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Abductive Inference and Theory of the Inversion of Probability in the Causal Representation of Dynamic Economic Phenomena

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Abstract: In this probabilistic analysis on the treatment of abductive inference in the dynamic economic phenomena it is demonstrated that its foundations coincide with the reasons that gave rise to the "Bernoulli problem" and to the consequent Bayes, Laplace and Poisson demonstrations of the probability theorem of the causes and, therefore, that the analysis of causal relations according to the postulates of the theory of the inversion of probability preceded its epistemological interpretation by two centuries. Likewise, it is demostrated that the presence of exogeneities consisting of rationally unjustifiable dogmatic principles in the field of the theory leads to hypocoded-creative abductions, which imply the negation of the selection rules suggested by Peirce and of the optimization criterion trough maximizing the likelihoods inherent in the theory of causal stochastic models.

Key words: decision under uncertainty, abduction, Bernoulli's theorem, probability theory, causality theory
JEL codes: B41, B23, C18

1. Abductive Inference and Dynamic Phenomena

Abduction is a type of inference directed to the discovery of causal hypotheses developed by Charles Sanders Peirce in the late nineteenth and early twentieth centuries in the field of his logic of scientific discovery. It is defined by a syllogism in which the major premise is true and the minor premise is only possible so the conclusion is uncertain (Peirce, 1878, 1893).

The inference process follows a path that starts from a set of information, passes through what could be called an intermediate state to conclude with the postulation of an initially unknown proposition. Depending on the nature of the set of information taken as a starting point and, above all, on the characteristic of the proposal that expresses the conclusion, it is possible to recognize three types of inference: induction (whose result is a synthesis), deduction (whose result is a thesis) an d abduction (whose result is a hypothesis)[1].

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[1] The first type of inference was the deduction, developed by Aristotle in his "*Analytical Primitives*" at the end of the third century and the beginning of the fourth century (BC), who called it "*syllogism*". The induction arose from the philosophical contributions of Marsilio of Padua, Francis Bacon and William de Ockham and of the experimental and scientific methods of Leonardo da Vinci and

In the field of dynamic economic phenomena induction consists, from the observation of their behavior and through associative reasoning, in the characterization of a presumably constant relationship-defined by an economic theory (S)- between a set of causes ($\{X(t)\}$) and an effect ($\{Y(t)\}$) that produces as a conclusion a probable synthesis ($p\big(Y(t)\ /\ X(\tau)\big),\ t > \tau$).

Deduction begins where induction ends. Its starting point is a (known) rule or law that considers the factors that make up the economic system as variables of mathematical analysis and the causal relationships that link them as functions of mathematical analysis. From this law the conclusions about the behavior of $\{Y(t)\}$ that constitute necessary derivations of the implication relations are obtained. So, assuming that the causal premises are true and the reasoning obeys a correct mechanics, the conclusion is a true thesis.

On the other hand, abduction is an inference method that allows going back towards a set of possible causes ($X(t)$) from an observed effect $Y(t)$ and in this going back it tries discover, from experience, the nexus that links this effect with causes[2], generating a conclusion consisting of a probable causal hypothesis represented by a conditional probability distribution, ($p\big(X(\tau)\ /\ Y(t)\big)$)[3]. This probability is characterized by Peircean "*fallibilism*" which is based on certain metaphysical principles: i) that the deterministic paradigm generates a type of hazard-ignorance inherent in the view that the observer has about the behavior of phenomena; ii) that there is no set of information that determines true behaviors and iii) that phenomena exist in a certain reality and that it experiences a state of continuous evolution (Haak, 1979; Rescher, 1998).

2. The Theory of the Inversion of Probability and the Abductive Stochastic Causality

2.1 Jakob Bernoulli

Although, as already mentioned in the preceding section, Peirce was the one who gave abduction an epistemological status, it should be taken into account that the treatment of the likelihoods of the set of presumably causal hypotheses of a phenomenon according to the postulates of the theory of the inversion of probability, preceded Peirce in two centuries.

It was the pragmatic rationality of partial certainty that forced the rationalist philosophers to employ an analytical scheme of empiricist reasoning: from "*obvious effects*" to "*hidden causes*". Now, given that this method, based on a limited set of observations, did not allow the characterization of the inexplicable nature of causality or metaphysical generalizations, the obtained inferences turned out to be unfailingly affected by different degrees of uncertainty.

In this sense, the fundamental contribution of Jakob Bernoulli — included in the "*Pars quarta: Fradens usum et applicationem procedentis doctrinæ in civilibus, moralibus et œconomicis*" of the "*Ars conjectandi*" — consisted in: i) demonstrating that, from the idea posed by the aforementioned English apologists of natural theology and by Arnauld and Nicole, the logicians of Port Royal, learning from experience was quantifiable through a process of transformation of objective experience into a degree of subjective belief and ii) assuming

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Galileo Galilei, which gave rise to the scientific revolution that led to the work of Isaac Newton. It was Peirce (1878) who linked abduction with the explanation of the behavior of phenomena, giving it an epistemological category capable of providing with its pragmatism a foundation to all heuristic processes and generating a movement whose influence today ranges from economic thought to the philosophy of religion (Houser, 2005a, 2005b, 2006).

