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Bühlmann & Bühlmann-Straub Credibility for Extreme Claims Applied on Non-Life Egyptian Insurance Market: An Actuarial Approach

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Abstract: Credibility theory is an actuarial approach used to calculate the short term insurance premiums. The aim of this article is to calculate the credibility premium based on real data from non-life Egyptian insurance industry during 10 years period (2006 to 2015), taking into consideration the amount of incurred claims from six insurance branches and the number of extreme losses in each branch. The analysis was based on the assumptions of Bühlmann and Bühlmann-Straub credibility models in order to estimate the net credible premium for the upcoming year as a linear function of the prior claims and the number of extreme events.

Key words: credibility theory; credibility premium; Bühlmann and Bühlmann-Straub credibility; R package
JEL codes: C13, C52, C58

1. Introduction

The idea of credibility theory is one of the most important and essential tool in actuarial mathematics, dealing with posterior distribution as a weighted average of prior distributions and likelihood. Credibility theory has been discovered and derived during the 18th century by Bayes (1763). Moreover, Bühlmann (1967) and Bühlmann-Straub (1970) introduced a multivariate generalization of the credibility model for claim reserving.

Insurance companies pay more attention on the tail behaviour of the distribution of claims (i.e., catastrophic claims). In Egypt each insurer deals with several branches, so they should focus on the risk of insolvency not only the profit since claims distributions have heavy tails. Furthermore, insurers are highly interested in meeting the regulators' requirements.

Bühlmann's approach is used to predict net premium based on the prior information for insurer's claims as a linear predictor assuming that data are identically distributed. This method is called the greatest accuracy approach. On the other hand, Bühlmann-Straub approach is an extent of Bühlmann credibility where data are not identically distributed, this method overcome the limitation of Bühlmann approach.

Our data used in this paper is collected from non-life insurance market based on records published in Financial Regulatory Authority (FRA) for six branches. These branches have been chosen according to claims that consist of extreme losses as shown in Table 1 and Table 2.

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Table 1 Amount of Claims for Each Branch

In Thousands

	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Fire	5497	65297	61483	115663	75427	107814	7263	63364	143552	278888
Marine	5864	-2353	96940	-42237	-35977	71269	-4200	35626	35772	56315
Aviation	-3881	5981	-7642	64401	51437	60300	56716	36316	35768	63877
Marine-Hull	1693	6364	23918	30870	29421	21157	16253	11613	9451	-2903
Oil	7789	31731	-95640	-26175	37721	50855	116960	132100	259030	94182
Engineering	10003	11944	35093	46771	29512	42453	23349	43299	60304	74950

Source: Financial Regulatory Authority (FRA).

The above table illustrates that claim amounts increase gradually for each branch. The table also shows negative claims referred to the amount of incurred claims are less than the estimated claims as presented in Figure 1.

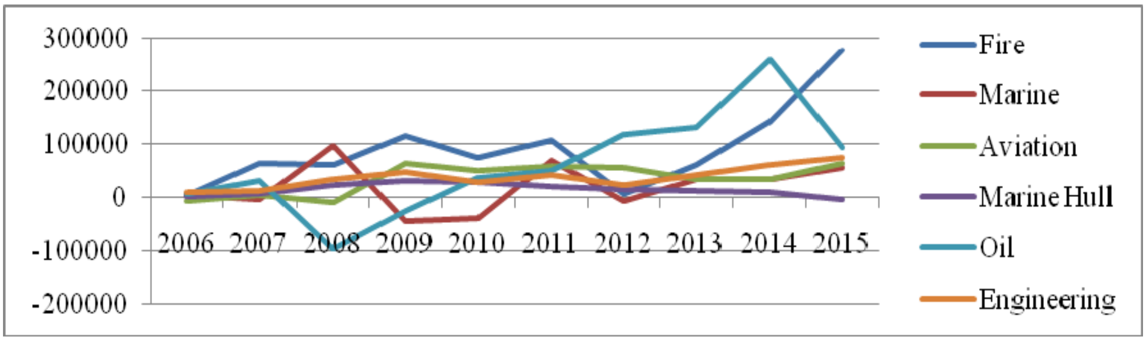

Figure 1 Amount of Claims for Each Branch

Table 2 Number of Extreme Losses for Each Branch

	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Fire	1	7	7	13	8	12	1	7	16	30
Marine	0	0	1	0	0	1	0	0	0	1
Aviation	0	0	0	1	0	1	0	0	0	1
Marin-Hull	0	0	0	0	0	0	0	0	0	0
Oil	0	0	0	0	0	0	1	1	1	1
Engineering	1	1	4	6	4	5	3	5	7	9

Source: Authors' calculations.

