Journal of Modern Education Review, ISSN 2155-7993, USA
November 2021, Volume 11, No. 11, pp. 1195–1202
Doi: 10.15341/jmer(2155-7993)/11.11.2021/005

http://www.academicstar.us

Mathematical Understanding of Learners Based on Constructivist Theory Undergraduate Partial Derivatives

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Abstract: The objective of the study was to investigate the mathematical understanding of learners based on Constructivist Theory undergraduate partial derivatives. The study employed qualitative research and targeted students enrolled in Calculus 2. The research instrument used in collecting the data was lesson plans of partial derivatives as adapted from Dubinsky & McDonald (2001), as well as field notes, semi-structured interview forms, and reflections on learning. The results of the study revealed Understanding of learners while solving the following classroom can be searched by ignoring the process of acquisition for the partial demonstration that the student understands. The action is that the students can confirm their understanding. They understand the procedures and solve the problem of understanding. The action on the idea of making calculations as many times until you understand predict the outcome of the making process and can create new batches of feed. It was shown that the learners had an understanding of the process level. In group discussion issues about the functions learned in Calculus 1, the concepts of function matter were connected to make it easier for the level of understanding concerning the process and related objects, together with creating a structure of knowledge (Schema).

Key words: mathematical understanding, constructivist theory, partial derivatives

1. Introduction

Mathematics understanding plays an important role in the development of human thinking, enabling human beings to be creative and think rationally, systematically, and orderly, thus be able to analyze problems. Further, the situation makes it possible to forecast, plan, and make decisions as well as solve problems accurately and appropriately. Mathematics is a tool for the study of science and technology as well as other related sciences. Mathematics also helps to develop human beings to be perfect human beings with balance physically, mentally, intellectually, and emotionally, meaning they are able to think, act, solve problems and live happily with others (Ministry of Education, 2015). However, a problem in mathematics teaching and learning is unsuccessful learning. Low academic achievement occurs if the teacher does not improve the teaching and learning to keep up with changes; students do not understand and do not want to learn what the teacher presents. The problem will be even more complex at every level as existing knowledge is very important to learning mathematics. This is because most of the learning and teaching of mathematics relies on exercises in textbooks. This is characterized by the fact that only one correct answer does not allow students the different abilities needed to participate. From the analysis

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of Thai mathematics textbooks, it was found that most consisted of exercises used to practice skills, especially computational skills, and review previously learned rules or principles (Inprasitha, 1997). Mathematics learning and teaching is therefore not teaching that focuses only on computational and mathematical skills. The steps to follow rely on memorization but no understanding.

Students must learn mathematics with understanding and be able to create new knowledge from experiences and existing knowledge. Learning math with understanding is essential to helping learners succeed in solving problems they have never encountered before. It is adapting prior knowledge, which is an internal structure that existed before, to create meaning (NCTM, 2000) that corresponds to the mathematical concepts that must be learned. The results that will occur for learners when learners have a mathematical understanding are as follows. Understanding enables new knowledge to be created. When learners have an understanding of the material, they are able to create and organize their ideas of mathematical knowledge on their own instead of acquiring that knowledge from the teacher or the book; understanding promotes memory. When students understand the content, they will be able to remember the principles, formulas, and theorems of the content. Understanding reduces the number of things to remember. When learners understand, they will be able to create a whole body of knowledge or content without having to remember. Partial content makes it less necessary for learners to remember. Increasing the ability to link knowledge associations is necessary for solving math problems, and knowledge associations occur frequently in mathematics because solving new problems requires the structure of knowledge as previously learned (Hiebert & Carpenter, 1992, pp. 74–77) understanding influences belief. When learners learn math with understanding, they build confidence in math as well as a positive attitude towards math. Therefore, math comprehension is essential for learning activities so students can achieve. Mathematics comprehension gives learners more competence and durability than other teaching methods (Sheffield & Cruikshank, 2005). It should allow learners to relate mathematical concepts, improve reasoning and interaction skills in class. Management of learning to understand will help learners apply their knowledge effectively. Because comprehension is the basis of deep thinking, it affects the understanding of concepts.

