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Design and Analysis of the PID Control Technique for an Inverted Linear Pendulum in V-REALM®

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Abstract: The main purpose of this article is to divulge the results obtained analyzing a PID as a control technique applied on the inverted linear pendulum.

This paper describes mathematical analysis using MATLAB-SIMULINK® and virtual simulations using both V-REALM tools and LABVIEW® interface. Also a physical implementation of a dynamic system is achieve using ARDUINO® as a microcontroller and analogical electronics/mechatronics.The objective is to model dynamic systems with applications ranging from engineering to economics.

Key words: inverted llinear pendulum; mathematical models; simulation modeling; PID control; Matlab-Simulink-V REALM®

JEL codes: C6

1. Introduction

The analysis of the behavior of dynamic systems using simulators is becoming more widespread and its areas of application are many. The mathematical model of a dynamic system allows anticipating the behavior of the system through equations (Ogata, 2002). These aspects constitute a research effort to continuously advance the understanding of methodologies that provide the ability to control systems (Lewis & Chang, 1997). The control systems consist of input signals, control system components and output signals (Kuo, 1996).

Most of the applications used use the so-called Proportional-Integral-Derivative control (PID) due to its wide range of application (Hdez-Gaviño, 2010). The content of this work is oriented to solve the problem controlling a linear inverse pendulum that is displaced on a rail. The approach is didactic and integrating between simulation, physical implementation and analysis of results using specialized mathematical software.

The dynamic system is an inverse pendulum that is placed on a base that moves linearly on a rail. On the base there is a pivot and attached to this a cylindrical bar (Figure 1).

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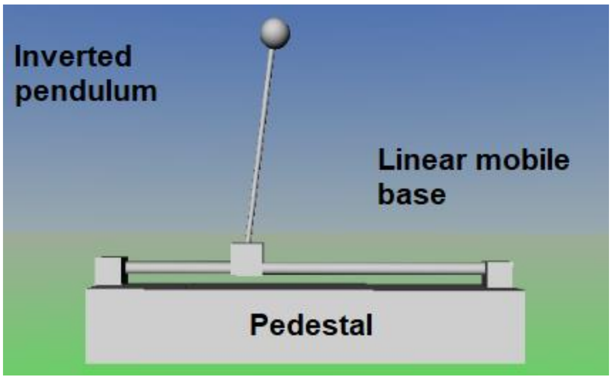

Figure 1 Inverted Linear Pendulum Modeling in V-Realm of MATLAB®

Scopes are excellent for educational didactics and the reverse pendulum is a classic problem that is addressed in the first Control Theory classes. The methodology is easily verifiable in this dynamic system because it is known that by definition, it is unstable and non-linear.

The system will seek to preserve the verticality of the pendulum and for this it is important to emphasize that there is no universal method to achieve this. It will depend on the approach of each proposal and the combination of methods and techniques that will enrich the experience when solving this classic of Theory of Control.

The project consists of the theoretical analysis and mathematical modeling of the system comprising the inverted pendulum on a mobile basis. It is desired that the pendulum maintain verticality despite possible disturbances present and for this the mathematical model that describes the variables that act on it must be performed.

For this purpose, the mathematical calculations supported by the SIMULINK® simulation tool of MATLAB® are used, where the obtained signals can be visualized and interpreted and the virtual behavior of the system can be established through the V-REALM tool.

In addition to the above, the dynamic system design proposal is made for its physical implementation. This stage includes the mechatronics section, which combines mechanical, electrical and electronic controlled elements using an ARDUINO® device as a central processor and a control console with a program designed using the program called LAB VIEW®.

2. Metodology

The inverted pendulum represents a problem of physical type and is precisely to ensure that the pendulum does not fall. For that, it is necessary to achieve what is known as local stabilization and which consists in balancing the pendulum from one side to the other depending on the disturbances present in order to maintain stability (analogously, a parallelism can be made with the game of maintaining a broom in balance resting one end on the palm of the hand while the other, with the bristles pointing upwards, swings vertically). Local stabilization can be achieved through linearization around the equilibrium position (Hdez-Gaviño, 2010; Lewis, 1997; Levine, 2000). However, this stabilization has a local character only and there are disturbances of capital magnitude the control of the pendulum is lost and it falls (Figure 2).

