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Analysis of Sunshine Duration Data Using Two-Parameter Weibull Distributions

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Abstract: Previous studies have shown that beta probability density functions (β-PDFs) are very good fits of the observed relative frequency distributions of data from various stations for dimensionless physical quantities such as the clearness index (k_t), the diffuse fraction (k_d), the relative wind speed (v_r) and the relative sunshine duration (s_r). In this work, attempts have been made to fit actual relative frequency distributions of sunshine duration (S) data with 2-parameter Weibull PDFs, which are otherwise often used to fit experimental frequency distributions of wind speed and wind power density data. The monthly and annual observed relative frequency distributions ($f_{i,obs}$) of S data from a 5-years period of records at six Burundian stations have been implemented and their main characteristics have been derived. The shape and scale parameters (k and c, respectively) of the theoretical (Weibull) PDFs ($f_{i,th}$) fitting those distributions ($f_{i,obs}$) have been determined by the means of the graphical method. The validity of the obtained formulations has been checked through three complementary statistical tests, namely the mean bias error (MBE), the root mean square error (RMSE) and the t-statistics. For any of the 78 S datasets, the MBE and the RMSE have been found very low as expected, since they lie in the ranges [-0.01;+0.009] and [0.040;0.052], respectively. Moreover, the values of t were lower than their critical counterparts (t_c). Those results show that the 2-parameter Weibull PDFs obtained so far are not statistically different from the related observed relative frequency distributions of S data. They should be therefore used as input data when planning solar energy systems for the stations of this study.

Key words: relative frequency distributions, statistical tests, sunshine duration data, 2-parameter Weibull PDFs

1. Introduction

The sunshine duration (S) is one of the physical quantities used to describe the climatic behaviour of a given geographical area and then to manage social and economic activities at that area, e.g., in tourism, transport, energy, agriculture, architecture and health. When designing solar energy conversion systems for example, S data can be used solely or together with data of other climatologically quantities to estimate the hourly, monthly or annual average of the beam (I), diffuse (D) and global (G) solar radiation components. This is achieved through statistically based formulations, e.g., Ångström-type simple linear relationships [1, 2] and linear regressions [3-5]. Furthermore, the statistical properties of I, D, G (and thus S) data are required when predicting the storage capacity, the long-term mean performance and the utilization of solar energy systems. The monthly and annual frequency distributions of actual I, D, G and S data are needed for that purpose. Nevertheless, most of the time, one rather prefers to use appropriate analytical expressions playing the role of probability density functions (PDFs) and representing the empirical frequency distributions of those data. Beta PDFs are one kind of such expressions which have been demonstrated to fit successfully actual frequency distributions of dimensionless physical quantities which range randomly from 0 to 1, e.g., the clearness index (k_t) and the diffuse fraction (k_d) [6, 7], the relative wind speed (v_r) [8], together with the relative sunshine duration (s_r) [9, 10].

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The sunshine duration S is not a dimensionless quantity. Its daily values (in hours) have been recorded continuously for a long period at many stations in Burundi. The wind speed (v) is another non dimensionless quantity for which long-term measurements (in m/s) have been performed on a 1-hour or a 3-hours period basis at different stations in Burundi. Data of those (and many other) climatologically quantities have been collected and kept by the Geographical Institute of Burundi (IGEBU). Any of the two quantities (S and v) ranges randomly between two boundary values: zero and the day length (S_o) for S, zero and the infinity for v (in theory). In practice, the working wind speeds for any wind machine range from a starting value and a cut-off one. Since 2-parameter Weibull PDFs are commonly used to fit frequency distributions of observed wind speed (and wind power density) data at various locations [11-15], attempts will be made in this work to fit the observed relative frequency distributions of S data from different Burundian stations, with such kinds of PDFs. The main characteristics of those distributions will be presented, together with the related fits and their statistical tests.

2. Basic Data and Selected Stations

The sunshine duration (S) data used in this study refer to a 5-years period of records (1990-1994) at the following six Burundian stations: Cankuzo (L= 30.38°E; Φ= 3.28°S'; z=1652 m); Gisozi (L= 29.68°E; Φ=3.57° S'; z=2097 m); Kirundo (L=30.12°E;φ= 2.58° S'; z=1449 m); Gitega-Airport (L=29.92°E; Φ= 3.42°S'; z= 1449 m); Makamba (L=29.82°E; Φ= 4.13°S'; z=1450 m) and Mparambo (L= 29.82°E; Φ= 2.83°S'; z =887m). The quantities L, Φ and z are the station's longitude, latitude and altitude, while E and S' hold for east and south, respectively. The previous stations and period have been selected for the representation of the main Burundian regions and the matching of reasonable continuous long-term data records. Owing to some record discontinuities, only the following numbers of daily S data were available amongst the 1826 ones expected for each station: Cankuzo (1681); Gisozi (1760); Gitega- Airport (1692); Kirundo (1517); Makamba (1595); Mparambo (1720).That means a total of 9965 data and thus 90.95% of data collecting.