[2] Peirce also referred to abduction as "*retroduction*" or "*presumption*".

[3] Burks (1946), "Induction is the method to testing hypotheses, and abduction is the method of discovering them" (p. 301). See also Burks (1943), Génova (1996).

certain hypothesis of simplicity and regularity, try to establish the nexus between the "a priori" or direct inference probabilities (defined from a reasoning that goes from causes to effects, from the hypothesis of symmetry of possible outcomes to the concept of equiprobability) and the "a posteriori" or inverse probabilities (defined from a reasoning that goes from the effects to causes) by creating a new model of causation[4].

Until the appearance of the "*Ars conjectandi*" the advances produced in the theory of probability had not been able to provide an effective response to the formalization of the process of abductive inference. The main treatises of the classical authors -using the reasoning method from the causes to the effects- referred exclusively to the resolution of problems of the type: given an urn that is known to contain a red balls and c blue balls, the probability of obtaining a red ball when performing a random extraction is $\theta = \frac{a}{a+c}$.

Bernoulli (probably influenced by the works of Graunt (1661) and Petty (1682)) was the first to treat the inverse empirical scheme: the asymptotic identification of the *a* and *c* values, based on the evidence provided by the results of the successive extractions and the first to conjecture "*... the relationship between the probabilistic 'conjectandum' and the inductive inference*" (Daston, 1988, p. 228). In addition, he proposed the replacement of the classical (deductive) concept of probability "*a priori*", based on the concept of equiprobability appropriate almost exclusively to solve problems related to gambling, by the idea of probability "*a posteriori*" ("*expectation*"'), defined as a measure of the knowledge that the observer has about the veracity of a proposition[5].

Using modern notation, Bernoulli's theorem can be expressed as follows: Be $Y_n = \frac{X}{n}$ the relative frequency corresponding to the result "red ball", obtained after a succession of n random extractions with replacement of an urn whose composition — unknown to the observer — is a red balls and c blue balls. Then, given a positive and arbitrarily small value ε, and a positive and arbitrarily large t value, it is shown that it is possible to find a $n > n(\theta, \varepsilon, t)$ such that it can be ensured that, with a probability greater to $\frac{t^2-1}{t^2}$, the relative frequency of the result "red ball" will be found at a distance less than or equal to ε of the true value of the proportion $\theta = \frac{a}{a+c}$. So, known n and Y_n, it is possible to solve the equation $n(Y_n, \varepsilon, t) = n$ with respect to t, thus obtaining an approximation to the lower limit $\frac{t^2-1}{t^2}$ (lower bound of the "*residual uncertainty*") corresponding to the probability of occurrence of the event $|Y_n - \theta| \leq \varepsilon$ and, from this expression, determine the probability that the true value of θ is included in an interval of the form $|Y_n - \varepsilon, Y_n + \varepsilon|$[6].

[4] This Bernoullian demonstration of the intuitive principle that uncertainty decreased as the number of observations increased, and the quantification of this process of inductive inference — known as the first (weak) law of large numbers — constituted the first theorem limit of the theory of probability.

[5] Bernoulli (1713): "We have now reached the point where it seems that, to make correct conjecture about any event whatever, is necessary only calculate exactly the number of possible cases, and then to determine how much more likely it is that one case will occur than another. But here at once our main difficulty arises, for this procedure is applicable to only a very few phenomena, indeed almost exclusively to those connected with games of chance. The original inventor of these games designed them so that all the players would have equal prospects of winning fixing the number of cases that would result in gain or loss and lefting them be known beforehand, and also arranging matters so that each case would be equally likely. But this is by no means the situation as regards the great majority of the other phenomena that one governed by the laws of nature or the will man. (…) The results (...) depend on factors that are completely obscure, and which constantly deceive our senses by the endless complexity of their interrelationships, so that it would be quite pointless to attempt to proceed along this road. There is, however, another way that will lead us to what we are looking or an enable us at least to ascertain "a posteriori" what we cannot determine "a priori", that is, to ascertain it form the results observed in numerous similar instances. It must be assumed in this connection that, under similar conditions, the occurrence (o non occurrence) of an event in the future will follow the same pattern as was observed for like events in the past" (p. 226). The page numbers that appear in the references correspond to the English edition of the "Ars Conjectandi" by Sung (1966).

[6] Bernoulli (1713): "This type of prediction requires 'a large number of observations' (…) but though we all recognize this to be case form the very nature of the matter, the scientific proof of this principle is not at all simple (…) Instead there is something more that

The position, more theological than mathematical, of a convinced militant of metaphysical determinism led Bernoulli to identify the ignored causes of the behavior of phenomena with the parameter θ (determined and invariable) and limited the scope of his theorem to the demonstration that, under the (ontological) assumption of the existence of "…*a certain immutable law*", the sample frequency Y_n "…*will converge* (in-probability) *to that law*".

This stance allowed him to propose an extension of this result that implied an inverse proposition according to which, if the relative frequency "... *converges to a certain value*", θ, then this value will define the "*law*" that governs that event. This conjecture is unjustifiable given the insurmountable circularity of this scheme of reasoning in which the convergence in-probability of relative frequencies was verified because the events were governed by a determined law but, in turn, the conviction that the events were governed by a determined law was based on the postulate of inversion of the probability according to which the relative frequencies had to converge to θ.