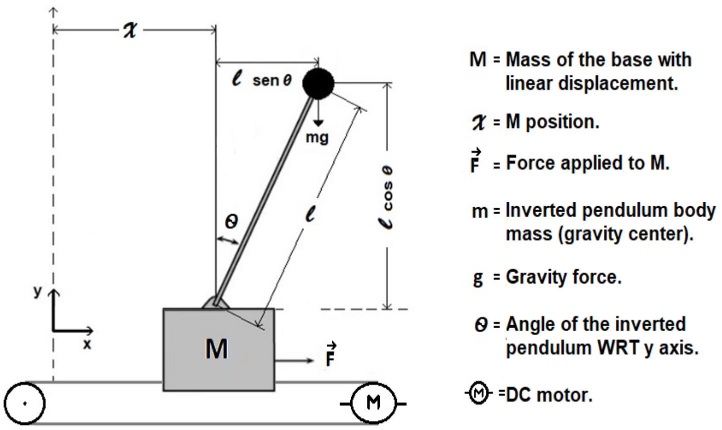

Figure 2 Free Body Diagram of the Inverted Linear Pendulum

The equations that describe the system are established. In the mathematical approach, the first step is to establish what is related to the description of the physical effects that act in the system (García et al., 2016; Szidarovszky, 2000). The first of these is the equilibrium point of the pendulum. This is obtained by determining the center of gravity (ɣ_XY) in the Cartesian plane (x, y), with respect to the upper end of the pendulum and is given by the following statements:

$$\mathrm{x}_{\gamma} = \ell\,\mathrm{sen}\,\theta + \boldsymbol{x} \quad (1)$$

$$\mathrm{y}_{\gamma} = \ell \cos\theta \quad (2)$$

Where x_ɤ and y_ɤ represent the components that make up the center of gravity ɣ_XY. Since it is observed that the mass (M) of the mobile base moves perpendicular to the x axis, the equation of motion can be obtained by applying Newton's second Law on the horizontal axis obtaining:

$$\vec{F}(t) = F_x + M\ddot{x} \quad (3)$$

In equation (3) we have that $\overrightarrow{F}(t)$ represents the input force in the system, while *Fx* refers to the movement in the x axis. Concerning the center of gravity located in the upper part of the pendulum, one must take into account the rotational movement and the second Newton's Law that applies to it. In combination, the description of the inertia (Ine) is obtained in the coordinate axes (x, y) represented by *Fx* and *Fy* respectively:

$$Ine\ddot{\theta} = F_y \ell sen\theta - F_x \ell \cos\theta \quad (4)$$

Thus, the horizontal movement of the pendulum can be expressed by F_x through the expression:

$$F_x = m\mathrm{x}_{\gamma} \quad (5)$$

This is how knowing that the horizontal movement for the center of gravity ɣ already has an assigned expression (x_ɤ = lsen〖θ + x〗), it can be asserted that:

$$F_x = m\ddot{\mathrm{x}} + m\ddot{\mathrm{a}} \quad (6)$$

Then:

$$\ddot{a} = \frac{d^2}{dt^2} \quad (7)$$

And in that way:

$$a = (cos\theta)\ddot{\theta} - (sen\theta)\dot{\theta}^2 \quad (8)$$

Therefore, the equation that describes the movement of the pendulum on the x-axis is:

$$F_x = m\ddot{x} + m\ell\ddot{\theta}cos\theta - m\ell\dot{\theta}^2 sen\theta \quad (9)$$

With respect to the y axis, the process is similar, only that the mass of the pendulum body must be taken into account as well as the gravity that acts on it. This is how you get the equation that describes the behavior in y:

$$F_y = -m\ell\ddot{\theta}sen\theta - m\ell\dot{\theta}^2cos\theta + mg \tag{10}$$

With respecUsing the equations that describe the movement of the pendulum both on the x axis and on the y axis, we propose the equation that represents the rotational movement of the inverted pendulum:

$$Ine\ddot{\theta} = \{\left[\left(-m\ell\ddot{\theta}sen\theta - m\ell\dot{\theta}^2cos\theta + mg\right)F_y\ell sen\theta\right] - \left[\left(m\ddot{x} + m\ell\ddot{\theta}cos\theta - m\ell\dot{\theta}^2sen\theta\right)F_x\ell\cos\theta\right]\} \tag{11}$$