3. Methodology

For each station, the S data of the considered period have been divided into twelve monthly sets and an annual one. That has resulted in 78 datasets for the whole database. The relative frequency distribution of each dataset has been constructed and its main characteristics have been derived. Then, as for fitting the relative frequency distribution of observed wind speed (v) data by a suitable 2-parameter Weibull PDF (which is derived from a Pearson III type distribution), a random variable S which is assumed to theoretically range from zero to the infinity has been considered. That distribution is given by the next expression [11, 16]:

$$F(S) = 1 - \exp\left[-\left(\frac{S}{c}\right)^k\right] \quad (1)$$

The related PDF has the next form:

$$f(S) = \frac{dF}{ds} = \frac{k}{c}\left(\frac{S}{c}\right)^{k-1} exp\left[-\left(\frac{S}{c}\right)^k\right];\ k>0,\ c>0 \quad (2)$$

where k and c are the Weibull shape and scale parameters, respectively. The main techniques used to determine empirically those parameters include the graphical (or least squares, or linear regression) method [16, 17], the maximum likehood method [18, 19], the modified maximum likehood method [12, 18], the standard deviation method, the moments method, the power density method, the equivalent energy method and the median rank regression method [11, 13, 20-26]. The graphical method has been used in this analysis for its simplicity. From Eq. (1), one gets:

$$1 - F(s) = \exp\left[-\left(\frac{S}{c}\right)^k\right] \quad (3)$$

and thus

$$ln\{-ln[1 - F(S)]\} = k \ln S - k \ln c \quad (4)$$

Eq. (4) looks like a straight line equation of the form

$$y = ax + b \quad (5)$$

with

$$y = \ln\{-\ln[1 - F(s)]\} \quad (6)$$

$$x = \ln s \quad (7)$$

$$a = k \quad (8)$$

$$b = -k \ln c \quad (9)$$

Therefore, from the observed S_i and $F(S_i)$ values, with $F(S_i)=\sum_j^i f(S_j)$ for each S dataset, a cloud of points (x_i, y_i) has been constructed. As the Weibull (and the exponential and Rayleigh) PDFs do not accurately represent the probability of observing zero or very low v values [8], [16], S_i values equal (or very close) to zero were excluded from this analysis. The linear regression's line fitting that cloud has been determined by deriving a = k as the slope of that line and $b = -k \ln c$ as its intercept on the y-axis. The correlation coefficient R (between the x and y variables) has also been computed.

The obtained k and c values have been used to plot the suitable theoretical (Weibull) PDF (points (S_i, $f_{i, th}$)) on the same system of axes as the related observed relative frequency distribution of S data (points (S_i, $f_{i, obs}$)). Then the checking of the fitting effectiveness has been made through three complementary statistical tests, namely the MBE, the RMSE and the t-statistics which are defined by the next relationships:

(i) $$\text{MBE} = \frac{1}{n}\sum_{i=1}^{n}\left(f_{i,obs} - f_{i,th}\right) \quad (10)$$

(ii) $$\text{RMSE} = \left[\frac{1}{n}\sum_{i=1}^{n}\left(f_{i,obs} - f_{i,th}\right)^2\right]^{\frac{1}{2}} \quad (11)$$

(iii) $$t = \left[\frac{(n-1)(MBE)^2}{(RMSE)^2-(MBE)^2}\right]^{1/2} \quad (12)$$

In Eqs. (10) to (12), n is the total number of ranges of 1.0 hour width each, into which any S dataset has been divided in order to construct its frequency distribution. The quantities $f_{i,obs}$ and $f_{i,th}$ are the experimental relative frequency distribution of that S dataset and the related theoretical counterpart (Weibull PDF), respectively. An extended discussion on those statistical tests can be found in some of the previous references, e.g., in Refs. [3, 5, 6, 10, 27, 28].

4. Results and Discussion

4.1 On the Observed Relative Frequency Distributions

Table 1 indicates for any of the 78 observed relative frequency distributions of S datasets, the values of the following quantities: mean ($\bar{S}$), standard deviation (σ), variation coefficient (c_v) and mode.