2.2 Abraham de Moivre

In 1733 de Moivre obtained the Normal approximation to the binomial distribution, according to which:

$$p(|X - n\theta| \leq \varepsilon) \approx \frac{2}{\sqrt{2\pi}} \int_0^{\varepsilon/\sqrt{n\theta(1-\theta)}} e^{-y^2/2}\, dy$$

This demonstration, published under the title of "*Approximatio ad summam terminorum binomii (a+b)^n in seriem expansi*", allowed to reduce the number of observations required to be able to affirm that the quotient $Y_n = \frac{X}{n}$ is contained in a given interval around the true value θ with a certain degree of confidence and to conclude that this degree of confidence increases proportionally to the square root of the number of independent observations made. Achieving effective quantification of the increase in confidence by an increase in empirical information constituted a great advance over Bernoulli's solution and a justification for his implicit model of combinatorial causality[7].

This result and his interpretation that the principle of stability of the frequencies was incontrovertible proof that a superior intelligence governed the behavior of natural phenomena led de Moivre to the conviction that he had demonstrated his own inverse version of Bernoulli's theorem which also could not resolve the circularity of reasoning. However, this proposal constituted a serious argument against the radical skepticism that maintained that regular causes did not necessarily have to produce regular effects. According to his interpretation, not only should regular causes be expected to produce regular long-term effects, but the observation of the effects should allow — asymptotically — to discover the causes under the ontological assumption that such causes existed.

De Moivre could also have shown that the apparent convergence of the relative frequencies was compatible with (and still caused by) the randomness of the observations, but obviously this possibility did not fit into his deterministic conception, which simply considered that, according to the principle of uniformity of nature the

must be taken into consideration (…) What is still to be investigated is whether by increasing the number of observations we thereby also keep increasing the probability that the recorded proportion of favourable to unfavourable instances will approach the true ratio, so that the probability will finally exceed any desired degree of certainty, or whether the problem has, as it were, an asymptote. This would imply that there exists a particular degree of certainty that the true ratio has been found which can never be exceeded by any increase n the number of observations" (p. 225).

[7] $\sqrt{n}$ constitutes ".. *the 'modulus' by which we are to regulate our estimation*" (De Moivre, 1733, p. 240). The page numbers that appear in the references correspond to the second edition of "*The doctrine of chances*" by Cass (1967).

series "*should*" converge (assuming like Bernoulli that, according to the principle of nature simplicity, the estimation of the "*true*" quotient resulting from convergence consisted in the adoption of the quotient "*simplest*" compatible with the finite set of observations)[8].

As in the Bernoulli proposal, the most important restriction of the de Moivre result is that the convergence of the relative frequency is justified only in the limit. That is to say that neither de Moivre achieved to solve the problem of the identification of the probability from a finite succession of observations (a solution in terms of probable inference not considered by de Moivre nor by Bernoulli)[9].

It can be concluded that, beyond the indisputable importance of their contributions to the development of probability theory, neither Bernoulli nor de Moivre managed to solve the problem of the inversion of probability due to they failed to define the link between the past observations and the probabilities of occurrence of future events, in other words, pass from the probability of direct inference $p(Y_n/\theta)$ — considered by classic writers- to inverse probability $p(\theta/Y_n)$.

Its failure was fundamentally due to the impossibility, in the context of its deterministic interpretation, of considering θ as a random variable. It should be borne in mind that both Bernoulli and de Moivre -like most scientists of the time- in their fidelity to Newtonian "*theology*"[10], saw in their limit theorems the argument that demonstrated the presence of *"Divine Providence"* in the stability of the statistical coefficients. In this philosophical framework θ could only be interpreted as a constant (of unknown value) and relative frequency as a random variable.

2.3 Thomas Bayes

The first rigorous attempt to solve the problem of the inversion of probability is due to Thomas Bayes ("An essay towards solving a problem in the doctrine of chance" (1764)) who, contrary to Bernoulli's proposal, considered θ as a continuous random variable with a known "a priori" probability distribution, which allows the characterization of the properties and the definition of the probability distribution of the conditioned variable (θ/Y_n), from a finite set of observations made under equal conditions and assuming "*a priori*" of the performanceof any trial that the results of the event are symmetric, obtaining the following definition:

$$p[(\theta_1 < \theta < \theta_2)/Y_n] = \frac{\int_{\theta_1}^{\theta_2} \binom{n}{n'} \theta^{n'}(1-\theta)^{n-n'} dF(\theta)}{\int_0^1 \binom{n}{n'} \theta^{n'}(1-\theta)^{n-n'} dF(\theta)}$$

The Bayesian axiomatic gave rise to certain adverse judgments reffering fundamentally to its confused and indefinite conceptual position regarding the notion of probability due to the use of the concept of rational subjectivity in an objectivist context. In this regard, it is necessary to point out that for the probabilists of the seventeenth and eighteenth centuries there were no rigid definitions of probability, but different methods of inference of their value whose characteristics depended on the context in which they were to be used. In the origins of the theory of probability, the contrast between the objectivist and subjectivist interpretations was less

[8] De Moivre (1733): "(...) and thus in all causes it will be found that althochance produces Irregularities, still the odds will be infinitely great, that in process of Time, those irregularities will bear no proportion to the recurrence of that Order which Naturally results from Original Design" (p. 252). According to Poisson an "Order" interpretable only in terms of expectation.

