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Personalized Learning Model of Trigonometric Functions in Secondary Education

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Abstract: This research aimed to optimize the teaching-learning process of mathematics to a personalized learning model and the use of technological tools in fifth grade secondary school students on the topic of trigonometric functions, aimed at achieving efficient learning. The theoretical framework that supports the study was pedagogical constructivism and the analysis of the fundamental concepts about trigonometric functions at the elementary level. The pedagogical experience was carried out during the classes of the topic of trigonometric functions in the pedagogical research-action mode, with interactive participation of the students and the teacher using mathematical software. The field work begins with an evaluation of requirements for the study of trigonometric functions, followed by process evaluations and concludes with an exit evaluation, the results of which are presented in tables and analyzed using descriptive statistics. Being empirically verified that the implemented didactic strategy resulted in the achievement of significant learning of trigonometric concepts and procedures of the students, fosters motivation and the development of positive attitudes for the learning of mathematics in general and of trigonometric functions in particular. This is corroborated by the level of satisfaction shown by the participants.

Key words: pedagogical constructivism, personalized learning, trigonometric functions, significant learning

1. Introduction

Research in mathematics education aims to understand the nature of mathematical thinking, teaching and learning in order to use such understandings to improve the instruction of mathematics as a set of ideas, knowledge, processes, attitudes and, in general, of activities involved in the construction, representation, transmission and assessment of mathematical knowledge that takes place on an intentional basis (Rico, Sierra & Castro, 2000).

The learning of Mathematics occurs mainly through the confrontation with examples, rather than through formal and technical definitions (in fact, they affirm, it is through the examples that the definitions make some sense, since the words mathematical techniques describe classes of objects or relationships with which the learner must become familiar). Through examples that satisfy certain restrictions, you can encourage students to extend their thinking beyond the "typical" examples. Great strength is seen in its effectiveness as a teaching strategy when students are faced with a new definition. In addition, they propose groups of tasks that require students to

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generate examples with given combinations of properties (Watson & Mason, 2005).

One of the most important topics but at the same time that it brings with it difficulties for the assimilation of the concepts and properties is the one referred to the transcendent functions, of these, the trigonometric functions being the most important at the basic level. In the educational reality where the study is carried out, there are shortcomings in the actors of the educational process, both in the mathematics teacher and in the student.

Students show serious deficiencies in: understanding the symbolic language of mathematics, differentiating a function from a relationship, identifying the domain and range of a function, identifying symmetry relationships between points on the Cartesian plane, drawing the graph of a real function, identifying odd and even functions, periodic and monotonous, etc.

Most teachers begin the study of trigonometric functions, in algorithmic form, as the ratio between the sides of a right triangle, with little analysis of their properties; They do not use updated texts of the intermediate and superior level to reinforce their knowledge; They do not make use of prior knowledge to address the subject, nor do they promote active learning strategies, in the task setting stage they promote collaborative learning; they also use ICTs sporadically to stimulate learning.

For this reason, the present work is focused on developing a didactic proposal oriented to personalize learning of trigonometric functions. This strategy takes into account that students learn in different ways and at different rates. Each student has a “learning plan” based on how they learn, what they know, and what their abilities and interests are of the students as subjects of the learning process. They collaborate with their teachers to set both short-term and long-term goals, and are in charge of their learning.

2. Guiding Theories of the Study

2.1 Pedagogical Constructivism

According to constructivism, development occurs articulated according to the factors of maturation, experience, transmission and balance, within a process in which biological maturation is followed by the immediate experience of the individual who, being linked to a socio-cultural context, incorporates new knowledge based on previous assumptions (social transmission), true learning occurs when the individual manages to transform and diversify the initial stimuli, thus balancing himself internally, with each cognitive alteration (Piaget, 1992). For constructivist interpretation of the teaching and learning processes, it can be located on a continuum that places the construction of knowledge in the individual subject, between the subject and the context, between the individual and the social (Bruning, Schraw & Ronning, 2002).

The characteristics of constructivist teaching are based on the precept that human learning is always the product of an interior mental construction, whether one is the first or the last to understand the new knowledge (Flórez, 1994), defines them in four fundamental actions student-centered: part of the student’s previous ideas and schemes, foresees the conceptual change and its repercussion on the mental structure, from the active construction of new concepts; confronts ideas and preconceptions related to the concept being taught; and applies the new concept to concrete situations and relates it to previous ones in order to expand its transfer.