The curves of the relative frequency distributions of some S datasets of this analysis are shown in red lines on the Figures 1.a to 1.f. Those distributions exhibit the following mean features. They are generally spread as indicated by the high values of c_v in table 1.They are also wavering, with one mode in general or two modes in particular. They present in general a high negative asymmetry (mode > mean). In seasonal trends, the highest S means values are noticed in the dry season, i.e., from June to September, with the maximum in July. At their turn, the lowest S means are observed during the rainy season (the remaining period of the year). Moreover, owing to the annual average of S from Table 1, and using $\bar{S}_o$ =12.00 h as the mean day length at Burundian stations [27], one should term the six stations of this analysis as following: Kirundo and Makamba are fairly sunny ($\frac{\bar{s}}{\bar{s}_0} \in$ [0.48; 0.50]) while Cankuzo, Gisozi, Gitega-Airport and Mparambo are rightly sunny ($\frac{\bar{s}}{\bar{s}_0} \in$ [0.51; 0.60]).

4.2 On the Fitting Process

For the whole 78 S datasets, Table 2 exhibits the results of the following quantities related to the fitting process: the regression coefficients (a and b) and the correlation coefficient (R) (between the x and y variables), the Weibull shape and scale parameters (k and c, respectively), the 3 statistical tests (MBE, RMSE, and t). The critical values of t (t_c) at different numbers of degrees of freedom (n-1) and confidence levels can be found in standard tables [29]. In this work, t_c = 3.055 at n = 13 and 99.5% of confidence level.

Table 1 Values of the means, standard deviations, variation coefficients and modes of the 78 observed relative frequency distributions of S datasets, period 1990-1994.

(i) Cankuzo

Month → Quantity↓	J	F	M	A	M	J	J	A	S	O	N	D	Year
$\bar{S}$(h)	6.17	5.39	6.30	6.85	8.17	9.01	9.53	8.86	8.17	6.60	5.88	4.93	7.21
σ(h)	3.26	3.00	3.29	3.14	3.03	2.54	1.94	2.34	2.68	3.19	3.10	2.78	3.22
c_v	0.53	0.56	0.52	0.46	0.37	0.28	0.20	0.26	0.33	0.48	0.53	0.57	0.45
Modes (h)	4.5	5.5;6.5	4.5;5.5	8.5	10.5	10.5	10.5	10.5	10.5	9.5	7.5	4.5	10.5

(ii) Gisozi

Month → Quantity↓	J	F	M	A	M	J	J	A	S	O	N	D	Year
$\bar{S}$(h)	5.51	4.85	5.64	5.71	6.23	7.42	8.68	7.91	7.37	6.02	4.98	5.02	6.31
σ(h)	3.13	2.89	3.04	2.47	3.12	3.03	2.60	2.67	2.95	2.93	2.63	2.78	3.12
c_v	0.57	0.60	0.54	0.43	0.50	0.41	0.30	0.34	0.40	0.49	0.53	0.55	0.49
Modes(h)	4.5	0.5	6.5	3.5	7.5	9.5	10.5	11.5	9.5	9.5	4.5	3.5	8.5

(iii) Gitega-Airport

Month → Quantity↓	J	F	M	A	M	J	J	A	S	O	N	D	Year
$\bar{S}$(h)	5.94	5.20	6.00	6.20	6.51	8.07	9.47	8.65	7.65	5.99	5.26	5.28	6.63
σ(h)	3.17	2.86	3.12	2.81	3.09	2.80	2.42	2.56	2.89	3.04	2.85	2.67	3.11
c_v	0.53	0.55	0.52	0.45	048	0.35	0.26	0.30	0.38	0.51	0.54	0.51	047
Modes(h)	9.5	6.5	10.5	5.5;7.5	7.5	10.5	11.5	10.5	10.5	5.5	3.5	4.5	10.5

(iv)Kirundo

Month → Quantity↓	J	F	M	A	M	J	J	A	S	O	N	D	Year
$\bar{S}$(h)	5.43	5.20	5.07	5.43	5.31	6.47	7.42	7.23	6.17	5.19	5.45	4.62	5.78
σ(h)	2.53	2.61	2.59	2.50	2.55	2.56	2.26	2.22	2.40	2.30	2.43	2.39	2.58
c_v	0.47	0.50	0.51	0.46	0.48	0.40	0.30	0.31	0.39	0.44	0.45	0.52	0.45
Modes(h)	3.5	7.5	5.5	5.5	5.5	8.5	8.5	9.5	7.5;8.5	6.5	4.5;5.5	4.5;6.5	8.5

(v)Makamba

Month → Quantity↓	J	F	M	A	M	J	J	A	S	O	N	D	Year
$\bar{S}$(h)	5.01	4.97	5.25	5.13	5.85	7.10	8.91	8.27	7.34	6.06	4.98	4.25	6.04
σ(h)	3.12	2.60	3.00	2.63	2.75	2.76	1.99	2.29	2.77	2.87	2.84	2.72	3.06
c_v	0.62	0.52	0.57	0.51	0.47	0.39	0.22	0.28	0.38	0.47	0.57	0.64	0.51
Modes(h)	1.5;5.5	7.5	3.5	4.5	6.5	8.5	10.5	10.5	8.5;10.5	7.5	2.5;5.5	3.5	9.5