Mathematics is a subject that deals with problem solving processes in which the problem that the learner brings to practice thinking should be a problem that must be sought for the truth. It is a new conclusion that students have not learned before and an academic problem such as rational proof. Mathematics content is based on definitions, theorems, and problems that require mathematical processes to solve mathematical problems, thus requiring knowledge and understanding of the steps of the problem solving process. The Constructivist Theory proposed the main idea that learning comes from experience (Baruque & Melo, 2004, p. 346) rather than acquiring knowledge. The goal of learning management is to support knowledge creation rather than knowledge transfer. The emphasis is on creating new knowledge appropriately for each individual. The environment is important in creating the meaning of reality. Learning emphasises that learners take action to create knowledge. Individuals learn by creating knowledge in different ways based on prior experience. Bell (1993) mentioned the nature of knowledge-building learning that learners own an idea rather than being a receiver or receiving information. Teacher communication is the nature of encouraging students to think. Without telling or answering questions directly to learners, learners must learn how to interpret what the teacher says, which can be used to be used to answer questions that students want learners to learn by understanding what the learner understands. The learners create understanding themselves, not copy the ideas of the teachers.

Researchers such as calculus instructors surveyed the mathematics content, finding that the learning unit of partial derivatives is what learners find the most difficult because there are no tools to solve problems successfully or improve their learning style. Corresponding to courses in various disciplines must provide learner-centred instruction based on the belief that everyone can learn and develop to their full potential, which is consistent with learning management according to the concept of the constructivist theory and the principles of mathematical understanding.

2. Research Objective

To investigate the mathematical understanding of learners based on the Constructivist Theory undergraduate partial derivatives: Case study of Rajamangala University of Technology Suvarnabhumi Suphanburi campus.

3. Conceptual Framework

A study of mathematical understanding of learners was done according to the constructivist theory about the sub-differentials of undergraduate students. A case study of Rajamangala University of Technology Suvarnabhumi Suphanburi Campus was carried out, which consisted of lesson plans and questionnaires, including the role of an instructor, alearner’s role, and the atmosphere of the classroom by the order of the learning management process. There are 6 steps as follows:

1) Step for the introduction to the lesson. It is a step that prepares students and encourages them to recall the subject specific experiences that form the basis for building new intellectual structures.
2) Step for the process of presenting problems to students is related to the lesson in accordance with daily life, appropriate age and ability.
3) Step for the exploration stage is where students observe, consider, collect, analyze, and search for knowledge that will be used to solve problems from the media prepared by the teacher. It could be an individual survey or a small group.
4) Step for problem solving is the stage where students present their ideas to the mathematics classroom with discussion. Ask questions about the guidelines of the proposed group. Verify accuracy and reasonableness, wherein the teacher guides the students who have not yet presented, then collects the correct and reasonable ideas. In addition, the teacher also observes the mathematical understanding level while solving problem. According to the concept of Dubinsky & McDonald (2001), who proposed the APOS Theory, mathematical understanding is classified into 3 levels: Action level, Process level and Object level. The process level and Object level are related to each other for building a cognitive structure (Schema).
5) Step for the conclusion of the new intellectual structure. This is the stage where students join together to summarize concepts about the subject. The teacher will assist in further conclusions if the learners see that the summary does not cover the content or the concept is not correct.
6) Step for skills training and implementation is the stage where learners can integrate their experiences and mathematical concepts to create meaningful steps that can be applied to different situations. The teacher plays a role as a motivator to provide an appropriate environment for learners. Teachers are responsible for finding students’ opinions in order to help learners to understand. The role of the learner is to take the initiative, take action, and think as well as describe things that have already been done to others, leading to a change in thinking with each other. That is, learners can build their own body of knowledge and the teacher assesses the learning outcomes of the learners from the learning achievement

test.

4. Research Scope

1) The target groups used in this research were students in the Faculty of Engineering and Architecture at Rajamangala University of Technology, Suvarnabhumi Suphanburi Campus. Students were registered for Calculus 2, Semester 2, in academic years between 2017 and 2020.
2) Mathematics content used in this research is Calculus 2 Credits 3 (3-0-6)
   Subject learning unit partial derivatives.

5. Research Methodology

This research was conducted using qualitative data at the stage of problem-solving. Students presented their own ideas and concepts of small groups to the class to study the level of mathematical comprehension while solving problems based on the concept of Dubinsky & McDonald (2001) proposed APOS Theory.

6. Target Group

The target group comprised students in the Faculty of Engineering and Architecture at Rajamangala University of Technology, Suvarnabhumi Suphanburi Centre Calculus 2 Course Registration 2nd Semester academic years 2517–2521 by means of selected purposive sampling.

7. Research Tools

1) The lesson plan has content according to TQF 3 Calculus 2 units of learning with partial function of the three lesson plans including:
   a) Partial derivatives, chain rule
   b) Partial derivative of an implicit function
   c) Application of partial derivatives for two variable and multivariate functions.
2) Each learning management plan uses learning management based on the constructivist theory.
3) Field notes from the researchers created using the participant observation to collect data on the behavior of the participants during the event in the process of the plunger to learn six steps.
4) Asemi-structured interview form was used to confirm the level of mathematical understanding while solving problems.
5) Reflection on learning, so that the researcher can use it to improve learning management the next time. In the case of reflecting a point of view that is a disadvantage, a solution must be proposed. The merits that should be retained in practice and developed in future learning arrangements should be recorded.