[9] De Moivre also did not indicate a practical method for obtaining a confidence interval for θ as a function of the values of Y_n, ε and t. On the other hand, it should be borne in mind that his attempts to univocally determine the value of θ from a mathematical argument such as that of condensation points in a finite succession of observations ended in failure.

[10] Pearson, K. (1925): "Post-Newtonian English mathematicians experienced a greater influence of Newtonian theology than of their mathematics" (p. 202).

profound than in the philosophy of the time[11].

The so-called doctrine of the association of ideas — which, from the linking of psychology and epistemology, tried to explain the psychological processes underlying rational behavior- provided the conceptual arguments that made possible the transitions between objectivist and subjectivist interpretations. The foundations of this principle of "*philosophy cum psychology*" of science, which undoubtedly influenced the thought of Bayes, were established by Locke (1689) who associated the qualitative and quantitative interpretations of objective evidence and linked them to the interpretation subjectivist of probability (an interpretation almost exclusively philosophical, not quantitative) based on degrees of belief, generating in this way a relationship of the type *experience = belief*. Thus, the greater the frequency of the correlation observed, the stronger the corresponding mental association would be and, therefore, the more intense the degree of belief, the greater the probability and, consequently, the reliability of the abductive generalizations[12].

The principle of total indifference of Hume (1718), according to which from an operational characterization of an event whose results were symmetrical it was possible to justify the assumption of "a priori" equiprobability, led Bayes to conjecture a uniform distribution of the variable θ $(dF(\theta) = d\theta)$ and to demonstrate the following equality referred to the behavior of the absolute frequency of those results:

$$p(X_n = n') = \int_0^1 \binom{n}{n'} \theta^{n'}(1-\theta)^{n-n'} d\theta = \frac{1}{n+1} (\forall n')$$

The independence of this result from n' was considered by Bayes as the justification of the "*a priori*" hypothesis of the uniform distribution of θ (expression known as the "*Bayes postulate*").

Now then, for this operationalization to overcome its condition of intuitively acceptable simple conjecture a rigorous demonstration is required that the uniform distribution of θ is not only a necessary condition but also sufficient for the fulfillment of Bayes' postulate.

As a corollary of de Finetti's representation theorem (1937) (Landro González, 2016), doing $n = n'$ in the previous expression and given a distribution function $F(\theta)$, the following definition of the moment of order *n* of the density function $dF(\theta)$ is obtained:

$$\int_0^1 \theta^n dF(\theta) = \frac{1}{n+1}$$

This allows us to conclude that Bayes postulate univocally determines the succession of infinite moments of $dF(\theta)$. On the other hand, since the function $dF(\theta)$ is concentrated in a compact set, according to the Hausdorff theorem (1914), it can be assured that it is strictly defined by the sequence of its moments and, according to Murray's (1930) theorem, it is shown that the only density function that satisfies the sequence of moments that Bayes postulate prescribes must be such that $dF(\theta) = d\theta$. That is, the Bayes postulate is verified if and only if the variable θ, conditioned by the binomial distribution of the variable X_n, is distributed uniformly[13]. So, it can

[11] Daston (1988): "Philosophers still puzzle over how probability can mean both a degree of certainty and a number of observed instances, but Christiaan Huygens, Gotfried Wilhelm Leibniz, and other seventeenth century probabilists identified the two without hesitation or justification" (p. 191).

[12] In other words, confirming the conjecture about the relation between reasonableness and probability theory of the Port Royal logicians and the English apologists of the Royal Society, the associationist psychology made the mind a type of machinery capable of automatically measuring frequencies of past events and calculate, accordingly, degrees of belief about their future recurrence.

[13] In the presentation of the "Essay" Price interprets the Bayes proposal postulating that "...in the constitution of things there are

be concluded that Bayes' theorem failed to avoid the metaphysical assumptions contained in its foundations and, consequently, to obtain a general solution to the problem of the identification of the density function of θ.

2.4 Pierre Simon Laplace

Subsequently Laplace ("*Mémoire sur les probabilité des causes par les événements*" (1774)) attempted formally to treat the methodological intuitions of naturalist philosophers by defining a set of mathematical rules aimed at discussing the objections of the skeptics of induction. He differed from de Moivre and Price in that he did not use the probability of causes as the fundament of the "*Original Design*" principle. He agreed with de Moivre and Price that the "*natural order*" was stable, but not that "*stable causes*" should produce "*stable effects*".

Laplace considered a nature composed of "*regular causes*" and "*irregular causes*", postulating that the latter observed a "*regular*" joint behavior whose long-term symmetrical effects were nullified. The result was a new model of causation "*... in which it was possible to conceive a world in which the macroscopic order was produced by a microscopic chaos*" (Daston, 1988, p. 267).

