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Similarity Solution of Heat and Mass Transfer for the Falling Film Flow on a Porous Medium in Presence of Heat Generation or Absorption

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Abstract: In this research work, similarity solution of heat and mass transfer for the falling film flow on a porous medium in presence of heat generation or absorption has been modeled by Darcy-Brinkman equations and solved by using similarity technique. Heat generation, thermal radiation and chemical reaction effects are considered. By using appropriate transformations, the governing nonlinear partial equations are transformed into coupled nonlinear ordinary differential equations. Graphs are decorated to explore the influence of physical parameters on the non-dimensional velocity, temperature and concentration distributions. The local Nusselt number and the local Sherwood number are computed and analyzed numerically.

Key words: similarity solution, heat generation, absorption, falling film, porous medium

1. Introduction

There are many applications in heat pumps, chillers and air-conditioners which is the gas absorption process taking place on a falling liquid film has received much attention. In these absorption machines, the refrigerant vapor from evaporator is absorbed by a falling film of an absorbent solution in an absorber, and then the absorbent solution is regenerated by releasing the refrigerant vapor in a regenerator (boiler). Since these absorption machines are driven mainly by low-grade energy, heat (Jones and Hawkins, 1986), rather than by high-grade energy, electricity, their application is especially interesting in areas where the electric power supply is limited. The performance of the absorption machine is controlled by the heat and mass transfer rates of the absorption process. Therefore, it is important to study the means of enhancing the heat and mass transfer rates of an absorption process. The absorption process for an absorption process is taking place on a falling film flow in a porous medium which is considered by Yang and Jou [2]. The application of porous media in a falling film absorption process is mainly to enhance the wetting conditions which are also discussed by Yang and Jou [3].

Gebhart and Pera [4] and Chen and Yuh [5] treated the vaporizing liquid film as the boundary condition for the gas stream and Shembharkar and Pai [6] and Baumann and Thiele [7] assumed the temperature distribution across the film to be linear. Recently, researches with more rigorous treatments of the equations governing the liquid film and liquid–gas interface have been published. Yan and Lin [8] studied the evaporative cooling of liquid film through interfacial heat and mass transfer in a vertical channel. A. Miyara [9] investigated the flow dynamics and heat transfer characteristics of wavy liquid films. Leu et al.

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[10] analyzed the liquid film evaporation flow along a vertical isothermal plate covered with a thin liquid-saturated porous layer.

Khader and Megahed [11] are presented a numerical technique which is the implicit finite difference method to the search for the numerical solutions for the given equations. Their technique reduces the problem to a system of algebraic equations. Recently, M. Hasanuzzaman and A. Miyara [12] have been studied a possible similarity solution of unsteady natural convection laminar boundary layer flow of viscous incompressible fluid caused by a heated (or cooled) axi-symmetric slender body of finite axial length immersed vertically in a viscous incompressible fluid. The basic theme of the present study is to investigate the effect of heat generation or absorption, thermal radiation and chemical reaction on the velocity, temperature and concentration fields in the thin liquid film on a porous medium. Mathematical modelling is developed under the considerations of heat generation or absorption, thermal radiation and chemical reaction stratification effects. The effects of various emerging parameters on velocity, temperature as well as concentration fields are presented graphically. The local Nusselt number and the local Sherwood numbers are computed and analyzed both numerically and graphically.

2. Governing Equations

The physical model and coordinate system are shown in Fig. 1. Two-dimensional wavy film on a vertical wall is considered. With the usual boundary layer approximations, the gas flow is assumed as laminar and steady.

The two-dimensional laminar continuity equation, momentum equation, energy equation and mass balance equations are the governing equations.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\nu}{K}u+g \tag{2}$$

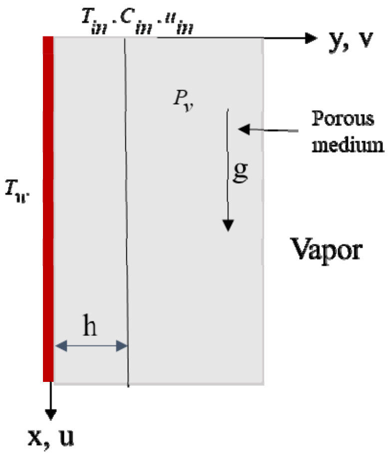

Fig. 1 Physical model and coordinate system.