8. Data Collection

The researcher conducted data collection using the steps as follows.

1) Collection of data from classroom laboratory research. The researcher experimented with the target group. Time spent in Calculus 2 course hours (401-12-07) for learners enrolled in Calculus 2 courses by implementing a learning management model based on the constructivist theory that promotes

mathematical understanding. On the subject of the partial derivatives of learners, there were 3 lesson plans.

2) Data collection from the field record form was done by Participant Observation in 6 steps of learning management. The researcher collected the students' problem-solving methods together with video recording.
3) Collecting Information from Unstructured Interviews helped to verify the level of mathematics understanding while solving problems.
4) Data collection from the learning management reflection form was used so that the researcher could use it to improve learning management the next time. If reflected in the view that was a disadvantage, a solution was proposed. The merits that should be retained in practice and developed in future learning arrangements were recorded.

9. Data Analysis

Qualitative data analysis uses the information obtained from the students' writing work, recording forms, unstructured interviews, learning management outcome reflection forms to analyze the students' level of mathematics understanding. Derivatives that arise at the stage of solving problems, according to the theoretical framework of Dubinsky & McDonald (2001), are presented in an analytical narrative format and verify the reliability of the data by the triangular data validation method from comparing data from field notes with interviews.

10. Research Results

The results of the study of mathematical understanding among students were assessed according to the concept of the constructivist theory on the sub-differentials of undergraduate students. A case study of Rajamangala University of Technology Suvarnabhumi Suphanburi Campus was carried out as follows:

1) Steps to introduce into the lesson involved instructors talking to bring into the lesson. To prepare students to motivate learners, recall previous experiences only on fundamental subjects, i.e., functions and derivatives of functions. In the course of Calculus 1, the creation of new intellectual structures was possible.

2) The process of presenting problems to learners was related to the lesson and in accordance with daily life appropriate to the age, ability and tools that students learned previously.

Examples of presenting problems to students can be given by:

$$f(x, y) = e^{xy} \cos x \sin y \tag{1}$$

3) The survey step is the stage where learners observe, consider, collect, analyze, and seek knowledge that will be used to solve problems, first by individual surveys and then in small groups.

4) For problem solving steps, students present their own ideas and small group ideas to the class with discussion. Ask questions about the guidelines of the proposed group. Verify accuracy and reasonableness, wherein the teacher guides the students who have not yet presented, and then collects the correct and reasonable ideas. The teacher observes the level of understanding of the learners while solving the problems according to the concept of Dubinsky & McDonald (2001), who proposed the APOS Theory as follows: learners can search for

answers ignoring the process of obtaining $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$. It shows that the learners have an understanding of the Action level, that is, the learners are able to confirm their understanding that they understand how to find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$. This corresponds to the information obtained from the interviews with the target learners, which is “a function has two variables, so the subdivision must be compared with x and y”; learners can solve problems. Develop an action-level understanding of multiple computations until you can use your knowledge to predict the outcome of an action and create new processes of action. The action method does not necessarily show the action in each step. This demonstrates that learners understand the process level, that is, how students solve mathematical problems, as follows:

(1) Finding $\frac{\partial f}{\partial x}$ is y a constant and $\frac{\partial f}{\partial y}$ is x a constant.

(2) Use the definition given $z = f(x, y)$ is a function of two variables and (x_0, y_0) is an internal point of the domain of f.

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{h \to 0} \frac{f(x_0 + h, y_0) - f(x_0, y_0)}{h} \tag{2}$$

$$\frac{\partial f}{\partial y}(x_0, y_0) = \lim_{k \to 0} \frac{f(x_0, y_0 + k) - f(x_0, y_0)}{k} \tag{3}$$

Learners cannot say that the limit is worth it. Based on the discussion of the class and the role of the teacher in the discussion together to expand the concept of the definition of the derivative that the value of the limit can be classified as understanding at the object level (Object), learners can present the issue of using derivative formulas in calculations to link concepts to solve problems. It was found that learners had difficulty using differential formulas,

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx} \tag{4}$$

The instructor provides an in-group discussion of the functions learned in Calculus 1. Some students try to link connect the concept of function to make it easier to calculate by understanding that the process-level and the object-level are related to each other. A schema is formed by having students share a summary of what they have learned and connecting it to find the partial derivative.