At this point it can be assumed that if the center of mass located at the upper end of the pendulum equals zero on the understanding that it reaches equilibrium, we can say that the expression $Ine\ddot{\boldsymbol{\theta}}$ would result in:

$$mg\left(F_y\ell sen\theta\right) = gm\ddot{x} + (F_x\ell\cos\theta)\, m\ell^2\ddot{\theta} \tag{12}$$

To obtain the equation that describes the movement of the base on which the pendulum acts, it must be remembered that it acts in a lateral horizontal movement whose formulation is proposed with:

$$\vec{F}(t) = F_x + M\ddot{x}m\ddot{x} + m\ell\ddot{\theta}cos\theta - m\ell\dot{\theta}^2sen\theta \tag{13}$$

Taking into account that the objective of the system is to keep the pendulum in equilibrium, the equations that represent the movement in the x and y axes can be linearized, resulting in:

$$mg\theta = m\ddot{x} + m\ell\ddot{\theta} \tag{14}$$

$$\vec{F}(t) = \left(m\ell\ddot{\theta}\right) + [(M+m)\ddot{x}] \tag{15}$$

Having linearized these expressions, the terms of interest θ¨ as well as x¨ are cleared, obtaining by result:

$$\ddot{x} = \left(\vec{F}(t)\frac{1}{M}\right) - \left(\theta\frac{m}{M}g\right) \tag{17}$$

$$\ddot{\theta} = -\left(\vec{F}(t)\frac{1}{M\ell}\right) + \left(\theta\frac{(M-m)}{M\ell}g\right) \tag{18}$$

Finally, the Inverse Pendulum Transfer Function can be obtained by applying the Laplace Transform:

$$\frac{\theta(s)}{\vec{F}(s)} = \left(-\frac{1}{M\ell}\right)\left(\frac{1}{s^2 - \frac{(M+m)}{M\ell}g}\right) \tag{19}$$

The next step is to establish the state and output equations of the system, for that it is necessary to represent state spaces, which is obtained with the following assignment:

$$x_1 = \theta,\ x_2 = \dot{\theta},\ x_3 = x,\ x_4 = \dot{x} \tag{20}$$

Making the substitutions of the state variables you get:

$$\dot{x}_2 = \left[\frac{(M-m)}{M\ell}gx_1 - \vec{F}(t)\frac{1}{M\ell}\right] \tag{21}$$

$$\dot{x}_4 = \left[-\frac{m}{M}gx_1 + \vec{F}(t)\frac{1}{M}\right] \tag{22}$$

Obtaining the output equations of the inverted pendulum system on a mobile basis, as well as its state equations, it is stated that:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ \frac{(M-m)}{M\ell}g & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ -\frac{m}{M}g & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \vec{F}(t)\begin{bmatrix}0\\ -\frac{1}{M\ell}\\ 0\\ \frac{1}{M}\end{bmatrix} \tag{23}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{24}$$

It is shown that the system is controllable when the substitution of the following equation is solved |M|≠0 in:

$$M = [B \vdots AB \vdots \cdots \vdots A^{n-1}B] \tag{25}$$

Since the system is controllable, the poles are determined in the closed loop with the characteristic equation:

$$s^n + \alpha_1 s^{n-1} + \cdots + \alpha_{n-1} s + \alpha_n = 0 \tag{26}$$

Applying Ackerman's formula, we obtain that:

$$K = [0\ 0\ \cdots 0\ 1][B \vdots AB \vdots \cdots \vdots A^{n-1}B]^{-1}\,\phi(A) \tag{27}$$

Where φ (A) represents the Cayley-Hamilton equation:

$$\phi(A) = A^n + \alpha_1 A^{n-1} + \cdots \alpha_{n-1} A + \alpha_n I = 0 \tag{28}$$

Therefore:

$$\vec{F}(t) = -Kx \tag{29}$$

Having obtained the previous equations, we can start designing in V-REALM of Matlab-Simulink® the virtual model that will graphically describe the behavior of the Linear Inverted Pendulum system.