Table 2 shows the number of extreme claims (i.e., catastrophic claims) incurred on each branch. In addition, fire and engineering branches have more extreme losses than other branches according to the incurred claims exceed the estimated claims.

In This paper we consider a development of Bühlmann and Bühlmann-Straub credibility models as a joint distribution of the claim amounts and the number of observed extreme losses, in order to predict the upcoming year net credible premium.

One of the most important concerns for the insurance industry is the tail behaviour of loss distributions, where the tail of loss distribution is demonstrated by extreme events. This article introduced an approach that modified Bühlmann and Bühlmann-Straub credibility models by taking into account the tail behaviour of claims in order to predict credible premiums.

The aim of this paper is to improve Bühlmann and Bühlmann-Straub credibility models in order to estimate the net credible premium for as a linear function of the prior claims and the number of extreme events.

The remainder of this paper is organized as follows. Section 2 Literature Review, Section 3 Data and Methodology, Section 4 Models, Section 5 Empirical Findings. Finally, Section 6 Conclusions

2. Review of the Literature

Norberg (2011) presented the greatest accuracy credibility theory and explained the basic ideas, techniques and practical application of the Bayesian credibility theory.

Loisel and Trufin (2013), they considered the discrete-time ruin model to determine the characteristics of the ruin probability in the heavy tailed claim amounts. Then apply the Bühlmann credibility to estimate net premiums.

Happ et al. (2014), applied Bühlmann-Straub credibility to claim reserving nonlife chain-ladder. They used a multivariate credibility of N correlated portfolios to estimate the conditional mean square error of the ultimate claims and compared the estimated results with the multivariate additive method.

Jindrová and Kopeck (2017), considered Bühlmann and Bühlmann-Straub empirical credibility to estimate the credibility premiums and net premiums for catastrophic claim amounts and economic losses for different regions. They applied Bühlmann and Bühlmann-Straub credibility for short term insurance taking into consideration two types of data, i.e., past data from risk itself and collateral data from other sources considered to be relevant.

Jindrová and Pacáková (2015), demonstrated classical Bayesian approach to estimate the probability of critical illness insurance. Thus, they used age and sex for different groups in order to calculate risk premiums.

Jindrová and Seinerová (2015), applied the Bühlmann Straub model to measure the cost of healthcare insurance and the corresponding credibility factor, and concluded the for large companies the estimation of the healthcare cost is reliable while for small sized companies its meaningless without the referral to data from the whole market. In addition, Bayesian analysis is considered a useful technique for healthcare insurance.

Gao (2016) illustrated modelling claim reserving using Bayesian analysis, this study classified into two sessions. First session, introduced Bayesian methodology for claim reserving. Second session, proposed a compound model as a probabilistic approach and the Bayesian expansion models by applying Monte Carlo simulation for claim reserving.

Pacáková (2013) applied Bayesian credibility analysis to estimate parameters for several statistical distributions with given prior distribution. Furthermore, estimate the credibility premium or credibility number of claims in insurance.

Linda and Kubanová (2012) used real data from five insurance companies to calculate premiums for motor third-party liability insurance based on Bühlmann-Straub credibility as a methodology to improve the quality of net premium estimation.

Seinerová (2015) illustrated and application of Bühlmann Straub model to estimate the credibility costs by combining individual and collective experience throughout the credibility factor as a confidence level. This model is applied for health care insurance.

Atanasiu (2005) illustrated how to estimate the insurance cost using the greatest accuracy theory, by applying Bühlmann-Straub model. Furthermore, this study considered the development of Bayesian credibility theory to measure the credibility net premium.

Boland (2007) illustrated an introduction on credibility theory; he applied Bayesian analysis in different fields (i.e., insurance and actuarial sciences). Furthermore, he used R package as statistical software to analyse the empirical results.

Hendrych and Cipra (2017) demonstrated the dynamic linear system of simultaneous equations for non-life insurance market in Czech; they used these equations to estimate the desired variables (i.e., outstanding claims, unearned premiums, other technical provision and loadings). This approach might motivate development of internal models applicable in the Solvency II framework.

