are eight patients in the system waiting to have an SO with a Waiting Time (WT) of two days and a hospitalization (HS) cost of $300 daily for each patient. Turnover time is equal to one hour for each SO [40]. Priority Level (PL) is 1 for each patient, except Patient 2, who has a higher PL, and equals 2.

Table 7 DOs in Hours for Each Scenario and Each Type of SOs

SO	DO (hours)				Patients	PL	WT (days)	HS ($/day)	Turnover (hr)
Cardio-Vascular	4	4	4	4	1	1	2	$300	1
Ear-Nose-Throat	2	2	2	2	2	2	2	$300	1
General Surgery	2	2	3	3	3	1	2	$300	1
Neurology	3	3	3	3	4	1	2	$300	1
OB/GYN	2	2	2	2	5	1	2	$300	1
Ophthalmology	2	3	2	3	6	1	2	$300	1
Orthopedics	4	4	4	4	7	1	2	$300	1
Hand	1	1	1	1	8	1	2	$300	1
Scenarios	1	2	3	4					
CP	0.25	0.25	0.25	0.25					

Table 8 shows the planning cycle, which is one day, the number of available ORs, which is three, and the available time for SOs, which is eight hours for regular time and two hours for overtime. The available number of surgical teams for SOs is shown in Table 9.

Table 8 The Planning Cycle for the First Model

		Regular Time								Overtime	
		1	2	3	4	5	6	7	8	**1**	**2**
	ORs	08:00-09:00	09:00-10:00	10:00-11:00	11:00-12:00	12:00-13:00	13:00-14:00	14:00-15:00	15:00-16:00	**16:00-17:00**	**17:00-18:00**
Day 1	OR1										
	OR2										
	OR3										

Table 9 Available Number of Surgical Teams

	Day 1 Available Number of the Surgical Teams									
SO	1	2	3	4	5	6	7	8	9	10
Cardio-Vascular	1	1	1	1	1	1	1	1	1	1
Ear-Nose-Throat	1	1	1	1	1	1	1	1	1	1
General Surgery	1	1	1	1	1	1	1	1	1	1
Neurology	1	1	1	1	1	1	1	1	1	1
OB/GYN	1	1	1	1	1	1	1	1	1	1
Ophthalmology	1	1	1	1	1	1	1	1	1	1
Orthopedics	1	1	1	1	1	1	1	1	1	1
Hand	1	1	1	1	1	1	1	1	1	1

The cost of repeating or the penalty cost of having the same completion time for SOs is $5,000. The fixed cost of opening an OR is $2,500. The cost of completing the last SO in each OR and the cost of overtime are $1,000 per hour. The current capacity of the PACU is three beds and the length of stay is one hour for each SO. Bed expansion cost in PACU is $4,000 per bed and the upper limit is one bed. The cost of deferring a patient to the next planning cycle is $15,000.

Table 10 provides the scheduling and sequencing results of the SM. It takes around 28 minutes to solve in Lingo 18.

Table 10 Scheduling and Sequencing Results of the First Model

		Regular Time								Overtime	
		1	2	3	4	5	6	7	8	**1**	**2**
	ORs	08:00-09:00	09:00-10:00	10:00-11:00	11:00-12:00	12:00-13:00	13:00-14:00	14:00-15:00	15:00-16:00	**16:00-17:00**	**17:00-18:00**
Day 1	OR1	Patient 8		Patient 7	Patient 7	Patient 7	Patient 7		Patient 3	Patient 3	*Patient 3*
	OR2	Patient 2	Patient 2		Patient 5	Patient 5		Patient 6	Patient 6	*Patient 6*	
	OR3	Patient 4	Patient 4	Patient 4		Patient 1	Patient 1	Patient 1	Patient 1		

BIM BIM BIM BIM BIM BIM BIM BIM

Then, the first model is updated based on the arrival of emergency patients. Assuming that four emergency patients arrive at the same time between 09:00 am and 10:00 am. After emergency patients arrive, they directly go to the emergency department and are checked for emergency conditions. They are categorized as two of them being high-level, one being medium-level, and one being low-level. Since emergency patients arrive between

09:00 am and 10:00 am and spend some time in the emergency department to be checked for the emergency levels, the earliest time they can start for an SO is at 10:00 or time 3. Thus, in the second model, it is assumed that emergency patients arrive at time 3. Table 11 shows the elective and arriving emergency patients with the type of surgery they need.