5) For the conclusion of the new intellectual structure, instructors take the mathematical concepts that arise in the classroom and arrange them from simple concepts to complex ones. The mathematical concepts are then brought to a conclusion together. The teacher will help to summarize more if the learner sees that the summary does not cover the content or does not address the right concept as follows. Consideration of dividing involves two functions, namely exponential function and trigonometric function.

6) Skill training and directing learners to integrate experiences and mathematical concepts that create meaningful steps to be applied to different situations by having learners perform teacher-created skills exercises where the instructor plays a motivating role. Promote and provide an appropriate environment for learners. Instructors are responsible for discovering the mathematical concepts of students to lead learners to understand the role of the learner. The learner must take the initiative to solve problems, including thinking and describing things that have already been done for others to know, leading to a change in thinking.

11. Discussion and Conclusions

The investigation of learners' mathematics understanding according to the theory framework of Dubinsky & McDonald (2001) found that the target groups had different levels of mathematics understanding, namely Schema Action and Object Process, because learners had learning tools. Learning from Calculus 1 and Mathematics Problems allow all learners to engage in their own potential to express mathematical concepts with their own understanding, resulting in different concepts. Learners learn from the ideas of classroom members.

The conceptual discussion is in line with Yackel & Hanna (2003, pp. 22–44), Who Found that comprehension can be developed by encouraging student interaction in the classroom and by allowing learners to propose ideas or mathematical deductions. From teaching and learning based on the concept of the constructivist theory, mathematics classes have changed from the teacher not being the one who conveys the content to the learners. However, the instructor acts as a coach to help students so that they can learn using their own understanding. Comprehension learning is the basis for the effective application of knowledge, in line with [NCTM] (2000),who said that encouraging students to learn math with understanding is essential for math classes, allowing learners to apply their knowledge effectively. In addition, the concept of the constructivist theory has confirmed that all knowledge can be created by a person. Understanding by interacting with the environment and exchanging ideas with others allows learners to interpret them for themselves (Nodding, 1990; Gadanidis, 1994; Goldin, 1996). It gives mathematics classrooms the perspective for considering the knowledge that students have. Learn to create and interpret by yourself.

References

Baruque L. B. and Melo R. N. (2004). "Learning theory and instructional design using learning objects", *Journal of Educational Multimedia and Hypermedia*, Vol. 13, No. 4, pp. 343–370.

Bell A. (1993). "Some experiments in diagnostic teaching", *Educational Studies in Mathematics*, Vol. 24, pp. 115–137.

Carpenter P. and Lehrer R. (1999). "Teaching and learning mathematics with understanding", in: *Mathematics Classrooms That Promote Understanding*, London: Lawrence Erlbaum Associates, pp. 19–32.

Dubinsky E. and McDonald M. (2001). "APOS: A Constructivist theory of learning", in D. Holton (Ed), *The Teaching and Learning of Mathematics at University Level*: *An ICMI Study*. pp. 275-282. Dordrecht: Kluwer Academic.

Hiebert J. and Carpenter P. (1992). "Learning and teaching with understanding", in *Handbook of Research on Mathematics Teaching and Learning*. New York: Macmillan.

Inprasitha M. (1997). "Problem solving: A basis to reform mathematics instruction", *Journal of the National Research Council of Thailand*, Vol. 29, No. 2, July-December 1997, Khon Kaen. Khon Kaen University.

Gadanidis G. (1994). *Deconstructing Constructivism Mathematics Teacher*, Vol. 87, No. 2, pp. 91–96.

Goldin G. A. (1996). "Theory of mathematics education: The contributions of constructivism", In L.P. Steffe (Ed.). *Theories of Mathematical Learning* (pp. 303-306). Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Ministry of Education. (2015). *TIMSS 2015 Project Research Report*. Bangkok, Institute for the Promotion of Teaching Science and Technology. July29, 2021, available online at: http://timssthailand.ipst.ac.th/timss/reports/timss2015report.

National Council of Teachers of Mathematics (NCTM) (2000). *Principles and Standards for School Mathematics*, Reston, Virginia: National Council of Teachers of Mathematics.

Noddings N. (1990). "Constructivism in mathematics education", in: R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist Views on the Teaching and Learning of Mathematics*, Reston, VA: National Council of Teachers of Mathematics.

Sheffield L. J. and Cruikshank D. E. (2005). *Teaching and Learning Mathematics Pre-Kindergarten Through Middle School* (5th ed.), New York: John Wiley & Sons.

Yackel E. and Hanna G. (2003). "Reasoning and proof", in: J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to Principles and Standards for School Mathematics*, National Council of Teachers of Mathematics, pp. 22–44.