Merz and Wüthrich (2009) discussed the issue of claim reserving position in the financial statements and discovered the huge portion of claims reserves. Moreover, they estimate and adequate amount of claim reserves for all branches and entire market.

3. Data and Methodology

Credibility theory is an actuarial approach used to calculate the short term insurance premiums. This approach is used to estimate premiums for each risk based on two ingredients: past data from the risk itself and data collected from other sources.

Data adopted in this paper are incurred claims from six different branches and the number of extreme losses observed from each branch for the period from 2006 to 2015. These data are reported in the Financial Regulatory Authority (FRA) for non-life insurance companies.

In this research we fit a statistical distribution for each risk (i.e., each branch) and estimate the statistical characteristics for each branch. Also we simulate the incurred claims in order to observe the number of extreme losses to apply Bühlmann-Straub credibility to predict the net premium as a linear function of the prior claims and the number of extreme events.

All statistical methods and calculations will be performed using the R Package that is an open source environment for mathematical and statistical computations. The {actuar}package is used to applyBühlmann and Bühlmann-Straub credibility.

4. Models

4.1 Bühlmann Credibility Model

The Bühlmann credibility model assumes that statistical distribution, random variables $\{X_1, X_2... X_N, X_{N+1} \ldots\}$ are independently and identically distributed (*i.i.d*).

As mentioned above the credibility premium consists of two ingredients, then

$$C = ZP_r + (1 - Z)\mu \tag{1}$$

where Crepresents the estimated pure premium, P_r referred to the estimation based on the prior data for each branch and Z is the credibility factor which is a number between 0 and 1 that measure how much reliance the insurer is ready to face its own risk.

The Bühlmann credibility approach estimates the credibility premium from each risk; this model has been derived in Bühlmann (1976), and concluded the following results

$$E(m(\theta)\big|x) = Z\overline{x} + (1-Z)E(m(\theta)) \tag{2}$$

and the credibility factor

$$Z = \frac{n}{n + \frac{E(s^2(\theta))}{VAR(E(m(\theta)))}} \tag{3}$$

Where $E(m(\theta))$ is the collective premium, $E(s^2(\theta))$ is within risk variance and $VAR(E(m(\theta)))$ between risk variance.

$$E(m(\theta)) = \overline{X} \tag{4}$$

$$E(s^2(\theta)) = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{n-1}\sum_{j=1}^{n}(X_{ij} - \overline{X}_i)^2 \tag{5}$$

Where

$$m(\theta) = E[X_{ij} | \theta_i]$$

$$s^2(\theta) = VAR[X_{ij} | \theta_i]$$

And

$$VAR(E(m(\theta))) = \frac{1}{N-1}\sum_{i=1}^{N}(\overline{X}_i - \overline{X})^2 - \frac{1}{Nn}\sum_{i=1}^{n}(X_{ij} - \overline{X}_i)^2 \tag{6}$$

The derivations of the above quantities can be found in Bühlmann (1967).

4.2 Bühlmann-straub Credibility Model

The Bühlmann-Straub model doesn't assume that the random variables $\{X_1, X_2 ... X_N, X_{N+1} ...\}$ are independently and identically distributed (*i.i.d*) as an extent to Bühlmann credibility model.

According to Bühlmann-Straub (1970) the estimation of the net credible premium for the *i*-th risk can be expressed as follows:

$$E(m(\theta) | x) = Z_i \overline{X}_i + (1 - Z_i)\overline{X} \tag{7}$$

The credibility factor for each i risk Z_i is calculated from the formula

$$Z_i = \frac{P_i}{P_i + \frac{E(s^2(\theta))}{VAR(E(m(\theta)))}} \tag{8}$$

where

$$E(s^2(\theta)) = \frac{1}{N(n-1)}\sum_{i=1}^{N}\sum_{j=1}^{n}P_{ij}(X_{ij} - \overline{X}_i)^2 \tag{9}$$

$$VAR(E(m(\theta))) = P * \left\{ \begin{array}{l} \frac{1}{nN-1}\sum_{i=1}^{N}\sum_{j=1}^{n}P_{ij}(X_{ij} - \overline{X}_i)^2 \\ -\frac{1}{N(n-1)}\sum_{i=1}^{N}\sum_{j=1}^{n}P_{ij}(X_{ij} - \overline{X}_i)^2 \end{array} \right\} \tag{10}$$

$$\overline{X}_i = \tfrac{1}{P_i}\sum_{j=1}^{n} P_{ij} X_{ij} = \tfrac{1}{P_i}\sum_{j=1}^{n} P_{ij} Y_{ij} \tag{11}$$

$$\overline{X} = \tfrac{1}{P}\sum_{i=1}^{N} P_i \overline{X}_i \tag{12}$$

$$P^8 = \frac{1}{Nn-1}\sum_{i=1}^{N} P_i \left(1-\frac{P_i}{P}\right) \tag{13}$$

and P_{ij} is the number of extreme losses of the i-th risk in year j.