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho C_p}(T-T_s)-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-Kr'C \tag{4}$$

where u and v are the velocity components along the x and y directions, respectively. ρ is the fluid density, T is the temperature and C is the concentration of the fluid, υ is the kinematic viscosity, h is the thickness of porous médium, K is the permeability of the porous medium, g is the gravitational acceleration, α is the thermal diffusivity, $Q_0(T\text{-}T_S)$ are heat generated or absorbed per unit volume (Q_0 is constant), q_r is the radiation heat flux, D is the mass diffusivity and Kr' is the chemical reaction rate of species concentration.

3. Boundary Conditions

Subject to the following boundary conditions are:

$$u=0,\quad v=0,\quad T=T_w(x) \text{ and } \frac{\partial C}{\partial y} \text{ at } y=0 \tag{5}$$

$$\frac{\partial u}{\partial y}=0,\ T=T_S \text{ and } \quad C=C_S \text{ at } y\rightarrow h \tag{6}$$

where T_w is the wall temperature, T_S and C_S are the surface temperature concentration, respectively.

According to Rossel and approximation [13], the radiation heat flux q_r is given by

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{7}$$

Where σ^* is termed as Stafan-Boltzman constant and k^* is as the mean absorption coefficient, Following Raptis (1998) [13], we assume that the temperature difference within the flow is small such that may be expressed as a linear function of the temperature. Expanding T^4 in a Taylor series about T_0 and neglecting higher order terms, we have:

$$T^4 \cong 4T_0^3 T - 3T_0^4 \tag{8}$$

In this paper, we used the relation between the velocity components as well as the stream functions which are given by:

$$u = \frac{\partial\psi(x,y)}{\partial y},\ v = -\frac{\partial\psi(x,y)}{\partial x} \tag{9}$$

4. Similarity Transforms

The similarity transformations which are given by:

$$\eta = \frac{y}{h},\ \psi = \sqrt{v}\, xf(\eta),\ \theta = \frac{T - T_S}{T_w - T_S}, \phi = C - C_\infty \tag{10}$$

Using the Eqs. (5)-(10), the problems defined in Eqs. (1)-(4) are then transformed into the following set of ordinary differential equations:

$$f'''(\eta) + \gamma\{f(\eta)f''(\eta) - f'^2(\eta) - Daf'(\eta) + Fr\} = 0 \tag{11}$$

$$\theta''(\eta) + \frac{Pr}{1+N_R}[f(\eta)\theta'(\eta) + \Delta\,\theta(\eta) - f'(\eta)\theta(\eta)] = 0 \tag{12}$$

$$\phi''(\eta) + Sc[f(\eta)\phi'(\eta) - f'(\eta)\phi(\eta) - Kr_x(Nc + \phi(\eta))] = 0 \tag{13}$$

with the boundary conditions

$$f(0) = 0,\ f'(0) = 0, \theta(0) = 1,\ \phi'(0) = 0 \tag{14}$$

$$f''(1) = 0,\ \theta(1) = 0,\ \phi(1) = 0 \tag{15}$$

where primes denote differentiation with respect to η, $Da = \upsilon/K$ is the Darcy number, $\gamma = h/\sqrt{\upsilon}$ is the dimensionless film thickness, $Fr = \rho^2 gh^3/\mu^2$ is the Froude number, $\Pr = \upsilon/\alpha$ is the Prandtl number, $R = 16\sigma^* T_0^3 / k^* k$ is the radiation parameter, $\Delta = \frac{Q_0}{\rho C_P}$ is heat generation/absorption coefficients, $Sc = \upsilon/D$ is the Schmidt number, $Kr_x = Kr'/a$ is the local chemical reaction.