(vi) Mparambo

Month → Quantity↓	J	F	M	A	M	J	J	A	S	O	N	D	Year
$\bar{S}$(h)	5.55	5.53	5.51	5.82	6.03	7.05	7.50	7.05	6.21	5.95	5.37	4.92	6.08
σ(h)	2.85	2.60	2.86	2.33	2.80	2.72	2.42	2.50	2.62	2.64	2.29	2.55	2.72
c_v	0.51	0.47	0.52	0.40	0.46	0.39	0.32	0.33	0.42	0.44	0.43	0.52	0.45
Modes(h)	6.5	5.5	4.5;7.5	5.5	6.5	9.5	8.5	9.5	7.5	6.5	5.5	3.5;4.5	8.5

Table 2 Results of the various quantities related to the fitting process for all the 78 S datasets, period 1990-1994

(i)Cankuzo

Months→ Quantities↓	J	F	M	A	M	J	J	A	S	O	N	D	year
a = k	1.289	1.241	1.520	1.495	1.401	1.673	3.382	1.395	1.781	1.372	1.361	1.279	1.412
b	-2.467	-2.090	-2.638	-3.058	-3.431	-3.936	-8.156	3.883	-4.168	-2.745	-2.382	-2.0251	-2.879
c	6.779	5.388	5.672	7.733	11.576	10.513	11.152	16.176	10.384	7.394	5.756	4.871	7.682
R	0.980	0.970	0.928	0.972	0.965	0.781	0.967	0.839	0.973	0.981	0.965	0.939	0.927
MBE	0.0100	0.0051	0.0018	0.0118	0.0289	0.0184	0.0278	0.0457	0.0376	0.0117	-0.0288	0.0085	0.0089
RMSE	0.0409	0.0351	0.0427	0.0416	0.0696	0.0905	0.0934	0.1239	0.0733	0.0389	0.1310	0.0386	0.0505
t	0.8363	0.4871	0.1461	0.9600	1.5140	0.7193	0.9353	1.3161	1.9819	1.0460	0.7807	0.7487	0.6202

(ii) Gisozi

Months→ Quantities↓	J	F	M	A	M	J	J	A	S	O	N	D	year
a = k	1.186	1.167	1.251	1.753	1.321	1.465	1.592	1.823	1.651	1.417	1.467	1.547	1.395
b	-2.032	-1.747	-2.189	-3.153	-2.280	-3.241	-4.072	-4.123	-3.225	-2.593	-2.364	-2.299	-2.595
c	5.547	4.468	9.520	6.041	5.618	9.136	12.907	9.599	7.052	6.233	5.010	4.420	6.425
R	0.960	0.950	0.956	0.933	0.950	0.973	0.932	0.975	0.890	0.969	0.992	0.963	0.943
MBE	0.0245	0.0009	0.0222	0.0027	0.0032	0.0186	0.0347	0.0193	0.0047	0.0064	0.0030	-0.0004	0.0072
RMSE	0.0389	0.0446	0.0327	0.0283	0.0495	0.0481	0.0667	0.0546	0.0568	0.0376	0.0377	0.0363	0.0418
t	2.6893	0.0669	3.0667	0.3179	0.2244	1.3910	2.0203	1.2533	1.651	1.417	1.467	1.547	1.395

(iii) Gitega-Airport

Months→ Quantities↓	J	F	M	A	M	J	J	A	S	O	N	D	year
a = k	1.319	1.444	1.339	1.845	1.668	2.143	1.508	1.866	1.688	1.289	1.550	1.512	1.507
b	-2.306	-2.186	-2.483	-3.237	-2.960	-4.762	-4.413	-4.639	-3.733	-2.370	-2.332	-2.542	62.847
c	5.745	4.563	6.388	5.780	5.898	9.227	18.660	12.014	9.129	6.288	4.502	5.372	6.614
R	0.944	0.936	0.974	0.968	0.939	0.983	0.934	0.963	0.943	0.961	0.898	0.7000	0.948
MBE	-0.0152	0.0007	0.0075	0.0164	0.0014	0.0162	0.0557	0.0344	0.0168	0.0098	0.0002	0.0042	0.0025
RMSE	0.0500	0.0416	0.0398	0.0649	0.0473	0.0579	0.1110	0.0715	0.573	0.0416	0.0388	0.0375	0.0381
t	1.1054	0.0558	0.6364	0.9047	0.1026	0.9216	0.9241	1.7355	1.0623	0.8040	0.0179	0.3562	0.2278