5. Empirical Findings

5.1 Descriptive Analysis

We will start this section by determining the statistical characteristics of each risk (i.e., branch of insurance).

Table 3 Descriptive Analysis for Claim Amounts per Branch

	Mean	Median	Q_1	Q_3	St.Dev.	Skewness	Kurtosis
Fire	92424.8	70362	61953.25	113700.8	78736.62	1.50	3.20
Marine	21701.9	20745	-3738.25	51179.25	45488.71	0.16	-0.86
Aviation	36327.3	43876.5	13427.75	59404	28356.91	-0.67	-1.32
Marine-hull	14783.7	13933	7135.75	23227.75	11512.32	-0.02	-1.17
Oil	60855.3	44288	13774.5	111265.5	97166.49	0.56	1.16
Engineering	37767.8	38773	24889.75	45903	20398.71	0.35	-0.22

Source: Authors' calculations.

Table 3 summarize the statistical characteristics of each insurance branch, showing the four moments and the quartiles.

Table 4 Summary of Parameters Estimated for Each Risk

Branch	Distribution	Estimated Parameter	Standard Error
Fire	Lognormal	$\mu = 10.96$ $\sigma = 1.19$	0.1969544 0.1392662
Marine	Normal	$\mu = 21701.90$ $\sigma = 43154.38$	11863.28 11863.28
Aviation	Normal	$\mu = 36327.30$ $\sigma = 26901.73$	8507.601 5671.734
Marine-Hull	Normal	$\mu = 14783.70$ $\sigma = 10921.54$	3424.635 2473.666
Oil	Normal	$\mu = 60855.30$ $\sigma = 92180.22$	N/A N/A
Engineering	Lognormal	$\mu = 10.37$ $\sigma = 0.63$	0.1969544 0.1392662

Source: Authors' calculations based on results from R Package.

Table 4 summarizes the results obtained from fitting a statistical distribution of each insurance branch and the estimated parameters for each distribution after performing 1000 simulations. See Appendix.

5.2 Application of Bühlmann Credibility Approach

In this section we are about to estimate the quantities $E(m(\theta))$, $E(s^2(\theta))$ and $VAR(E(m(\theta)))$ in order to calculate the credibility factor Z, then estimate the credibility premiums for each branch of insurance based on data from Table 1.

From Equations (1) to (6), we calculate the quantities mentioned above and these results are summarized in Table 5.

Table 5 Results of Bühlmann Credibility Model

Branch	Mean ($\overline{X}_i$)	Credibility Premium (C)
Fire	92424.8	73550.13
Marine	21701.9	30379.89
Aviation	36327.3	39307.44
Marine-Hull	14783.7	26156.92
Oil	60855.3	54279.67
Engineering	37767.8	40186.74

Source: Authors' calculations based on results from R Package.

where

$E(m(\theta)) = 43976.8$, $E(s^2(\theta)) = 3177126666$, $VAR(E(m(\theta))) = 49780078$

and the credibility factor Z = 0.6104139.

5.3 Application of Bühlmann-Straub Credibility Approach

Bühlmann-Straub credibility approach is applied to calculate the net credible premium for each branch i, (i = 1, 2 ...N, N = 6) based on the number of extreme events per year j, (j = 1, 2…n, n = 10).

From Equations (11) to (13), we obtain the results in Table 6.

Table 6 Computed Characteristics of Branches

Branch	Total number of extremes (P_i)	Total amountof Claims (Y_i)	$\overline{X}_i$
Fire	102	924248	9061
Marine	3	217019	72340
Aviation	3	363273	121091
Marine-Hull	1	147837	147837
Oil	4	608553	152138
Engineering	45	377678	8393

Source: Authors' calculations.

where $P_i = \sum_{j=1}^{n} P_{ij}, Y_i = \sum_{j=1}^{n} Y_{ij}$

Table 6 presents the number of extreme events for each branch of insurance (P_i), also the total amount of claims incurred per branch (Y_i) in order to calculate the average insured extreme events ($\overline{X}_i$).