Within the scope of this new model Laplace postulated that every problem in the field of "*chance theory*" belonged to one of the following two classes: i) the former in which the result of the phenomenon being analyzed was eventual but the cause that conditioned the assignment of probabilities on its occurrence was known ("*direct*" or inductive probability) and ii) the one in which the result of the phenomenon was known, but its cause was unknown ("*indirect*" or "*inverse*" or abductive probability) and it devoted its attention exclusively to the study of the phenomena of the second class.

Laplace's fundamental principle about probability is summarized in the following paragraph: "If an event can be produced by a number n of different causes, then the probability of these causes, given the event, are with respect to each of the other causes, as the probabilities of the event given those causes, and the probability of existence of each of these is equal to the probability of the event, given that cause, divided by the sum of the probabilities of the event, given each one of the causes." (pp. 384-385).

Representing by E_1 the occurrence of an event when performing a first trial, by E_2 the occurrence of the same event when performing a second trial, by $\{h_i; i = 1,2, \dots n\}$ the set of all causes (mutually exclusive) that condition the assignment of probabilities on the occurrence of E_1 and E_2, by E the occurrence of the event in any of the individual trials and assuming that events E_1 and E_2 are conditionally independent with respect to each cause h_i, Laplace concluded that:

$$p(E_2/E_1) = \sum_i p(E/h_i)p(h_i/E)$$

From the additional (sujective) assumption of equality of probabilities for all causes, $p(h_i) = \frac{1}{n}$ $(i = 1,2,\dots,n)$, the following system of $n-1$ linear equations in $p(h_i/E)$ remains defined:

$$\sum_i [p(E/h_i)]^k p(h_i/E) = \frac{\sum_i [p(E/h_i)]^{k+1}}{\sum_i p(E/h_i)} (k = 1,2, \dots, n-1)$$

fixed laws that govern the occurrence of events and that, therefore, the frame of the world must be the effect of the wisdom and power of an intelligent cause and, consequently, it allows to confirm the argument about the existence of the Deity from final causes (...).The inverse problem solved in this essay is directly applicable to this purpose, it demonstrates clearly and precisely for any order of recurrence of events, that there are reasons to suppose that such order or recurrence derives from causes or stable regulations of nature and not from any irregularities of chance" (p. 297).

to which must be added the equation $\sum_i p(h_i/E) = 1$. The solution of this system coincides with the postulates of Bayes' theorem (Josang, 2008).

In turn, Laplace (1781, 1812) considered the case of assignment of different probabilities to the different causes: "The probability of most simple events is unknown and 'a priori' seem equally likely to assume any value between 0 and 1; but it is from observing the results of several such events that some of these values become more probable than the others" (1781, p. 228).

It should be noted that thefundamental difference between Bayes and Laplace solutions lies in the fact that, while the Bayes' objective consisted in the estimation of a probability, the (dynamic) proposal of Laplace was aimed at predicting the behavior of a phenomenon.

2.5 Siméon Denis Poisson

Finally, it was Poisson ("Recherches sur la probabilité des jugements en matière criminelle et en matière civile, précédées des règles générales du calcul des probabilités" (1837)) — along with Cournot (1843), Ellis (1849), Venn (1866), the principle of association of ideas of Locke and the reaction of British empiricism against the continental rationalism of Laplace — who proposed an interpretation attempt that reconciled the theory of probability with the principles of Fechner's indeterminism.

In the chapter "*Sur las probabilités des résultats moyens des observations*" of the "*Recherches*" Poisson showed that the probability θ can be approximated by the relative frequency according to the following version of Bernoulli's theorem:

$$p\left(|\theta - Y_n| \leq \frac{z}{n}\sqrt{\frac{2n_E(n-n_E)}{n}}\right) = 1 - \frac{2}{\pi}\int_z^\infty e^{-v^2}dv + e^{-z^2}\sqrt{\frac{n}{2\pi n_E(n-n_E)}}$$

(where n_E denotes the absolute frequency of the result in question) and in the chapter "*Calcul des probabilités qui dépendant de très grands nombres*" proposed the first generalization of the law of large numbers for binomial addends not-identically distributed according to which, given the random variable:

$$Y_n = \frac{X_{(n)}}{n} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

(where the $X_i (i = 1,2, \ldots, n)$ denote variables of type $b(1, \theta_i)$, in which θ_i represents the probability that an event E will occur in the *i*-th repetition due to the cause C_i), it is shown that:

$$\lim_{n\to\infty} p\left(\left|\frac{1}{n}\sum_{i=1}^{n} X_i - \frac{1}{n}\sum_{i=1}^{n}\theta_i\right| \leq \varepsilon\right) = 1$$

Poisson — like Bernoulli and de Moivre- could not conceive of a universe "*governed*" by chance. But, unlike them — which assumed the existence of a "*Providential Order*", Poisson maintained that the tendency of phenomena to exhibit regularities was inherent "*... to the natural state of things, which subsist by themselves, without the help of any strange cause and, on the contrary, would require such a cause to experience a significant change*" (pp. 144-145).