(iv) Kirundo

Months→ Quantities↓	J	F	M	A	M	J	J	A	S	O	N	D	year
a = k	1.529	1.653	1.215	1.814	1.292	1.488	1.285	2.506	2.070	1.673	1.599	1.615	1.619
b	-2.607	-2.395	-2.056	-2.858	-2.283	-2.947	-3.391	-5.086	-3.577	-2.739	-2.698	-2.141	-2.737
c	5.502	4.258	5.431	3.833	5.853	7.246	13.998	7.610	5.629	5.141	5.405	3.765	5.422
R	0.969	0.936	0.959	0.968	0.962	0.932	0.919	0.957	0.953	0.141	5.405	3.765	5.422
MBE	0.0415	0.0642	0.0520	0.0427	0.0479	0.0692	0.1090	0.0738	0.0877	0.0494	0.0458	0.0633	0.0500
RMSE	0.3372	0.02698	0.6803	0.01623	0.8954	0.6435	1.6244	0.3561	0.0877	O.O494	0.0458	0.0633	0.0500
t	0.3372	0.02698	0.6803	0.1623	0.8954	0.6435	1.6244	0.3561	0.00395	0.1410	0.1668	0.3831	0.0929

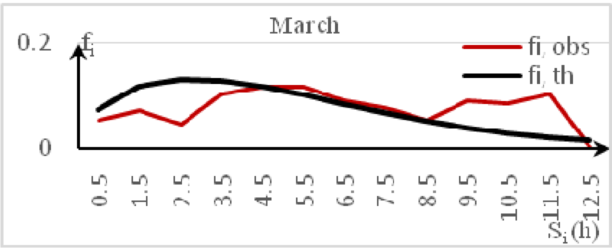
(v)Makamba

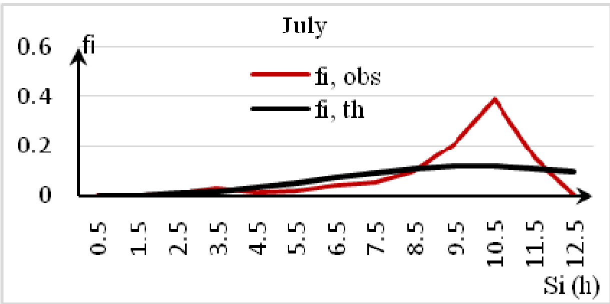
Months→ Quantities↓	J	F	M	A	M	J	J	A	S	O	N	D	year
a = k	1.178	1.338	1.174	1.501	1.536	1.334	1.892	2.348	1.585	1.372	1.294	1.167	1.266
b	-3.387	-2.108	-2.008	-2.440	-2.716	-3.031	-4.908	-5.475	-3.387	-2.529	-2.029	-1.584	-2.348
c	4.933	4.833	5.531	5.081	5.860	9.700	13.384	10.296	8.473	6.317	4.797	3.886	6.389
R	0.99.	0.959	0.985	0.984	0.967	0.947	0.876	0.982	0..935	0.951	0.978	0.978	0.944
MBE	0.0047	0.0032	0.0092	0.0031	0.0037	0.0278	0.0372	0.0314	0.0144	0.0070	0.0025	0.0015	0.0086
RMSE	0.0329	0.0517	0.0369	0.0353	0.0382	0.0620	0.1120	0.0801	0.0617	0.0429	0.0380	0.0469	0.0405
t	0.4787	0.1961	0.8141	0.2788	0.3228	1.5863	1.1679	1.2783	0.7960	0.5477	0.2187	0.10613	0.7211

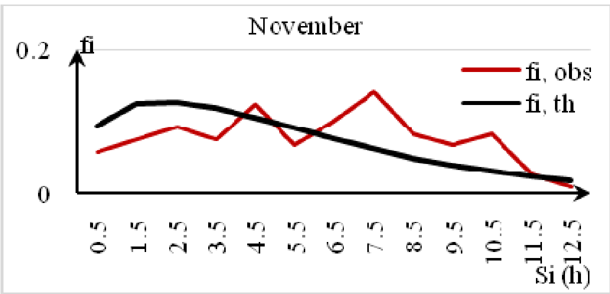
(vi) Mparambo

Months→ Quantities ↓	J	F	M	A	M	J	J	A	S	O	N	D	year
a = k	1.248	1.480	1.583	2.146	1.307	1.822	1.452	1.167	1.365	1.325	1.760	1.336	1.486
b	-2.123	-2.526	-2.417	-3.567	-2.511	-3.324	-3.426	2.943	-2.676	-2.500	-2.975	-2.100	-2.676
c	5.480	5.511	4.604	5.271	6.829	6.199	10.586	12.452	7.102	6.598	5.421	4.816	6.054
R	0.933	0.952	0.931	0.975	0.954	0.898	0.908	0.925	0.933	0.924	0.973	0.964	0.936
MBE	0.0052	0.0031	0.0002	0.0069	0.0137	0.0015	0.0314	0.0447	0.0143	0.0121	0.0023	0.0039	0.0048
RMSE	0.0529	0.0456	0.0534	0.0590	0.0481	0.0724	0.0984	0.0956	0.0575	0.0537	0.0412	0.0461	0.0491
t	0.3276	0.2260	0.012974	0.407923	0.88286	0.071785	1.0648	1.4027	0.78645	0.731437	0.1785	0.2685	0.3258