According to Equations (7) to (10) the credibility factor Z_i is calculated for each branch of insurance, then the value of net premiums are estimated using Bühlmann-Straub model in Table 7.

Table 7 Credibility Factors and Estimates of Net Insurance Premiums per Branch in 2016

Branch	Credibility factor (Z_i)	Net Insurance Premium
Fire	0.9606063	148894.09
Marine	0.4176563	87219.65
Aviation	0.4176563	82215.29
Marine-Hull	0.1929407	83512.35
Oil	0.4888219	122723.80
Engineering	0.9149514	52019.00

Source: Authors' calculations based on result from R Package.

Table 7 provides the net premiums for each branch of insurance showing how much money each branch will need to cover extreme events in year 2016.

where

$E(m(\theta)) = 96097.37$, $E(s^2(\theta)) = 15012766748$ and $VAR(E(m(\theta))) = 3589046514$

6. Conclusion

Bühlmann and Bühlmann-Straub credibility approaches represent a recent development of the Bayesian credibility theory; these models apply the greatest accuracy theory. In this study we improved Bühlmann and Bühlmann-Straub credibility models as a joint distribution in order to estimate the net credible premium using incurred claims and extreme losses. However, we adopt Bühlmann-Straub credibility because it doesn't assume that risks are independently and identically distributed (i.i.d). Our research concentrates on six branches of Egyptian non-life insurance market during the period from 2006 to 2015. Furthermore, these branches include extreme events to predict the credibility net premium for the upcoming year 2016 for each branch of insurance as shown Table 7. Moreover, this process show how much money each branch will need to cover extreme events in order to manage risks.

Future research may consider fluctuation in claim amounts resulted from extreme losses or lapses in insurance policies, based on the economic changes that happens in the Egyptian insurance market.