2.2 Personalized Learning

In a personalized learning model, the student is not just someone with characteristics to take into account and with learning needs that must be satisfied; Rather, it is above all someone with a voice and recognized and accepted capacity to participate, based on their characteristics, aspirations and interests. Within the approaches of the pedagogical proposals that place learning and the apprentice at the center of educational action, which recognizes the protagonism of the student in the learning process and that favor and promote this protagonism as a fundamental element to achieve deep school learning and significant (Coll, 2017). Thus, personalized learning is directly linked to the pedagogical tradition centered on the learner and the constructivist proposals of education. Where the school curriculum, teacher training, the organization and operation of the educational institution, and the use of digital information and communication technologies converge.

The personalized learning process fosters active, reflective, theoretical and pragmatic learning; and, contemplates four models: those that use the student profile, which maintains an updated record of the skills, needs, motivations and goals of each student that help teachers understand their students, they also help them make decisions to have a positive impact on student learning; Those that use personalized learning routes, where there are high expectations of each student, look for how each student manages to satisfy their expectations, here students can choose how they learn and have multiple options to complete a task with the permanent supervision of the teacher; those who consider progress to be aptitude-based, continually assess students to monitor their progress toward specific goals, allow the student to advance and receive credit by demonstrating ability to do what they set out to do, and developing various skills at the same time weather; those that use flexible learning places, adapts the place where students learn based on how they learn best, includes things like the physical conditions of the classroom, how teachers are assigned and how the school day is structured (Morín, w/d).

The personalization of learning is fundamental in current education, being a strategic need for the development of teaching activities at different educational levels, where we find the explanation of the prominence given to learning activities by students, constituting a proposal of innovation and improvement of mathematics education. The personalization of learning is understood as the manifestation of pedagogical approaches that adjust to the teaching and learning activities and the teaching action to the characteristics, needs and interests of the student, the same that takes force in the learning of mathematics.

2.3 Trigonometric Functions

Trigonometry is a branch of mathematics, whose etymological meaning is “measurement or treatise of triangles”. It is derived from the Greek terms τριγωνος trigōnos “triangle” and μετρον metron “measure”. Trigonometric functions are located within the transcendent functions, whose understanding by the student, causes difficulties due to their abstract nature. Being special functions whose values are extensions of the concept of trigonometric ratio in a right triangle drawn in a circle (of unit radius). Where the cosine and sine functions are defined, and from them it is possible to define the four remaining functions, without resorting to artifices on the right triangle.

Trigonometric functions are of great importance for the study of calculus and has wide application in physics, astronomy, cartography, nautical, and in recent times has led to the revolution in telecommunications, global satellite navigation systems, the representation of phenomena newspapers, and many other applications.

2.4 Learning Process of Trigonometric Functions

In the didactic strategy implemented for the learning of trigonometric functions, the four models of personalized learning were taken into account as a reference, which was important to motivate, stimulate and facilitate the learning process in the students and enable their active participation in the different educational settings, such as in the assimilation of algebraic language, development of inductive and deductive processes in learning, the interpretation of graphic language, resolution of exercises and in evaluations. The development of mathematical content that allowed students to assimilate definitions, properties, theorems, algorithms and analytical procedures, had a systemic sequence as shown in Figure 1.

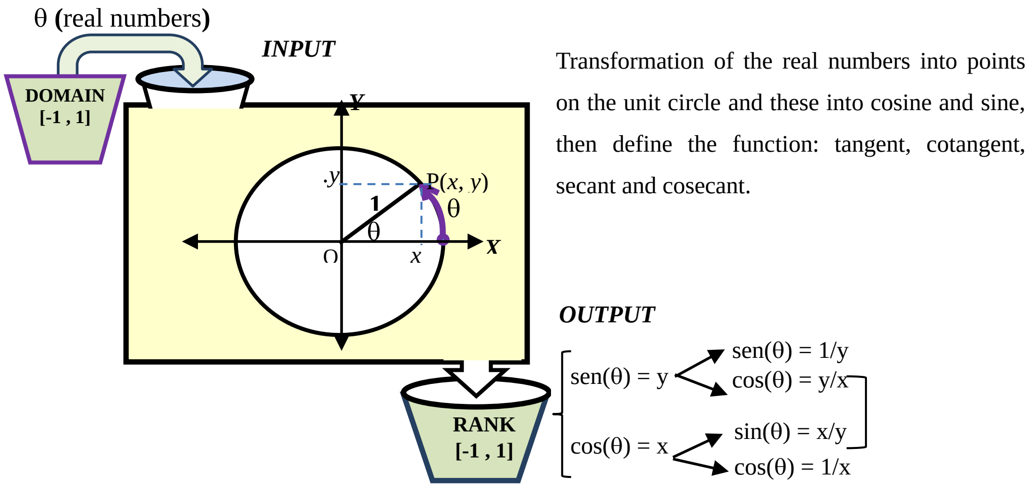

Figure 1 Producer Machine of the Sine and Cosine Functions of ℝ in [-1, 1]