5. Flow Parameters

The physical quantities of interest the local Nusselt number Nu_x and the local Sherwood number Sh_x which are given by

$$Nu_x = \frac{1}{h}\theta'(0),\ Sh_x = \frac{1}{h}\phi'(0) \tag{16}$$

6. Results and Discussion

By using the similarity solution technique in MATLAB, the set of ordinary differential Eqs. (11)-(13) with the boundary conditions (14)-(15) are solved numerically. Here the velocity, temperature and concentration are determined as a function of coordinate η. We have adopted a numerical procedure based on MATLAB for getting the solution of the differential Eqs. (11)-(13) with the boundary conditions (14)-(15). The fundamental parameters that governed the flow are the dimensionless film thickness, Froude number, Darcy number, Prandtl number, thermal radiation parameter, heat generation/absorption parameter, Schmidt number and chemical reaction parameter. According to study their effects, a MATLAB programe is written to enumerate and produce the graphs for the velocity, temperature and concentration for different values of these parameters. Few delegate results are given in Figs. 2-9.

Figs. 2(a), (b) and (c) are shown the effect of the Froude number Fr on the velocity, temperature and concentration profiles. From Fig. 2(a), it is observed that in all cases the velocity is started at 0 (zero) and then the velocity increase with the increase of η. After $\eta = 0.5$ again the velocity decrease with the increase in the similarity variable η. Also, the velocity increases with the increase of the Froude number Fr along the similarity variable η. The dimensionless temperature profiles shown as in Fig. 2(b) for different values of Froude number Fr. It is clearly seen that the temperature at any point decreases with the increase in Fr. The concentration increases with increase in Froude number Fr along the similarity variable η which as shown in Fig. 2(c). This is due to fact that influence of the gravitation force enhancing the velocity and concentration as well as reduce the temperature of the fluid.

