

## Learning to Understand the Notion of Limit of a Function

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**Abstract:** The following educational experience is related to the axis of solving and creating problems, it was carried out with students of the second cycle of the professional school of administration, who in their learning have difficulties when understanding the notion of limit of a real function, that is for this reason, our objective was to contribute to improving the understanding and learning of this mathematical object by using a didactic sequence designed based on the Theory of Didactic Situations, since it is developed situations of action, formulation, validation and institutionalization that the students managed to develop mostly. The results obtained are satisfactory, as the majority of students manage to understand the notion of limit and it is easier for them later to understand the formal definition of limit.

**Key words:** didactic situation, limit, function

### 1. Introduction

The students of the arts majors at the university often have difficulties with learning mathematics in the different general or specialty courses, and consequently their academic performance is affected and in many cases they have to repeat the subjects, due to experience own in the teaching of the Higher Mathematics course, that in its contents the limit mathematical object of a function is inserted, it is necessary for students to begin by understanding what a limit is in an intuitive and practical way, which contributes to the students understanding and internalize the concept of the limit of a real function in an intuitive way making use of situations and contents that are very familiar to them and then introduce them into the formal definition, because in this way they initially better understand the meaning of this object and then introduce it into the language more formal.

### 2. Purpose, Design and Implementation of the Proposal

#### 2.1 Purpose of the Proposal

Improve the understanding and understanding of the notion of the limit of a function, in Administration students, young people who generally do not like mathematics, but nevertheless have to carry these mathematical objects that are essential in the development of other subjects and specialty topics.

#### 2.2 Design of the Proposal

This proposal has been worked on with 28 students from the Management School of the Universidad Peruana Los Andes from the second cycle of studies.

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For the design of the sequence of activities, Guy Brousseau's Theory of Didactic Situations has been taken into account.

The topic of the limit of a function is part of the Higher Mathematics course of the second cycle and for the application of the design, 8 groups of 3 students and a group of 4 have been formed so that they can discuss their results among themselves and be able to issue their conclusions. regarding the situations presented.

Before working on the application of the proposal, they are reminded of notions of real numbers and functions, contents learned in the first cycle.

## 2.3 Sequence of Proposed Situations to Understand the Notion of Limit of a Function

### 2.3.1 Situation 1

Given the function  $f(x) = \frac{2}{x}$ , complete the following table and respond:

1) Determine the domain of the function

2) In the following table, the behavior of the images of the proposed function will be observed. To do this, values are given to the variable  $x$ , as shown below:

| $x$         | $f(x)$ |
|-------------|--------|
| 1           |        |
| 2           |        |
| 10          |        |
| 100         |        |
| 1000        |        |
| 10000       |        |
| 100000      |        |
| 1000000     |        |
| 10000000    |        |
| 100000000   |        |
| 1000000000  |        |
| 10000000000 |        |

- What do you observe regarding the proposed values of the variable  $x$ ?
- With respect to the previous question, following this trend, to what number do the values of the variable  $x$  approximate, as indicated by the arrow?
- Find the images of the function and complete the table. What happens to the values of the variable  $f(x)$ ? What value do they approach?
- Taking into account the situation in c), at some point or under certain circumstances, this value may be zero.
- Construct a graph with the results obtained in a, b and c.
- Express in your own words what was obtained in the graph constructed in d).
- What would happen if instead of giving large positive values, large negative values are given? Make a table like at the beginning and write down the behavior of the images.
- Analyze the table you have obtained and write a conclusion about it in your own words.
- With the data obtained so far, build a graph adhering to the one made in step e).
- Observe the obtained graph and analyze, it can cut to the Y axis, explain your answer.

- k) How would you analyze the behavior of the function taking into account the answer obtained in j)?
- l) Prepare a table as at the beginning to be able to carry out the respective analysis. Keep in mind that you are going to work values that are very close to the number zero on both the right and left sides.
- m) Observe and express in your own words the conclusion of the analysis that you have carried out with respect to the data obtained in the previous step.
- n) Build a graph attached to the one built in step i)

### 2.3.2 Situation 2

$$f(x) = \frac{x^2 - 36}{x - 6}$$

Given the function

- 1) Determine the domain of the function
- 2) Taking into account the domain of this function:
  - a) From the values that we have as the domain of the function, what happens to the images of the function, if the values of the variable  $x$  are close to six? Build a chart for this. Note that the values can be approximated from either the left or the right side.

| $x$ | $f(x)$ |
|-----|--------|
|     |        |
|     |        |
|     |        |
|     |        |
|     |        |
|     |        |

- b) Graph the function and observe its behavior for values close to  $x = 6$ .
- c) Values for  $x$  such as 1, 2, 3, 8, 9, 10 can be considered in the analysis. What happens if I consider these values?
- d) Express in your own words what was obtained in the analysis carried out in a) and b).