Table 11 Elective and Emergency Patients With the Surgeries They Request

SO	DO (hours)				Elective Patients	Emergency Patients High-Level	Medium-Level	Low-Level
Cardio-Vascular	4	4	4	4	1			
Ear-Node-Throat	2	2	2	2	2	**9**		
General Surgery	2	2	3	3	3			
Neurology	3	3	3	3	4	**11**		
OB/GYN	2	2	2	2	5		**10**	
Ophthalmology	2	3	2	3	6			
Orthopedics	4	4	4	4	7			
Hand	1	1	1	1	8			**12**
CP	**0.25**	**0.25**	**0.25**	**0.25**				

Waiting costs of emergency patients are \$15,000/hr for Patient 9, \$3,000/hr for patient 10, \$5,000/hr for Patient 11, and \$2,000/hr for Patient 12. The cost of transfers or loss of revenues for emergency patients is \$120,000 for patient 9, \$60,000 for Patient 10, \$180,000 for Patient 11, and \$20,000 for Patient 12. The cost of using dedicated rooms per hour is \$10,000. Table 12 shows the scheduling and sequencing results of the second model.

Table 12 Scheduling and Sequencing Results of the Second Model

		Regular Time								Overtime	
		1	2	3	4	5	6	7	8	**1**	**2**
	ORs	08:00-09:00	09:00-10:00	10:00-11:00	11:00-12:00	12:00-13:00	13:00-14:00	14:00-15:00	15:00-16:00	**16:00-17:00**	**17:00-18:00**
Day 1	OR1	Patient 8		**Patient 11**	**Patient 11**	**Patient 11**		Patient 6	Patient 6	*Patient 6*	
	OR2	Patient 2	Patient 2		**Patient 10**	**Patient 10**	**Patient 10**		Patient 3	Patient 3	*Patient 3*
	OR3	Patient 4	Patient 4	Patient 4		**Patient 12**		Patient 5	Patient 5		
	DR			**Patient 9**	**Patient 9**						

BIM BIM BIM BIM BIM

Since Patients 9 and 11 are high-level emergency patients and need to have a surgery immediately, the second model schedules Patient 9 in the dedicated room and Patient 11 in the OR1 at time 3 or at 10:00 am immediately. Patient 10 is a medium-level emergency patient who needs to have a surgery within two hours and is scheduled in the OR2 at time 4 or 11:00 am. Patient 12 is a low-level emergency patient who needs to have a surgery within six hours and is scheduled in the OR3 at time 5 or 12:00 pm. Elective Patients 1 and 7 are deferred to the next planning cycle due to unavailable capacity.

4. Conclusion

This study considers operational online level OR planning and scheduling problem by developing two MILP models. The first model considers scheduling and sequencing of elective patients in the ORs by minimizing cost of ORs and downstream units and improving the total waiting time of elective and emergency patients. When patients finish their surgeries in the ORs, they are transferred to downstream recovery units as PACU, before their discharge. If there is no available capacity in PACU, patients stay in ORs after their surgeries until there is available capacity in the recovery units. This will cause a delay in scheduling patients in ORs since non-transferred patients will be using OR resources. Then, the second model updates the first model based on the arrival of the emergency patients. The second model considers having dedicated rooms in addition to elective rooms in case some emergency patients may require an immediate operation. For this, the second model reschedules and reassigns elective patients and schedules and sequences emergency patients by taking dedicated rooms into consideration. The cost of using dedicated rooms and the cost of transfer of patients to nearby hospitals if there is no available capacity are additional cost factors considered by the second model.

There are two disruption sources considered in this study, namely having shorter or longer durations for SOs and arrival of the emergency patients. Future work includes considering another disruption source, such as no show-up by patients, for rescheduling the MILP model. When patients complete their SO in ORs, they are transferred to PACU to recover. However, some patients require a higher-than-normal level of care, and they need to be transferred to Surgical Intensive Care Unit (SICU) from ORs. The current MILP models only consider patients transferring to PACU. The SICU is left for future study.

References

Davila M. P. (2013). “A methodology for scheduling operating rooms under uncertainty”, doctoral dissertation, available online at: https://digitalcommons.usf.edu/cgi/viewcontent.cgi?article=5660&context=etd.

Fei H., Meskens N. and Chu C. (2010). “A planning and scheduling problem for an operating theatre using an open scheduling strategy”, *Computers & Industrial Engineering*, Vol. 58, No. 2, pp. 221-230, doi: https://doi.org/10.1016/j.cie.2009.02.012.

Erdem E. (2013). “Optimization models for scheduling and rescheduling elective surgery patients under the constraint of downstream units”, available online at: https://library.ndsu.edu/ir/handle/10365/27227.

Jeang A. and Chiang A. J. (2010). “Economic and quality scheduling for effective utilization of operating rooms”, *Journal of Medical Systems*, Vol. 36, No. 3, pp. 1205-1222, doi: https://doi.org/10.1007/s10916-010-9582-0.

Van Der Lans M., Hans E., Hurink J., Wullink G., Van Houdenhoven M. and Kazemier G. (2005). “Anticipating urgent surgery in operating room departments”, Vol. 158, available online at: https://pure.tue.nl/ws/files/4321936/615316.pdf.

Van Essen J., Hans E., Hurink J. and Oversberg A. (2012). “Minimizing the waiting time for emergency surgery”, *Operations Research for Health Care*, Vol. 1, No. 2-3, pp. 34-44, doi: https://doi.org/10.1016/j.orhc.2012.05.002.