Through an algorithm that uses the resolution of ordinary differential equations (ODE45) the solution that describes the mathematical model of the linear inverse pendulum is obtained. The program is lost vertically and is stationed at 180° from the original point (Pérez, 2002) as described in Figure 2.

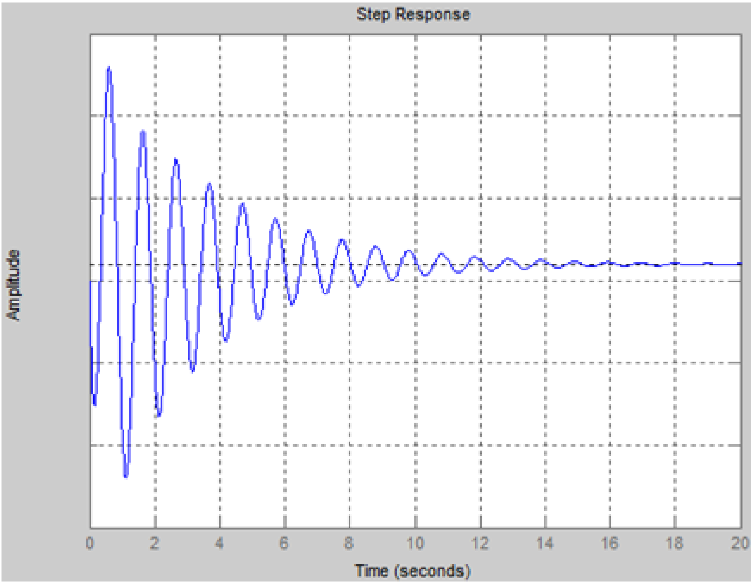

Figure 2 Behavior of the Non-linear Model of the Inverted Pendulum Performed in MATLAB.

For the implementation of the control system, a PID (Proportional-Integral-Differential) control is proposed. For this it is convenient to define the following:

- The proportional controller (P) has the objective of reducing the lifting time without completely eliminating the steady-state error.
- The integral control will have the effect of eliminating the steady-state error, however it could make the transient response an undesirable oscillation.
- The derivative control will tend to increase the stability of the system by reducing the overshoot and

improving the transient response.

In such a way that the combination of these three types of control is called PID (Seborg, 2011; Reyes, 2015) and it can be asserted that the proportional block is made up of the product between the error signal and the proportional constant, obtaining an almost null error.

The block diagram in Matlab-Simulink® shown in Figure 3 shows the description of the inverted pendulum system.

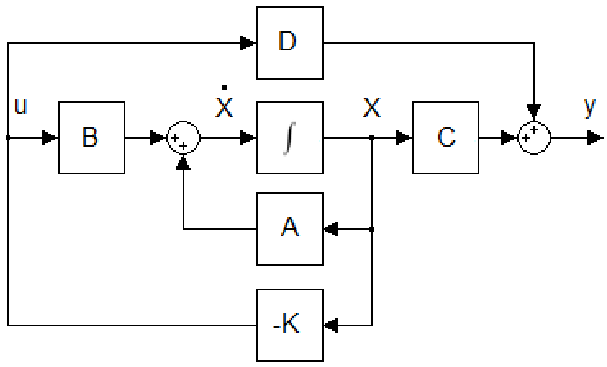

Figure 3 Block Diagram of a PID Control

Figure 4 shows the numerical values that were assigned taking into account the physical values with which the system works in a real implemented application.

```
function dx = penduloinvertido(x,u)
%Parámetros definidos

m=.10;    % masa expresada en kg
g=9.8;    % gravedad en SI m/s^2
L=30;     % Longitud de la barra del péndulo
b=0.4;    % Coeficiente de fricción viscosa
J=m*L^2;  % Inercia representada por la masa...
            % ...multiplicada por la longitud...
            % ...del péndulo elevada al cuadrado.

%Varaibles definidas
x1=x(1);
x2=x(2);
dx = [x2;m*g*L/J*sin(x1)-b/J*x2+1/J*u];
```

Figure 4 Numerical Values Assigned to the Inverted Pendulum System

This is how it is known that the nonlinear model provides a very close response to the physical action reality of the pendulum while the linearized model (Figure 5) will only work well around the point of operation, providing stability (Chapra & Canale, 1999).