### 2.3.3 Situation 3

$$f(x) = \frac{x-1}{2}$$

In the following function

- 1) Determine the domain of the function
- 2) What happens to the values of the function, to what value do they approach, if the values of the variable  $x$  approach the number 1? Build a table to record your observations
  - a) Graph the function and observe its behavior around the value  $x = 1$
  - b) Express in words what was obtained in conclusion of the analysis carried out

### 2.3.4 Situation 4

From the table, answer:

| $x$    | $f(x)$     |
|--------|------------|
| 1.90   | 8.90       |
| 1.93   | 8.9201     |
| 1.96   | 8.99201    |
| 1.99   | 8.999201   |
| 1.999  | 8.9999201  |
| 1.9999 | 8.99999201 |



| A      | B       |
|--------|---------|
| 2.0001 | 9.00008 |
| 2.001  | 9.0008  |
| 2.01   | 9.008   |
| 2.1    | 9.08    |

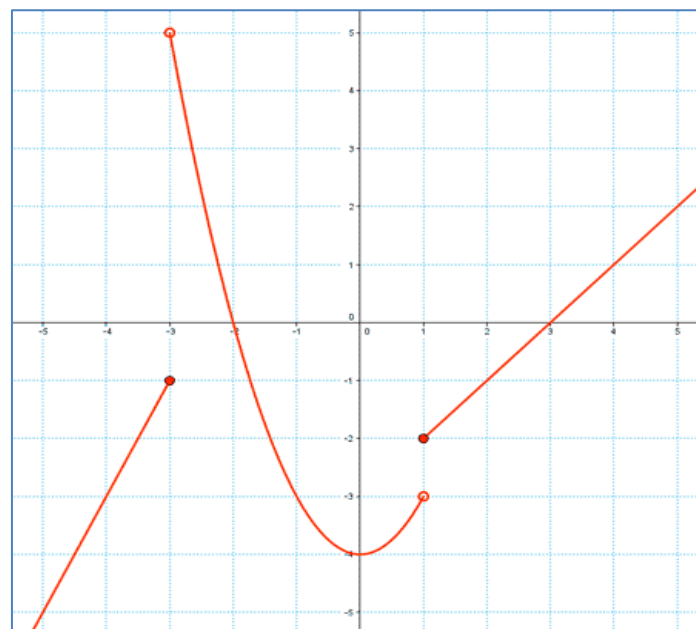
a) If the values of the variable are approximated to the right of A, what is the value of A? and if they approach from the left, what is the value of A?

b) What number do the values of the function  $f(x)$  approach, what value would B have?

c) Write a conclusion about the behavior of the function as it approaches A.

#### 2.3.5 Situation 5

The graph of a piecewise function is shown below; in which we are going to analyze the behavior of the images of the function taking into account the following:



a) If the values of  $x$  approach the number -3, what value do your images approach? (Take into account the side from which the values of the variable  $x$  are approximated so that you can give an answer.)

b) If the values of the variable  $x$  approach the number 1, your images approach what value? (Take into account the side from which the values of the variable  $x$  are approximated so that you can give an answer.)

### 3. Results Obtained

The understanding of the mathematical limit object of a function is improved, since they are students of the area of letters who have difficulty in learning mathematics, because by showing them this sequence of activities they can intuitively understand the meaning of a function's limit then to the formal definition in which terms such as epsilon and delta will be used.

#### 4. Final Thoughts

From the answers obtained after applying the sequence of didactic situations, mention was made of the limits that they had found in a practical way, for example in the case of situation 1 it was mentioned that they found, and this was mentioned in the other situations.

This proposal may vary taking into account the level of mastery of the students' learning in relation to the topics of real numbers and functions, since in order to carry out the respective analyzes it is necessary to have previous knowledge.

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