In the area of mathematics, secondary education students pay special attention to graphical representations, in this way the graph of trigonometric functions is carried out with great enthusiasm, permanently interacting with technological resources. Therefore, the intention of this work is not only to present a personalized learning strategy, but also to analyze the effect of the use of technological tools and the practice of collaborative work; The presentation of the topic follows a different sequence than usual, encourages self-study and personalized work in class through activities designed for an efficient study of trigonometric functions in an algebraic, analytical and graphic way.

For the training activity, it was decided to integrate emerging technologies as a tool to develop an environment that promotes effective, dynamic and different learning, in which the student becomes an active and responsible agent of their learning. Under this premise, free software is used to manipulate the behavior of the trigonometric functions, relating the points of the unit circle with the graphs of the trigonometric functions, identifying their period and other elementary and visible properties, which allowed us to recognize the importance of the trigonometric functions (Figure 2).

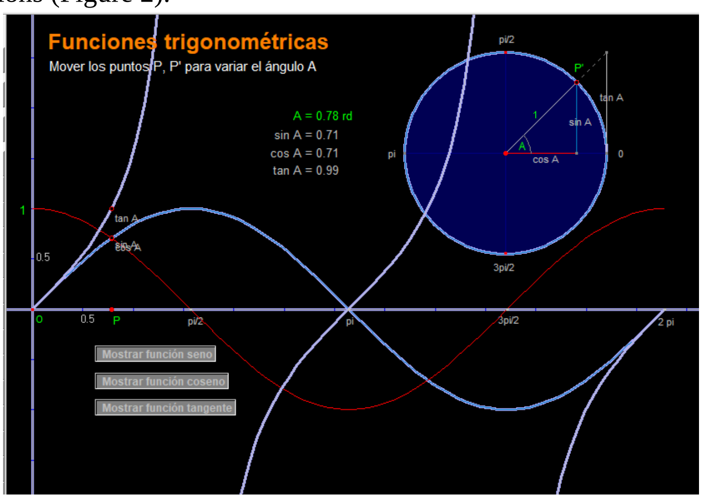

Figure 2 Dynamic Graph of the Cosine, Sine and Tangent Function in Correspondence With Points on the Unit Circle

3. Research Method

Taking into account the classification made by Sierra Bravo (2007), the type of research from the degree of abstraction is descriptive research since its main objective is based on describing and analyzing the academic performance of students during the teaching-learning process of trigonometry. In the degree of generalization it is an action research, because it focuses on generating changes in the learning process and does not place so much emphasis on the theoretical. Try to unite investigation with practice through problem solving making appropriate use of mathematical software. In relation to time it is a synchronous investigation, since an analytical study of the evolution of the didactic process is made in six class sessions. In relation to the place it is a field investigation because it focused on doing the study where the phenomenon occurs naturally.

The field work was supported by the activities of both the teacher and the student, where each one fulfills their roles according to a previously established plan, in order to achieve more efficient student learning. A set of traditions and methods intervenes in the research that entails at all times reflecting on our pedagogical practice allowing the administration and solution of problems of the learning process, from a participatory methodological perspective, through an alternative pedagogical proposal, based on a action plan, the same one that allowed to overcome the problems detected in the pedagogical and academic field Tójar (2006).

The research process had three well marked and concatenated stages: content planning and design and the didactic strategy, implementation process and development of activities. The field work of the experience was carried out during the second semester of the school year in accordance with the school curriculum and was in charge of the research teachers. The application of the proposal is governed exclusively by the sessions that are formulated through printed materials with personalized treatment; the same that serves as a guide to the teacher throughout the intervention period, guiding their daily class work, the use of technological resources are mainly to identify algebraically and graphically properties of trigonometric functions. For the design, information gathering, data analysis, execution, and subsequent evaluation of the proposal.

To collect information on the didactic process, a rubric was developed for the evaluation of conceptual learning and another for the evaluation of procedural learning that is structured in order to measure the level of learning achieved. An opinion test was also applied on the qualification of their learning on trigonometric functions with the personalized learning strategy. As the fourth data collection instrument developed and applied to the study group was a satisfaction survey. On the other hand, for the evaluation of the evolution of the students' production, an observation sheet was prepared and administered in each session to have evidence about the work in the classroom.