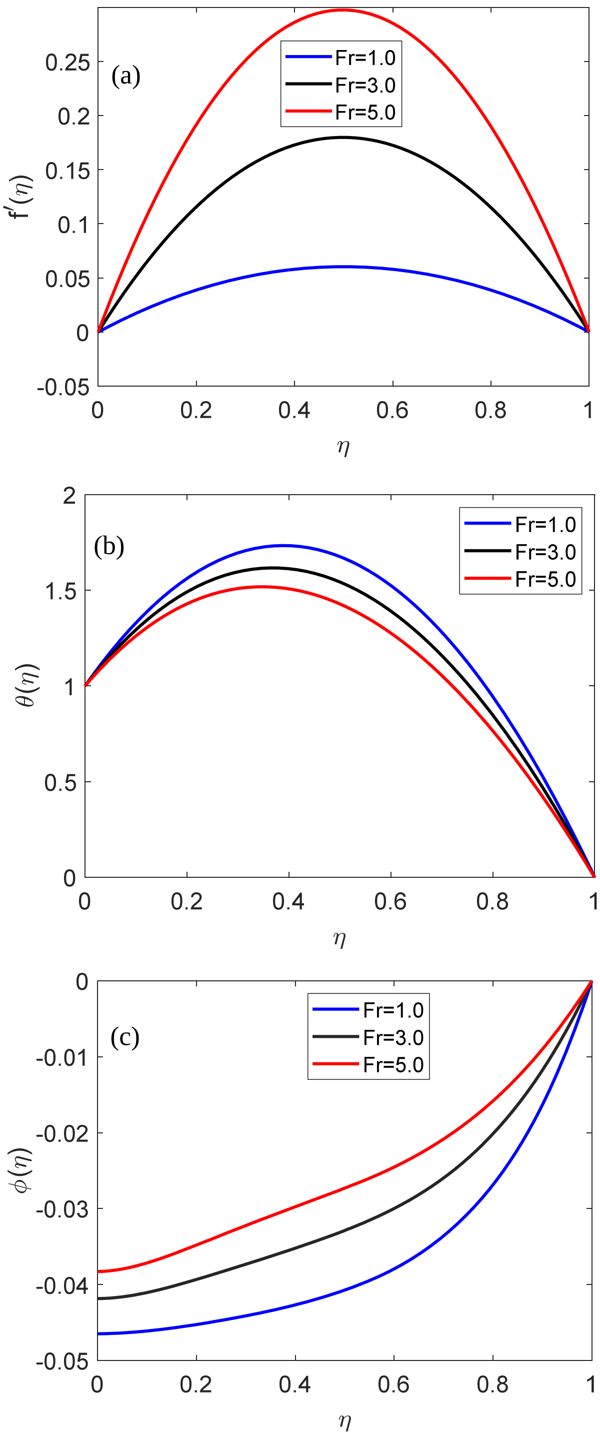

Fig. 2 (a) Velocity, (b) Temperature and (c) Concentration profiles for different values of *Fr* with fixed values of Pr = 10, R = 1, Da = 0.6, Sc = 50, γ = 0.5 and Kr_x = 0.5.

Figs. 3(a), (b) and (c) demonstrates the effect of the dimensionless film thickness γ on the velocity, temperature and concentration profiles, respectively. It is clearly observed from Fig. 3(a) that the fluid increases with the increase in dimensionless film thickness γ along the similarity variable η.

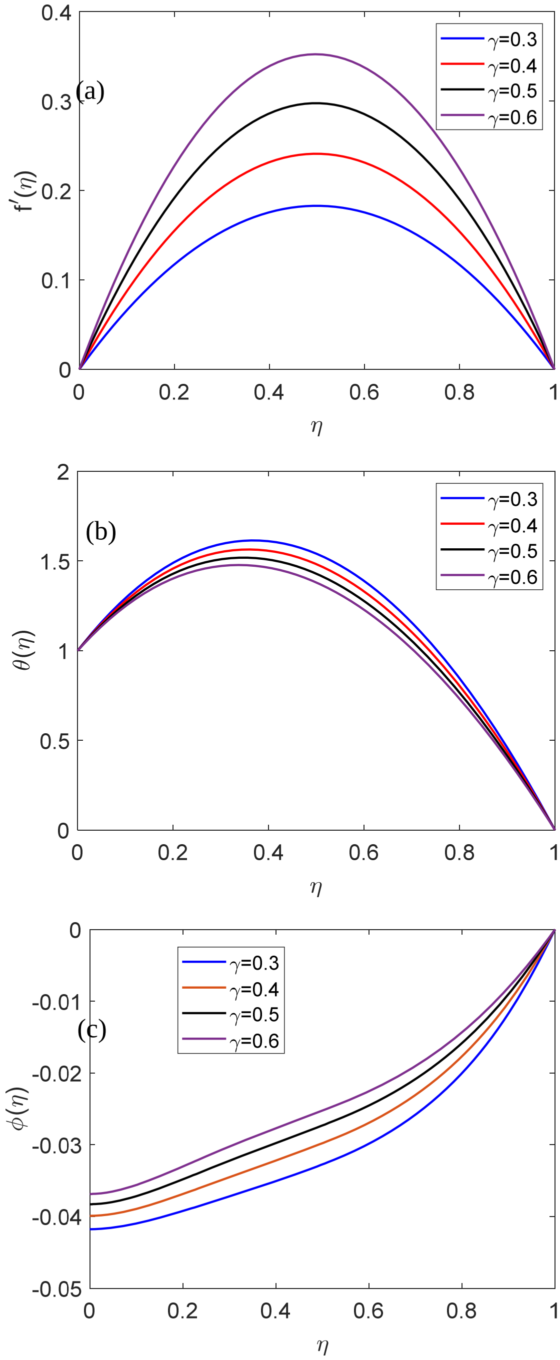

Fig. 3 (a) Velocity, (b) Temperature and (c) Concentration profiles for different values of γ with fixed values of Pr = 10, R = 1, Da = 0.6, Sc = 50, *Fr* = 0.5 and Kr_x = 0.5.

Temperature of the fluid decreases with the increase in γ along the similarity variable η which is shown in Fig. 3(b). Also, the concentration behavior is opposite for increasing dimensionless film thickness γ. With an increase of dimensionless film thickness γ the concentration increases along the similarity variable η.