Like Bernoulli, Leibniz, de Moivre, Price and Condorcet, Poisson considered that the eventual non-verification of the principle of stability of frequencies, did not mean a refutation of the "*principle of permanence of causes*" that governed nature, but the recognition of that some of these causes could have been replaced by others, causing the probabilities of occurrence of the event to vary.

3. The Probability of Causes in the Representation of Dynamic Economic Phenomena

Returning to the phenomena belonging to the field of factual sciences, let it be a dynamic economic phenomenon $Y(t,\omega)\ (t \in \mathbb{R})$ that admits a linear representation assimilable to a strictly stationary stochastic process and whose configuration varies in the continuous domain of the states, $\omega(t) \in \Omega(t,Y)\ (t \in \mathbb{R})$. In this domain each state $\omega(t)$ is supposed to be defined by the simultaneous realization at time *t* of the infinite strictly stationary stochastic processes that form its causal structure whose formal expression is given by the infinite countable set:

$$\Omega(\tau,Y) = \{Y(\tau), X_1(\tau_1), X_2(\tau_2), \dots\} \quad (t,\tau,\tau_i \in \mathbb{R}, \tau < t, \tau_i < t, i = 1,2,\dots)$$

and such that the temporal succession of states forms its necessary trajectory. So, the phenomenon $Y(t,\omega)$ is defined as a system that evolves in a space-time domain and that is supposed to be characterized by a chain of presumed causes and effects of the form:

$$\left(X_i(\tau_i) \Rightarrow Y(t)\right) \quad (t,\tau_i \in \mathbb{R}, \tau_i < t, i = 1,2,\dots)$$

representative of the principle of universal solidarity that relates to phenomena.

These hypotheses assumed as a starting point correspond to the following axiomatic system that defines the theoretical field in which the phenomenon $\{Y(t,\omega)\}$ evolves:

1) $\{Y(t,\omega)\}\ (t \in \mathbb{R})$ behaves according to a deterministic paradigm, which implies that its nature obeys certain metaphysical premises: i) that the field to which the phenomena belong is real; ii) that there are objective laws that rule their behavior and iii) that these laws are inherent to phenomena, rational and asymptotically cognoscible.

2) The representation of the behavior of the phenomenon is given by a (non-stochastic) function $f[\Omega^*(t,Y)]$ defined by a system formed by a finite set of presumed causal variables suggested by an economic theory assumed as a starting point:

$$\Omega^*(\tau,Y) = \{Y(\tau), X_1(\tau_1), X_2(\tau_2), \dots, X_k(\tau_k)\} \subset \Omega(\tau,Y)$$

$(t,\tau,\tau_i \in \mathbb{R}, \tau < t, \tau_i < t, i = 1,2,\dots,k; Y(t), X_1(t), \dots, X_k(t) \in \mathbb{R})$ and for which the observer has empirical information.

3) Since $f[\Omega^*(t,Y)]$ constitutes an inevitably insufficient representation, under conditions of strict stationarity, it follows that:

$$Y(t) = f[\Omega^*(\tau,Y)] + \left(\varepsilon(t) \,/\, \Omega^*(\tau,Y)\right)$$

where: i) $f[\Omega^*(\tau,Y)]$ denotes the expected behavior of $\{Y(t)\}$:

$$f[\Omega^*(\tau,Y)] = E[Y(t)/\Omega^*(s_1,Y), \Omega^*(s_2,Y), \dots] \quad (t, s_i \in \mathbb{R}; i = 1,2,\dots; t > s_1 > s_2, \dots)$$

assuming that the factors included in $\Omega^*(\tau,Y)$ are its only presumed causes and $f[\cdot]$ is the function that represents (according to the framework of an economic theory *S*) the true causal relationship, invariant in time, between these factors and $Y(t)$ and **ii)** $\{\varepsilon(t) \,/\, \Omega^*(\tau,Y)\}$: WN, formed by unobservable random "*shocks*"[14], denotes the stochastic process representative of the "*innovations*" that affect $\{Y(t)\}$ which defines the influence exerted on $\{Y(t)\}$ by the infinite factors of its causal structure not included in $\Omega^*(\tau,Y)$:

$$\left(\varepsilon(t) \,/\, \Omega^*(\tau,Y)\right) = Y(t) - f[\Omega^*(\tau,Y)]$$

4) The deterministic paradigm implies that:

[14] According to the nomenclature of Wold (1938).

$$\lim_{\Omega^*(\tau,Y)\to\Omega(\tau,Y)} f[\Omega^*(\tau,Y)] = Y(t)$$

and, therefore, that:

$$\lim_{\Omega^*(\tau,Y)\to\Omega(\tau,Y)} \big(\varepsilon(t) \,/\, \Omega^*(\tau,Y)\big) = 0$$

According to the considerations presented in the previous sections and from the assumption that the process $\{Y(t,\omega)\}\}$ behaves according to a deterministic paradigm, the objective of the theory of the inversion of probability consists of constructing an order of importance related to the explanatory capacity of the causal hypotheses -included in the set of hypotheses $\{X_1(t), X_2(t), \dots, X_k(t)\}$ postulated by a structure of economic thought (*S*) assumed as a starting point- using a selection criterion by maximization of likelihoods.