3.1 Development of Experience and Analysis of Results

Development of the experience. The development of the topic of trigonometric functions answers the following question: How are trigonometric functions defined, what are their properties, how are their graphs constructed, what are their main applications? The entire subject was developed with the help of printed text prepared with the purpose of achieving significant learning in the subject and with the availability of an individual computer with Derive6 and Geup7 mathematical software installed.

The classes on the topic of trigonometric functions were developed over six weeks, with six hours of class per week, distributed in blocks of two hours for each session:

• Week 1. Enveloping function on the unit circumference, from arcs oriented on the circumference and their respective measurements.

• Week 2. Trigonometric functions cosine and sine, identification of the domain and range, determination of values.

• Week 3. Tangent, cotangent, secant and cosecant trigonometric function, obtaining values, their fundamental properties, relationships between trigonometric functions.

• Week 4. Graph of trigonometric functions, identification of their properties, and calculation of values on the graph.

• Week 5. Definition of inverse trigonometric functions, identifying their properties and constructing their corresponding graphs.

• Week 6. Identities and trigonometric equations: applications.

The evaluation of performance in the process and of attitudes towards the established activities took place systematically during the development of the topic of trigonometric functions with pencil and paper, as well as through mathematical software. The personalized assistance to the student is carried out both in the theoretical and practical classes in the classroom, complemented with programming of extracurricular schedules, also all students have the opportunity to work with the computer equipped with mathematical software.

4. Results

In the application of the didactic strategy designed and elaborated for the learning of the trigonometric functions in a personalized way mediated by technological resources during six weeks, to a mixed group of fifteen students whose ages range between sixteen and eighteen years; To obtain empirical results, two rubrics were taken into account, an opinion survey and a satisfaction survey:

(A) **The rubric referred to the activities of evaluation of the conceptual learning of the trigonometric functions**, consisting of: identification of the range, period and graph of the trigonometric functions, from the manipulation of the Geup7 program, from which the correspondence between points of the unit circumference with the construction of the corresponding curve, the qualifiers obtained by the students are summarized in the Table 1.

Table 1 Result of the Evaluation of the Conceptual Learning of the Trigonometric Functions

Student	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
Qualifying	19	18	20	13	20	19	18	15	20	19	19	14	11	17	19
Statical	Mode = 19, Media = 17.40, Desv.Est. = 2,823, CV = 16.23%														

Source: application of rubric (learning concepts) to study subjects, 2017.

The results shown in Table 1 indicate that the implementation of learning activities with personalized treatment significantly influences the conceptual learning process during the study of trigonometric functions, since the student handles the concepts and properties of trigonometric functions, obtaining a global average for the group of 17.40, grading in the vigesimal system, a standard deviation of 2.823 and a coefficient of variation of 16.23%; the same that indicates that at the conceptual level they assimilated in a very acceptable way the concepts and basic properties of the trigonometric functions.

(B) **Rubric referring to the evaluation of procedural learning and the use of concepts in problem solving**, from an algebraic, graphical and analytical perspective of the trigonometric functions, the qualifiers obtained are summarized in Table 2, they indicate that the implementation of the strategy Personalized learning significantly influences the procedural learning process during the study of trigonometric functions, since the student successfully identifies properties and solves problems algebraically and graphically; obtaining a global average for the group of 17.60, with a score in the vigesimal system (from 0 to 20), a standard deviation of 2.971 and a coefficient of variation of 16.88%. The three statistics indicate that the students develop the exercises with a logical procedure of raisins until the results are obtained, interpreting efficiently through symbolic and graphic language.

Table 2 Result of the Evaluation of the Procedural Learning Activities of the Trigonometric Functions

Student	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
Qualifying	20	17	18	17	20	20	19	14	20	18	19	12	11	19	20
Statical	Moda = 20, Media = 17.60, Desv.Est. = 2,971, C.V. = 16.88%														

Source: rubric application (procedural learning) to study subjects, 2017.

(C) **Result of the Opinion Test**, of the 10 items that were formulated regarding their learning of trigonometric functions with personalized treatment, the majority responded positively in favor of the use of the didactic strategy and the use of technological resources, which had an impact on the degree of motivation towards learning mathematics, due to the fact that the interaction with the computer for the development of mathematical tasks was very new for them, affecting their mathematical competence. The results are shown in Table 3.