The influence of the Darcy number Da on the velocity profile is shown in Fig. 4. The results show that the velocity decreases as the Darcy parameter increases. This is because that the porous medium produces a resistive type of force which causes a reduction in the fluid velocity.

Fig. 5 illustrates the effect of the Prandtl number Pr on the temperature profiles. It is observed that the temperature increases with the increase of the Prandtl number. This is due to fact that a fluid with large Prandtl number possesses large heat capacity, and hence augments the heat transfer.

The effect of the radiation parameter R on the dimensionless temperature is shown in Fig. 6. It is observed that with an increase in the radiation parameter the temperature decreases along the similarity variable η. This is because the increase in the radiation parameter implies higher surface heat flux and there-by decreasing the temperature of the fluid.

Fig. 7 represents the effect of heat generation ($\Delta > 0$) and a heat absorption generation ($\Delta < 0$) on the temperature profile. It is clearly observed that with an

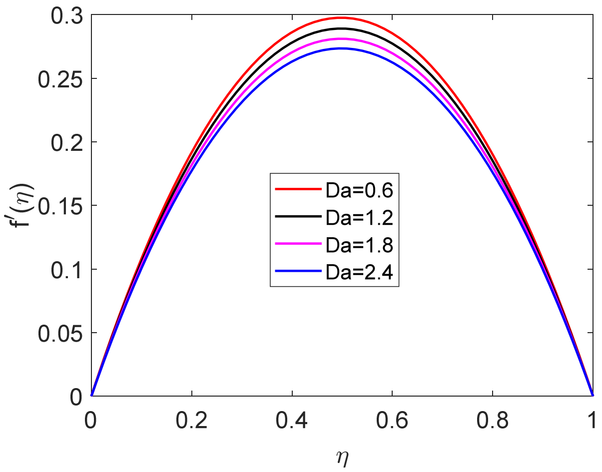

Fig. 4 Velocity profiles for different values of Darcy number *Da* with fixed values of Pr = 10, R = 1, γ = 0.5, Sc = 50, *Fr* = 0.5 and Kr_x = 0.5.

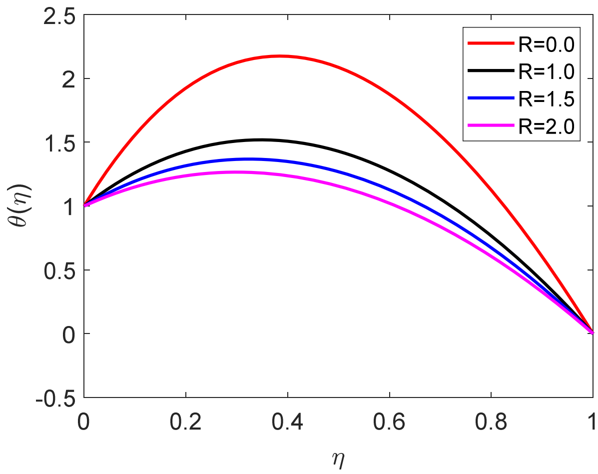

Fig. 6 Temperature profiles for different values of radiation parameter *R* with fixed values of *Da* = 10, *Pr* = 10, γ = 0.5, *Sc* = 50, *Fr* = 0.5 and Kr_x = 0.5.

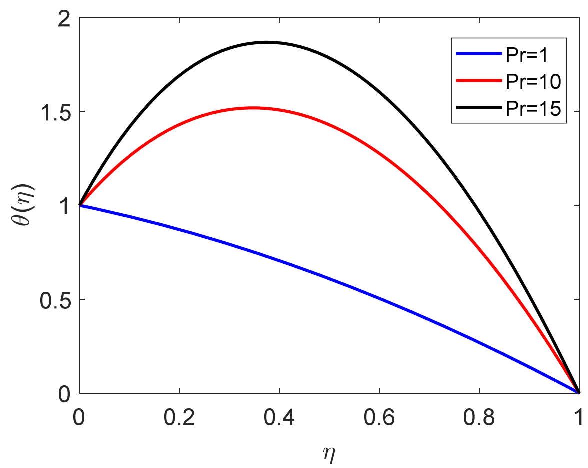

Fig. 5 Temperature profiles for different values of Prandtl number *Pr* with fixed values of *Da* = 10, *R* = 1, γ = 0.5, *Sc* = 50, *Fr* = 0.5 and Kr_x = 0.5.