4. Creative Abduction As A Negation of the Theory of the Inversion of Probability

From his pragmatist position, Peirce postulates that the only admissible causal hypotheses are those capable of generating empirical or practical consequences and manifests "*... a refusal to admit unnatural or supernatural sources in any descriptive or explanatory discourse that has to do with the truth*" (Margolis, 2002, p. 6).

According to these premises and with an operational purpose, Peirce (although fragmentarily) proposed some rules for the application of abductive reasoning[15]. "*What is a good abduction? How should a causal hypothesis be to deserve the category of hypothesis? Obviously, an abduction must explain the facts. But what other conditions must it meet to be a good abduction? Any hypothesis may be admissible in the absence of any special reason to the contrary if it can be proven that it admits an experimental verification and according to the extent to which it admits such verification. This is approximately the doctrine of pragmatism*" (CP 2786).

On the other hand, Peirce postulates that, according to the probability inversion theorems analyzed in the Section 2, it would seem better "*... to treat the hypothesis suggested by an experiment whose results approximate as much as possible the equiprobability*" (CP 2786)[16] and, given that "*Facts can never be better explained than by the same facts, from the various alternative hypotheses the least extraordinary must be adopted*" (CP 692) (Sebeok, 1981, p. 31), that is, the one that best satisfies the optimization criteria associated with the statistical methods inherent to the assumption of stability of causal relationships.

In the scope of this pragmatic naturalism Peirceano, Eco (1990) recognizes three types of abduction: the "*hypercoded*", the "*hypocoded*" and the "*creative*"[17].

The characteristics of the first two lie in: i) that the set of possible causal hypotheses obeys the dictates of an theoric structure of thought *(S)* and ii) that the provisional adoption of a causal hypothesis is based on the set of information $\Omega^*(t,Y)$ that has the observer about the behavior of the system $\{Y(t), X_1(t), X_2(t), \dots, X_k(t)\}$.

The fundamental difference is that, while hypercoded abduction is associated with an axiomatic that is supposed to represent the true behavior of the system ("*the laws of the possible as necessary*" (Riesz de Rivarola, 1989, p. 112), in the hypocoded one, the process of defining the causal link is based on a hypothesis that, although plausible, does not admit a full theoretical justification and consequently leads to the consideration of a presumed causal structure associated with the system, estimated from statistical methods according to a given optimization

[15] Rules of "*evaluative abduction*", according to Magnani (1998).

[16] The "*principle of economics in research*" according to Peirce or the "*principle of interpretative economics*" according to Davidson (1985, p. 349). See also Davidson (1986).

[17] Or "*inventive*" according to the Bonfantini (2006) denomination. See also Bonfantini Proni (1980).

criterion ("*the laws of the possible according to the absolute likelihood*" (p. 112)).

Creative abduction coincides with the hypocoded one in that it leads to the creation of a causal structure, but it is such that the law that justifies the formulation of a hypothesis obeys imaginary considerations that are not rationally justifiable and are associated with behaviors that, to the extent that they are based on metaphysical principles and beliefs of a dogmatic nature, are sujective or convergent to the intersubjectivity of a given collective ("*the laws of the possible according to the relative likelihood*" (p. 122)).

Now then in many cases the presence of exogeneities consisting of axiomatic principles frequently leads to hypocoded-creative abductions which imply the negation of the hypothesis selection rules suggested by Peirce. A circumstance justifiable in the literary universe of Eco because of its relationship with certain inherent precepts in the very essence of the aesthetic approach of thrillers, in which a great part of the interest of the investigative process is not in the clarification, but in the mysteries that it generates.

This stance which evidently contradicts the postulates of the theory of stochastic causality, based on the optimization criterion by maximizing the likelihoods, is also presented in the representations of dynamic systems in the field of factic sciences due to the influence of dogmatic concepts generated by the personal beliefs of the observer on the behavior of the causal system $\{Y(t), X_1(t), X_2(t), \ldots, X_k(t)\}$.

5. Conclusions

As an epilogue to this probabilistic analysis on inferential methods and the treatment of abductive inference in dynamic economic phenomena, it is shown, from the postulates of the probability inversion theorems, that:

1) since the decisions that result from the abductive method are assumed under conditions of uncertainty, the selection of causes is associated with the likelihood of the set of presumed hypotheses, which reveals the stochastic nature of causality;
2) the foundations of abductive inference coincide with the reasons that gave rise to the "Bernoulli problem" and the subsequent Bayes, Laplace and Poisson demonstrations of theorem of the probability of the causes;
3) the treatment of causal relationships according to the postulates of the theory of inversion of the probability preceded in two centuries to the recognition by Peirce work of an epistemological "status" to abduction;
4) the presence of exogeneities consisting of axiomatic principles based on rationally unjustifiable dogmatic precepts generated by the personal belief of the observer in the ambit of the theory, frequently leads to hypocoded-creative abductions that imply the denial of the results of Bernoulli's theorem and its generalizations and of the optimization criterion trough maximizing the likelihoods.

References

Bayes Th. (1764). "An essay toward solving a problem in the doctrine of chances", *Philosophical Transactions of the Royal Society*, Vol. 53, pp. 370-418, reprinted in Barnard (1958) and in Pearson Kendall (1970).