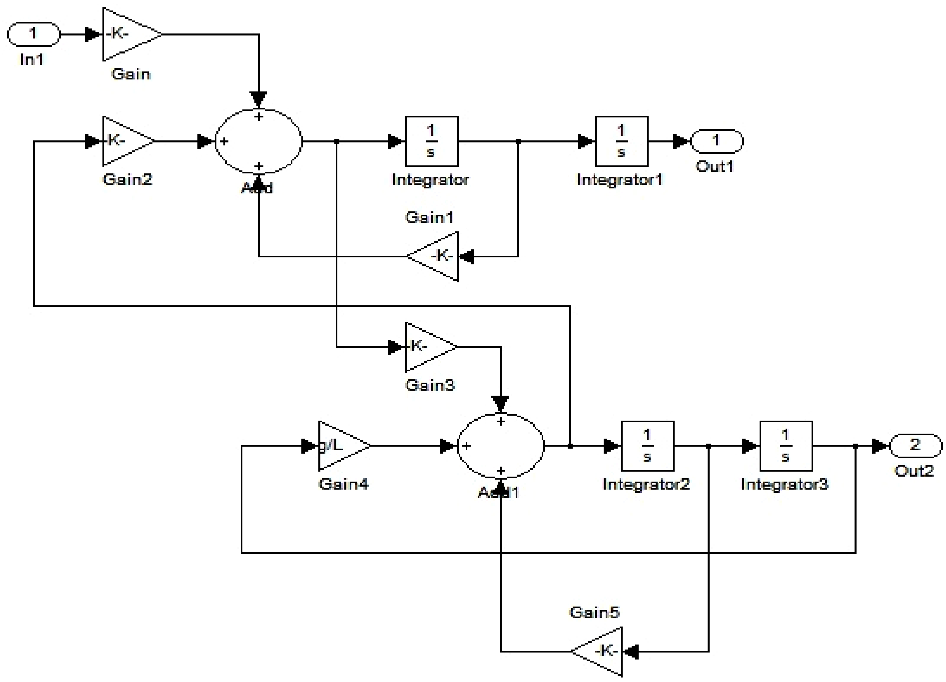

Figure 5　Linearization of Linear Inverse Pendulum in Simulink®

3. Results

With this model, you can graph through Simulink® and obtain the graphical responses of both the pendulum behavior and the mobile base (Figure 6).

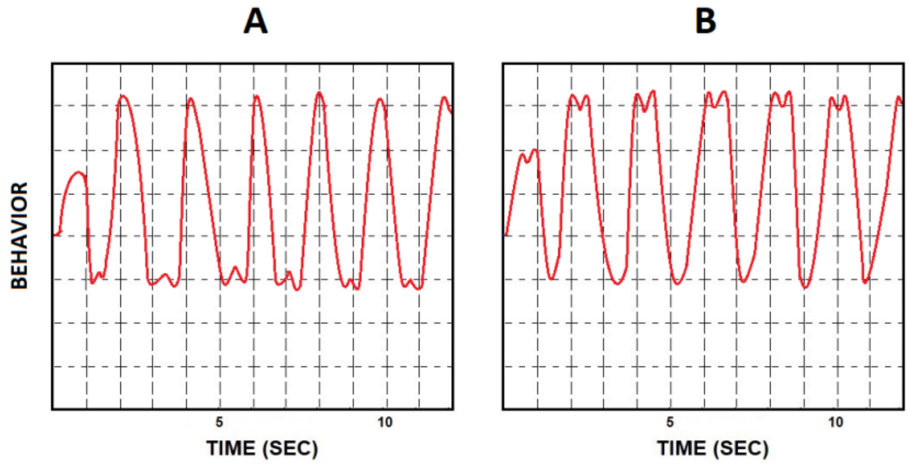

Figure 6　Graph (A) Represents the Behavior of the Pendulum in Real Time; Graph (B) Represents the Behavior of the Mobile Base in Real Time

The results obtained in V-Realm and Simulink-Matlab® are that the pendulum swings maximum 6° and the mobile base moves up to 3.1 cm with respect to the point of origin established to balance the pendulum (Figure 7). These are validated through the data provided by the sensor reading itself, which is reflected in the LabView® interface.