Table 3 Results of the Opinion Test Related to the Personalized Learning Process of Trigonometric Functions

Ítem	**Reagents**				
	Excellent	**Good**	**Regular**	**Bad**	**Total**
01	5	6	3	1	15
02	6	6	2	1	15
03	7	5	3	0	15
04	5	6	3	1	15
05	5	6	3	1	15
06	6	5	3	1	15
07	6	7	2	0	15
08	6	5	3	1	15
09	6	5	3	1	15
10	6	5	3	1	15
Total	**58**	**56**	**28**	**8**	**150**
Total	**38.67**	**37.33**	**18.67**	**5.33**	**100.00**

Source: application of the opinion test on the didactic strategy to the study subjects, 2017.

According to the summary that is made in table 3, the majority representing 38.67% of the students consider the personalized learning model as excellent; while 37.33% state that their learning achieved was good; 18.67% classify personalized learning as regular, and only 5.33% of the students stated that the learning strategy was bad.

(D) The **results obtained in the satisfaction questionnaire**, Figure 3, the items referring to *satisfaction with the learning content developed*, most of the students, plus 92%, responded to be very satisfied or satisfied, with respect to the development of the topic on the part of the teacher with the active participation of the students, the three remaining students consider themselves satisfied.

Regarding the *didactic strategy*, 60% surveyed students state that they are very satisfied, 26.7% state that they are satisfied and only 13.3 say that they are not very satisfied; Due to the restrictions on accessing the GEUP7 mathematical software, as it is a trial version that does not allow recording actions, capturing activities, among others, but if it motivates and enables the practice of personalized and interactive learning to reinforce the learnings.

Regarding the *performance of the Teacher* in the mathematics class using the didactic and technological resource, 73.3% of the participants of the experience, showed their full satisfaction and 26.7% said they were satisfied; Well, for them the first experience of taking a math class using GEUP7 as a tool, which allows the interactive and personalized study of trigonometric functions.

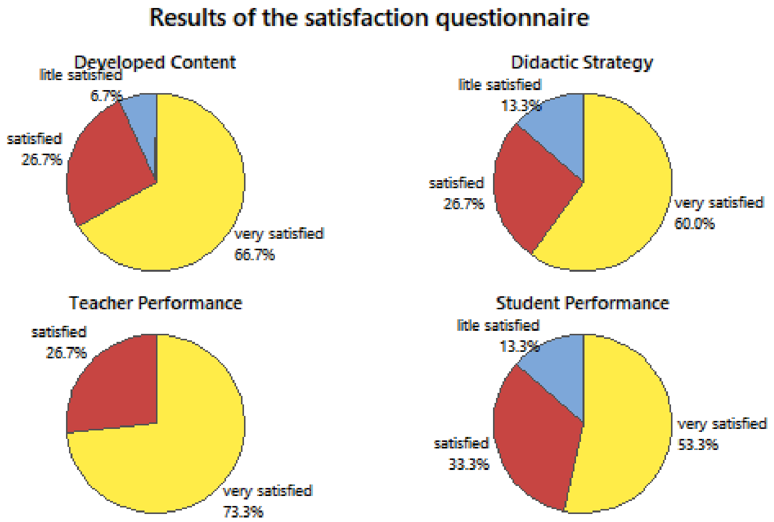

Figure 3 Level of Student Satisfaction With the Teaching Strategy Used

Regarding their *personal performance*, in the six items raised in this area, 53.3% of the surveyed students consider themselves to be very satisfied, 33.3% state they are satisfied and only 13.3% express be little satisfied. Overall, we can affirm that the majority of the students were very satisfied with the personalized learning strategy mediated by the GEUP7 of the trigonometric functions.

5. Conclusions

The empirical findings obtained in the research process indicate that the didactic strategy used showed an efficient understanding of the subject during the intervention, that is, the study of trigonometric functions from points on the unit circumference of the Cartesian plane, considering knowledge previous elementary geometry and algebra, is an alternative to the usual learning of trigonometric functions from ratios between the sides of a right triangle, where some concepts, properties, graphical representations, are insufficient and not very consistent.

An immediate implication of the didactic strategy used, with regard to procedural learning, allows direct teacher and student interaction, facilitating the development of intuition, abstraction and reasoning skills, relating to real situations and applications in the solution of problems, promoting active learning, it also enables the translation of algebraic language into graphic language using mathematical software, putting into practice viable procedures for learning. The achievements obtained in learning the trigonometric functions were satisfactory for both the student and the teacher.

In the attitudinal aspect, the results obtained indicate that the students were able to carry out autonomous activities, individually and in groups, empowering themselves from collaborative work as a natural way of learning, they solve problems on trigonometry by combining algebraic forms and graphic intuition with relevant use of mathematical software, in flexible performance; assuming with responsibility the study of the subject; that a posteriori you can use them in various situations of your personal and social activity.

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