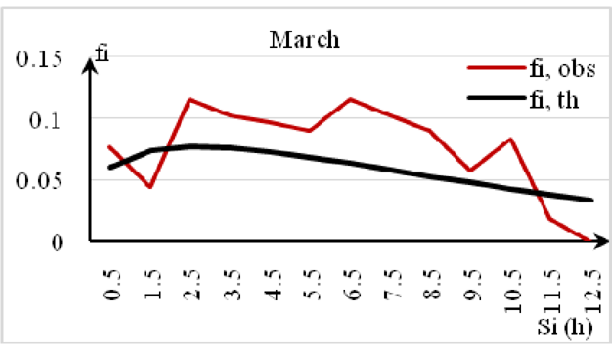
(i) Cankuzo

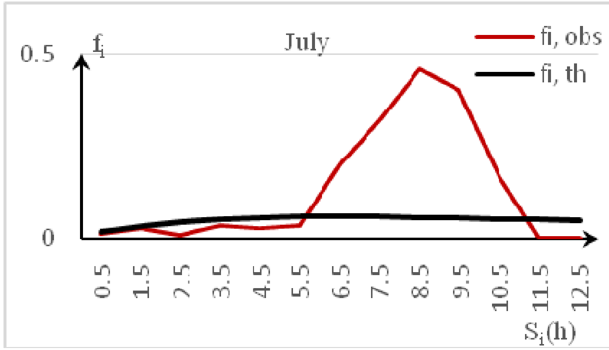


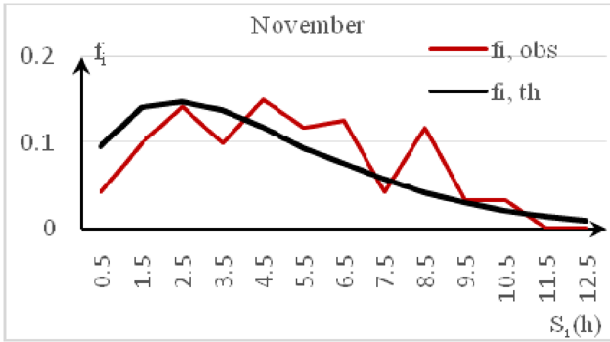




(ii) Gisozi







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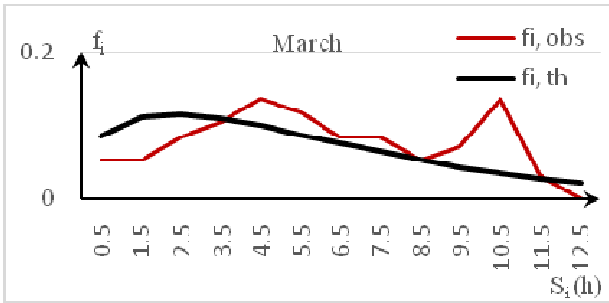