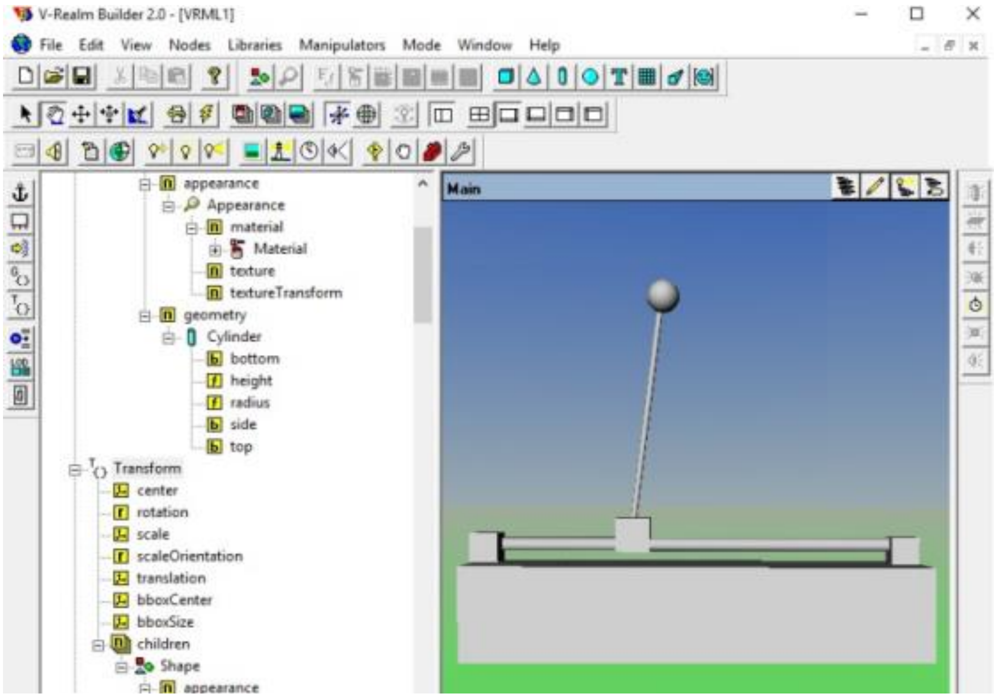

Figure 7 Inverted Pendulum Simulated in V-Realm

With regard to physical implementation, a system consisting of the pendulum attached to the mobile base through a pivot was designed. The mobile base can be moved laterally through a rail with carousel. Attached to the pivot is the position sensor. Both the ARDUINO® microcontroller and the H Bridge that controls the motor are placed on the pedestal that supports the entire structure. The details are shown in Figure 8.

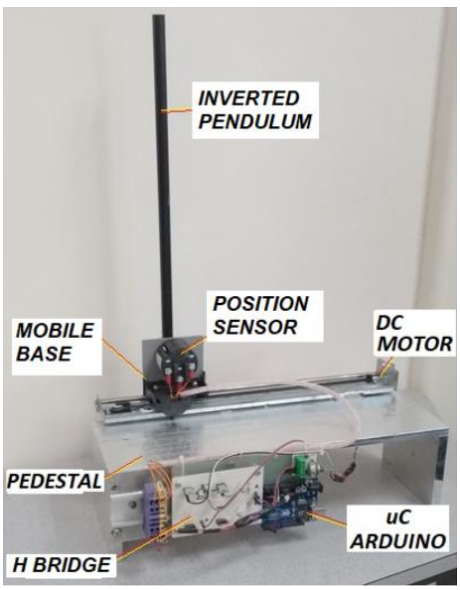

Figure 8 Linear Inverted Pendulum Implemented

The control system can be seen by means of a representation diagram in the "closed loop" configuration in Figure 9.