Bernoulli, J. (1713). "Ars conjectandi", in: *Die Werke von Jakob Bernoulli*, Basel Naturforschende. Gsellschaft, 1969-75. Translation to Englishby Sung, B., Harvard University Departmen of Statistics, Technical Report n° 2, 1966.

Bonfantini M. (1987). *La semiosi e l'abduzione*, Bonpiani.

Bonfantini M. (2006). *L'inventiva. Psómega vent'annidopo*, Moretti & Vitali.

Bonfantini M. and Proni G. (1980). "To guess or not to guess?", *Scienze Umane*, No. 6, pp. 249-260.

Burks A.W. (1943). "Peirce's conception of logic as a normative science", *The Philosophical Review*, Vol. 52, pp. 187-193.

Burks A. W. (1946). "Peirce's theory of abduction", *Philosophy of Science*, Vol. 13, No. 4, pp. 301-306.

Cournot A. A. (1843). *Exposition de la théorie des chances et des probabilités*, Hachette.

Daston L. (1988). *Classical Probability in the Enlightment*, Princeton University Press.

Davidson D. (1985). "Incoherence and irrationality", *Dialectica*, Vol. 39, pp. 345-354.

Davidson D. (1986). *A Nice Derangement of Epitaphs*, Grandy Warner (Eds.), reprinted in Le Pore (ed.).

De Moivre A. (1733). *The Doctrine of Chances*, Woodfall, reprinted by Cass, 1967.

Eco U. (1990). *I limiti dell' interpretazione*, Bonpiani.

Ellis R. L. (1849). "On the foundations of the theory of probabilities", *Transactions of the Cambridge Philosophical Society*, Vol. 8, pp. 1-6.

Génova G. (1996). "Tres modos de inferencia", *Anuario Filosófico*, Vol. XXIX, available online at: https://www.worldcat.org/issn/0066-5215.

Graunt J. (1661). "*Natural and Political Observations Mentioned in a Following Index and Made Upon the Bills of Mortality*", London.

Haak S. (1979). "Fallibilism and necessity", *Synthèse*, Vol. 41, pp. 37-63.

Houser N. (2005a). "The scent of truth", *Semiotica*, Vol. 153, pp. 455-466.

Houser N. (2005b): "Peirce in the 21th century", *Transactions of the Charles Peirce Society*, Vol. 41, pp. 729-739.

Houser N. (2006). *Peirce's Contrite Fallibilism*, Fabbrichesi Leo Marietti (Eds.).

Hume D. (1718). *An Inquiry Concerning Human Understanding*, reprinted by Handel (1955).

Josang A. (2008). "Conditional reasoningwith subjective logic", *Journal of Multiple Valued Logic and Soft Computing*, Vol. 15, pp. 5-38.

Landro A. H. and González M. L. (2016). "Acerca del problema de Bernoulli y la determinación del verdadero valor de una probabilidad", *Ediciones Cooperativas*, available online at: http://bibliotecadigital.econ.uba.ar/download/libros/Landro-Gonzalez_Acerca-del-problema-de-Bernoulli-2016.pdf.

Laplace, P. S. (1774). "Mémoire sur la probabilité des causes par les événements", *Mémoires de l'Académie Royal des Sciences*, Vol. 6, pp. 621-656.

Laplace P. S. (1781). "Mémoire sur les probabilités", *Mémoires de l'Académie Royal des Sciences*, Vol. 12, pp. 227-232.

Laplace P. S. (1812): "*Théorie analytique des probabilités*", Courcier.

Locke J. (1689). *An Essay Concerning Human Understanding*, London. Reprinted by A. Campbell Fraser, 1959.

Magnani L. (1998). "Abduction and hypothesis withdrawal in science", in: *20th World Congress of Philosophy*, available online at: https://www.bu.edu/wcp/Papers/Scie/ScieMagn.htm.

Margolis J. (2002): *Reinventing Pragmatism: American Philosophy at the End of the Twentieth Century*, Cornell University Press.

Pearson K. (1925). "James Bernoulli theorem", *Biometrika*, Vol. 17, pp. 201-210.

Peirce Ch. S. (1878). "Deducción, inducción e hipótesis", available online at: https://www.unav.es/gep/DeducInducHipotesis.html.

Peirce Ch. S. (1893). *Collected Papers*, Harvard University Press.

Petty W. (1682). *An Essay Concerning the Multiplication of Mankind*, London.

Poisson S. D. (1837): *Recherches sur la probabilité des jugements en matière criminelle et en matière civile, précédées des règles générales du calcul des probabilités*, Bachelier.

Rescher N. (1998). "Fallibilism", in: Craig (Ed.).

Riesz de Rivarola S. (1989). "Ficcionalidad, referencia, tipos de ficción literaria", in: *Teoría y análisisdel texto literario*, Hachette.

Sebeok, J. (1981). *The Play Musement*, Bloomington.

Venn, J. (1866). *The Logic of Chance*, MacMillan.

Wold, H. (1938). *A Study in the Analysis of Stationary Time Series*, Routledge-Kegan.