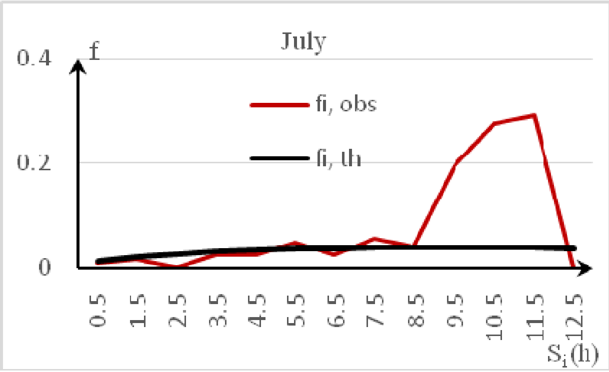

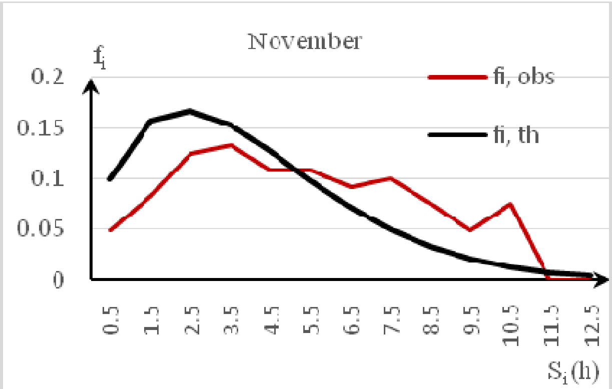

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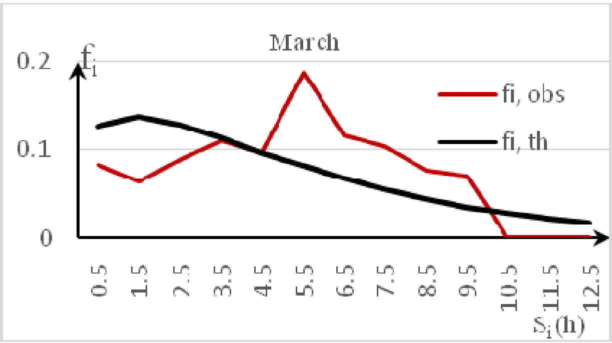
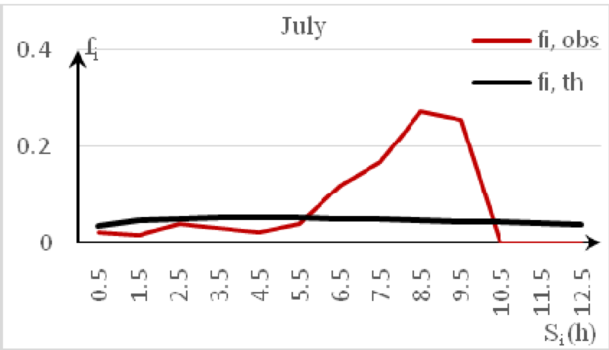
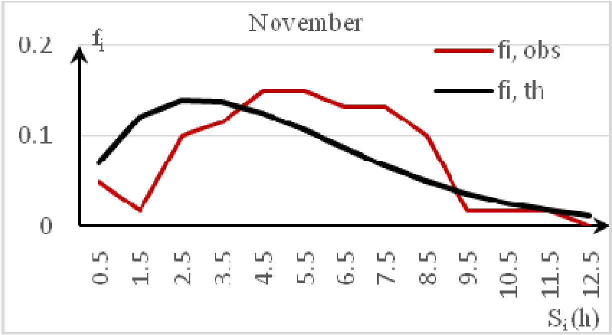
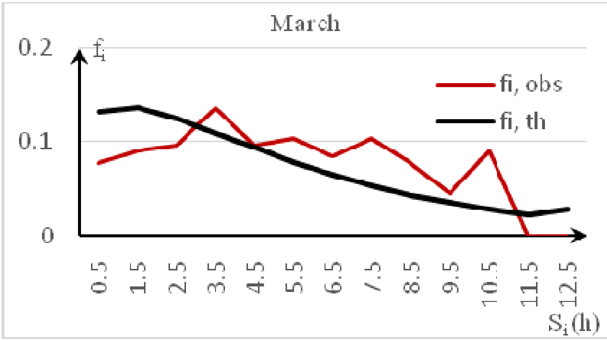
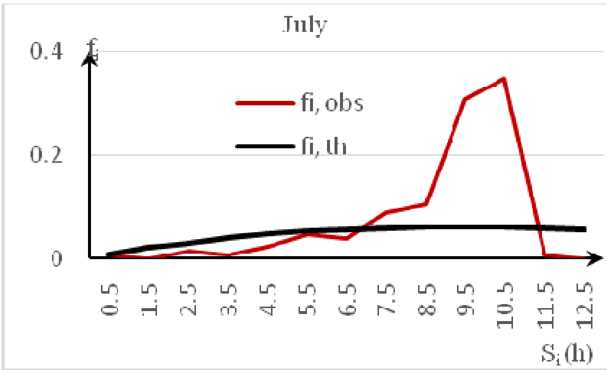