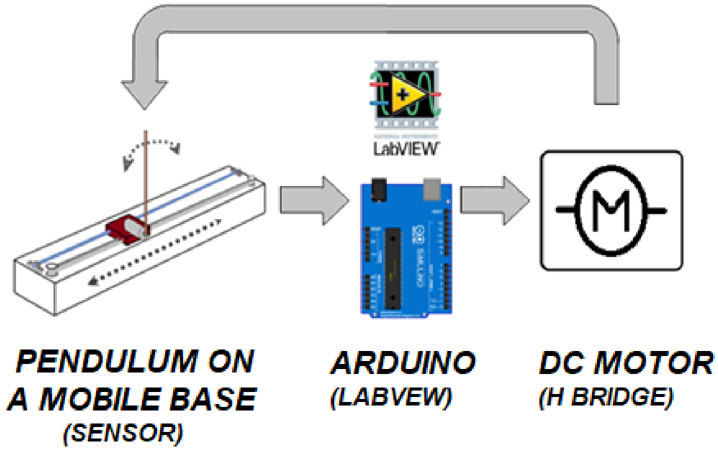

Figure 9 Representative Diagram of the System in Close Loop Configuration

Proportional-Integral-Derivative (PID) control is exercised from ARDUINO®. In this microcontroller the control action will be exercised as well as the processing of the input and comparison signals to be processed by the whole system. In Figure 10 the control routine observed on the compiler screen is partially appreciated.

The user interface of the control system is designed in the LabView® program of National Instruments and consists of a graph that describes the behavior of the pendulum and where its position can be appreciated (Figure 11). That is, it denotes whether the pendulum is in balance with the weight up or if it has rotated and is with the weight down. The results are also obtained that corroborate the calculations thrown by Matlab®, where the inclination in degrees of the pendulum is indicated numerically, as well as the distance that the mobile base moves with respect to a set point. It is with this virtual tool that the results obtained with the simulation made from the establishment of the equations that describe the system is corroborated, verifying the utility of the mathematical modeling.

Figure 10 ARDUINO® Compiler Window

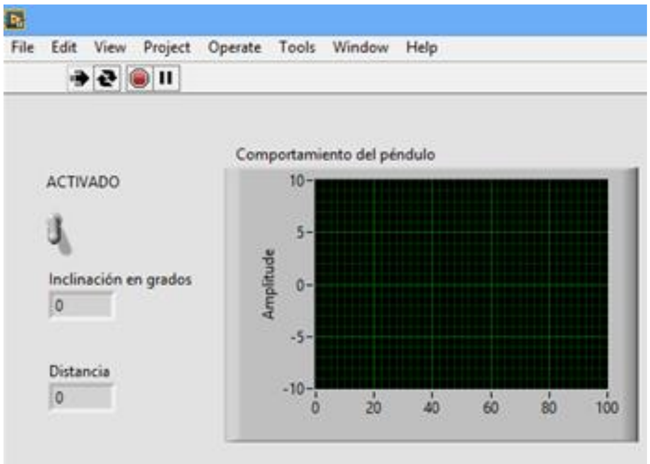

Figure 11 LabView® Indicator Panel

4. Conclusion

Currently, to achieve the best results it is imperative to know how to handle both the simulation and control tools that are available in the market and that are updated to the current technological age, as well as classical mathematical approaches.

- For this case, the PID control achieves its objective of maintaining the verticality of the inverse pendulum. Due to the nature of this method, the region of attraction of the controller can be considered small, so that the pendulum will become unstable before capital disturbances.
- The results obtained are as desired. Similarly opens the window to other techniques that includes the Control Theory for non-linear models such as sliding modes, a complementary technique to achieve even greater control over the system.
- The interface between the control system and the human being is made through the LabView® platform, while for the mathematical simulation of this, the V-REALM software from Matlab-Simulink® is used.
- Signal processing is achieved using the ARDUINO® microprocessor. Together, these programs and devices are useful tools that allow observing the behavior of the system before adjustments, tunings or design changes and allow the compilation of hard data to analyze them carefully.
- Practically the control technique can be exercised in any type of system with variables. A future application is focused on the economy.

In summary, the theme of the present system has been one of the most exposed to validate various methodologies in control systems. The didactic value of the documentation must be highlighted, as well as the integration and synchronization of the various computational, virtual and physical implementation tools as well as in economics (Keynes, 1936).

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