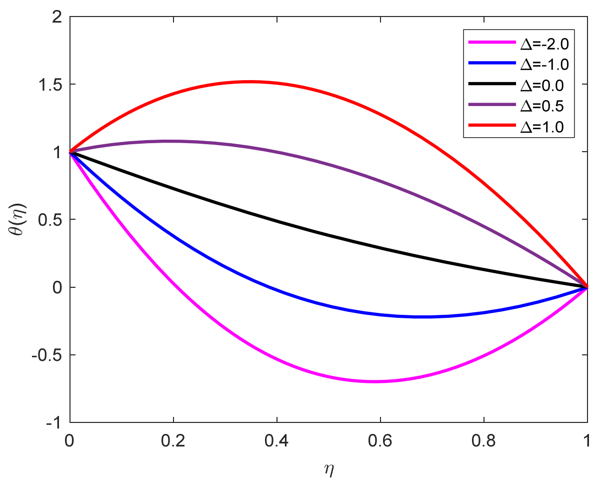

Fig. 7 Temperature profiles for different values of heat generation and absorption with fixed values of *Da* = 10, *R* = 1, γ = 0.5, *Sc* = 50, *Fr* = 0.5 and Kr_x = 0.5.

increase the heat generation the temperature increases. This increase in the fluid temperature causes more induced flow towards the plate through the thermal buoyancy effect. For the case of absorption, the temperature decreases with an increase the absorption.

The variation of the dimensionless concentration against the similarity variable η for various values of the Schmidt number Sc are displayed in Fig. 8. It is seen that the increase of the Schmidt number leads to decrease in the concentration. Schmidt number is inversely proportional to the diffusion coefficient. Hence with an increase in Schmidt number corresponding to a smaller diffusion coefficient. Such smaller diffusion coefficient creates a reduction in the concentration.

The influence of the chemical reaction rate constant Kr_x on the concentration profile within the boundary layer is given in Fig. 9. An increase in the chemical reaction effects increases the concentration within the thermal boundary layer region. This is because increasing the chemical reaction rate causes a thickening of the mass transfer boundary layer.

Fig. 10 is shown the variation of local Nusselt number Nu_x versus thickness of porous medium h for selected values of heat generation and absorption Δ. With an increase the heat generation parameter the local Nusselt number decrease within the boundary along h. Also, increases the absorption the local

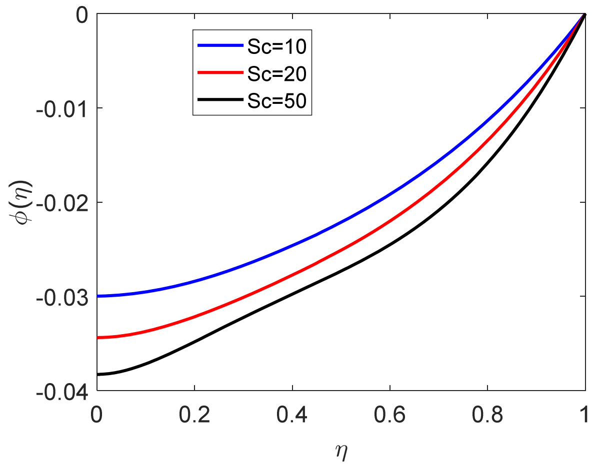

Fig. 8 Concentration profiles for different values of Schmidt number *Sc* with fixed values of Pr = 10, R = 1, Da = 0.6, γ = 0.5, *Fr* = 0.5 and Kr_x = 0.5.