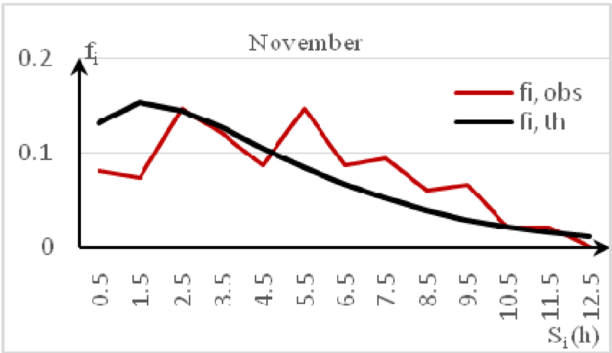




(v) Makamba







(vi) Mparambo

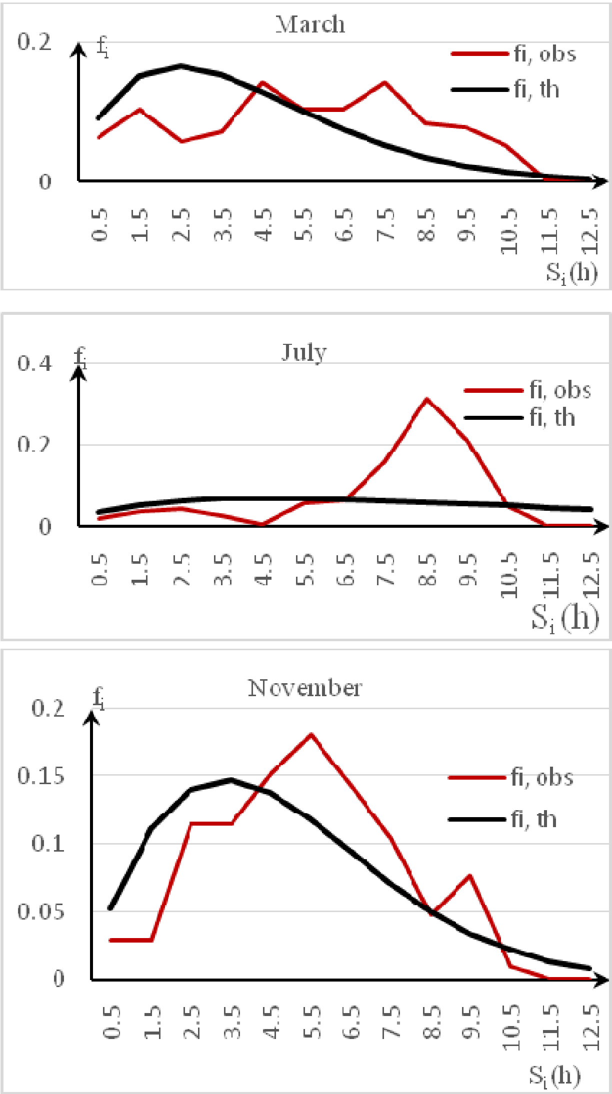


Fig. 1 Curves of the relative frequency distributions of daily sunshine duration data from six Burundian sites, period 1990-1994: observed data (red lines); Weibull PDFs fitting the observed data (black lines).

Some examples of the curves of the Weibull PDFs fitting the related observed relative frequency distributions of S datasets are shown in black lines on the Figs. 1a to 1f.

From such curves and the whole data of Table 2, the following features should be pointed out. The variables x and y of Eq.(5) are highly correlated since the quantity R lies in the range [0.90;0.99] for 73 S datasets and in the range [0.70;0.89] for the remaining 5 S datasets. As expected, k and c are positive. Moreover, except for four S datasets, k ranges from 1 to 2. This indicates that in general the Weibull PDFs obtained in this analysis range from exponential to Rayleigh PDFs. The values of the Weibull scale parameter (c) of this study exhibit similar seasonal trends as the means ($\bar{S}$) and the modes of the observed distributions, i.e., the highest values in the dry season— with the maximum in July— and the lowest values in the remaining period of the year. The values of MBE and RMSE have been found quite close to zero as expected. Nevertheless, MBE is observed positive for 75 S datasets out of 78. This means an average slight underestimation of the observed distributions by the related theoretical functions (Weibull PDFs). For all the 78 S datasets, t has been found lower than its critical value ($t_c = 3.055$) for the number of degrees of freedom n-1 = 12 and for 99.5% of confidence level. Despite the slight disparities noticeable between the black lines and the red ones on Figs. 1a to 1f, all the previous results indicate that the 2-parameter Weibull PDFs obtained so far fit in a satisfactory manner the related observed relative frequency distributions of daily S data for the period and stations of this study.

5. Conclusion

Based on a 5-years period of data records at six Burundian locations, the monthly and annual relative frequency distributions of the daily sunshine duration values have been computed in this study and their main characteristics have been pointed out. Furthermore, the 2-parameter Weibull PDFs fitting those distributions have been constructed by the means of the linear regression method. Three complementary statistical tests, namely the MBE, RMSE and t-statistics, have been implemented in order to check the efficiency of the fitting technique. For all the 78 S datasets, the MBE and RMSE have been found close to zero as expected, even if the MBE was slightly positive in most of the cases, indicating a slight underestimation of the observed frequencies ($f_{i,obs}$) by the theoretical (Weibull) counterparts ($f_{i,th}$). At its turn, the variable t of Student was lower than its critical value (t_c) for any of those S datasets. Therefore, all the Weibull PDFs obtained in this analysis are very good fits of the related observed relative frequency distributions of S datasets. They should constitute useful input data in any project of solar energy conversion systems at the considered stations.

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