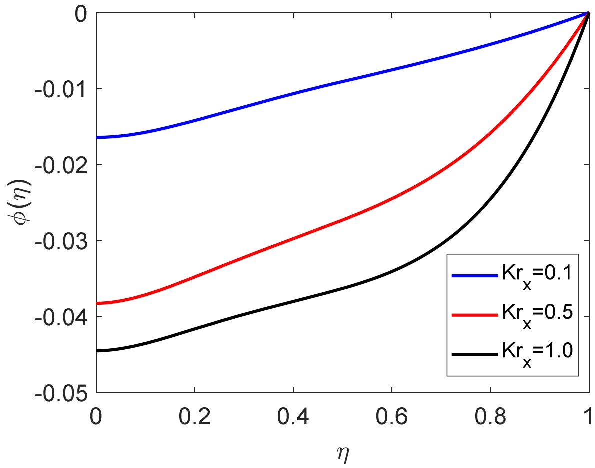

Fig. 9 Concentration profiles for different values of chemical reaction parameter Kr_x with fixed values of Pr = 10, R = 1, Da = 0.6, γ = 0.5, *Fr* = 0.5 and *Sc* =50.

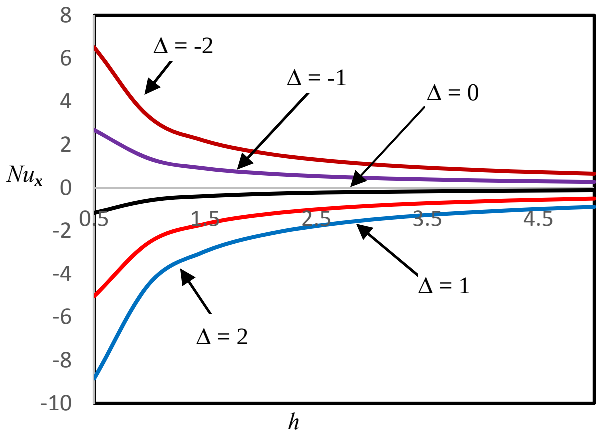

Fig. 10 Variation of local Nussult number Nu_x with *h* for various heat generation and absorption parameter Δ.

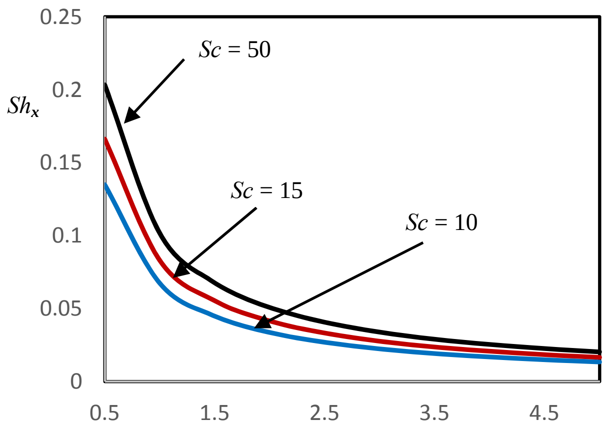

Fig. 11 Variation of local Sherwood number Sh_x with *h* for various Schmidt number *Sc*.

Nusselt number increase along h. The Schmidt number $Sc = \upsilon/D$ indicates the relative extent of the concentration field. The local Sherwood number Sh_x increases with the increase the Schmidt number Sc along h. This is because, for increasing Schmidt number, larger mass flow rate is achieved which is shown in Fig. 11.

7. Conclusions

Influence of triple stratification in the falling film flow on a porous medium with heat generation and absorption, thermal radiation and chemical reaction are examined. The velocity and concentration increase as well as temperature decreases with an increase in Froude number. This is due to the fact that influence of the gravitation force enhancing the velocity and concentration as well as reduce the temperature of the fluid. With an increase the heat generation the temperature increases. This increase in the fluid temperature causes more induced flow towards the plate through the thermal buoyancy effect. For the case of absorption, the temperature decreases with an increase the absorption. The increase in the radiation parameter implies higher surface heat flux and there-by decreasing the temperature of the fluid. An increase in the chemical reaction effects increases the concentration within the thermal boundary layer region. This is because increasing the chemical reaction rate causes a thickening of the mass transfer boundary layer. In addition, for the absorption and Schmidt number increase, larger heat transfer rate and mass flow rate are achieved.

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