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Appendix

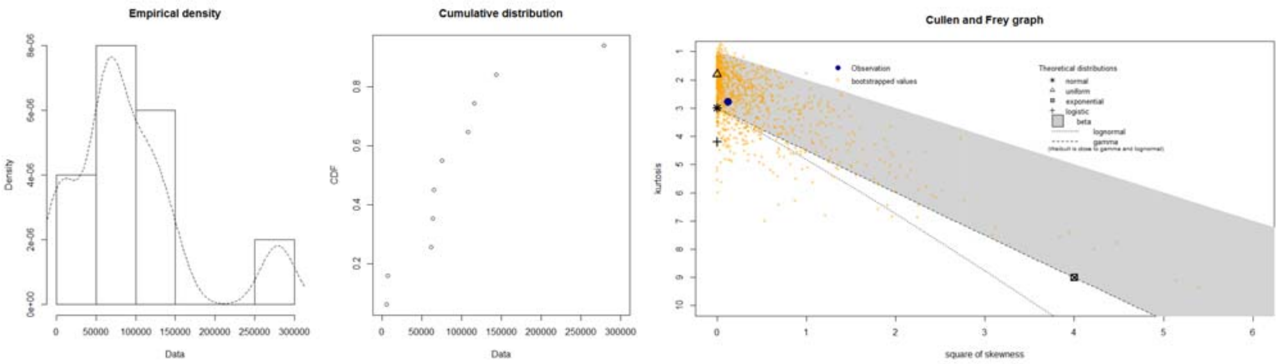

Figure 2 Pdf, CDF and Cullen and Frey Graph for Fire Claims

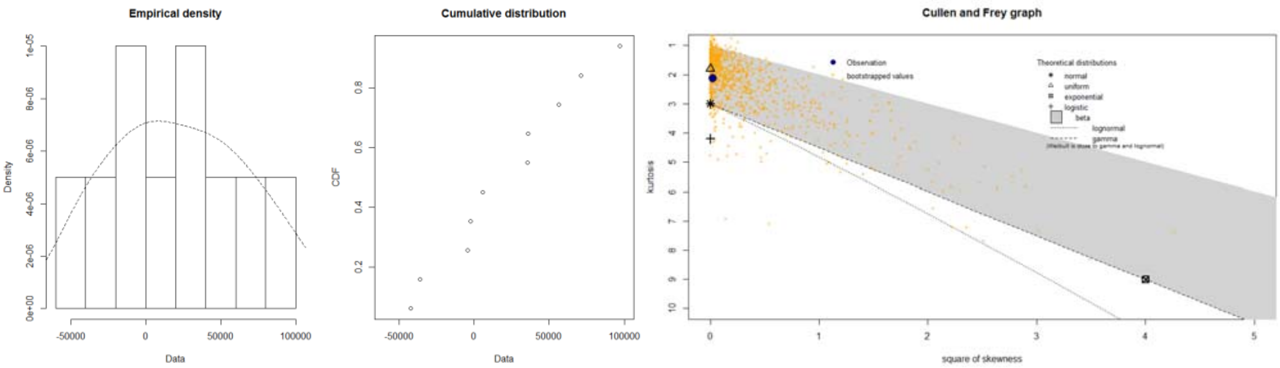

Figure 3 Pdf, CDF and Cullen and Frey Graph for Marine Claims

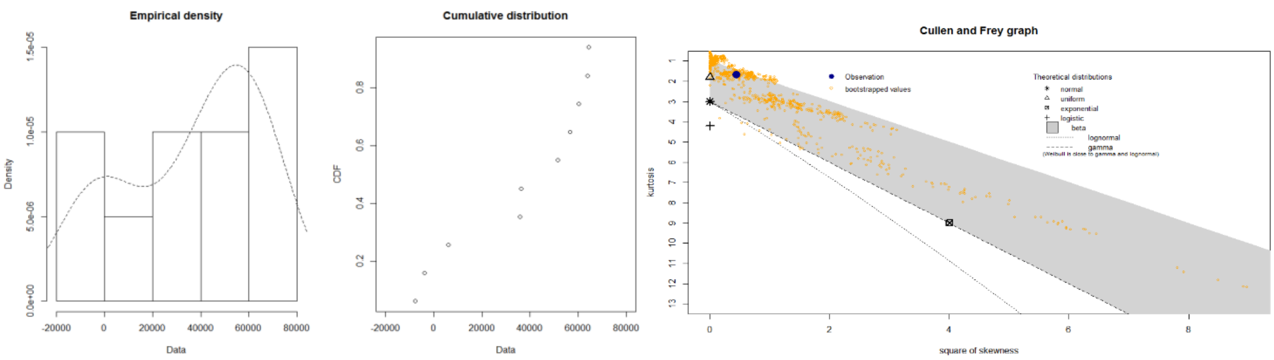

Figure 4 Pdf, CDF and Cullen and Frey Graph for Aviation Claims

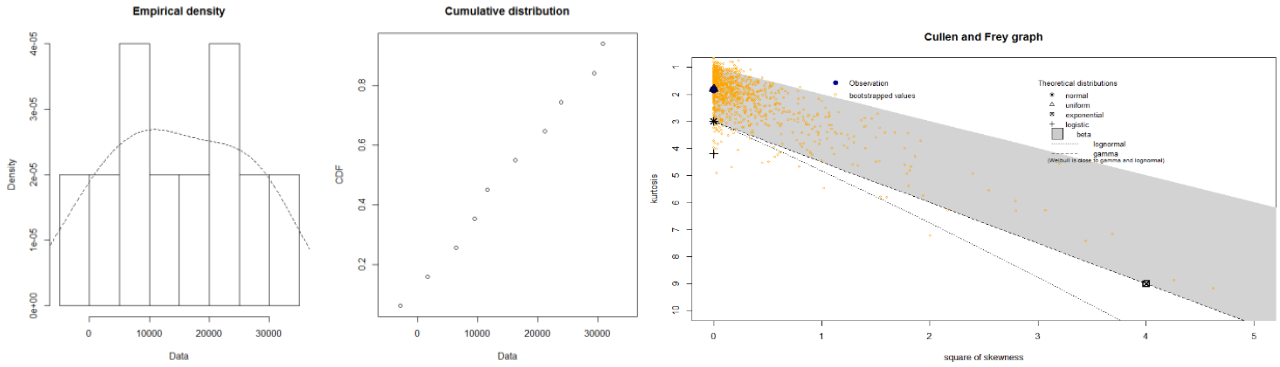

Figure 5 Pdf, CDF and Cullen and Frey Graph for Marine-Hull Claims

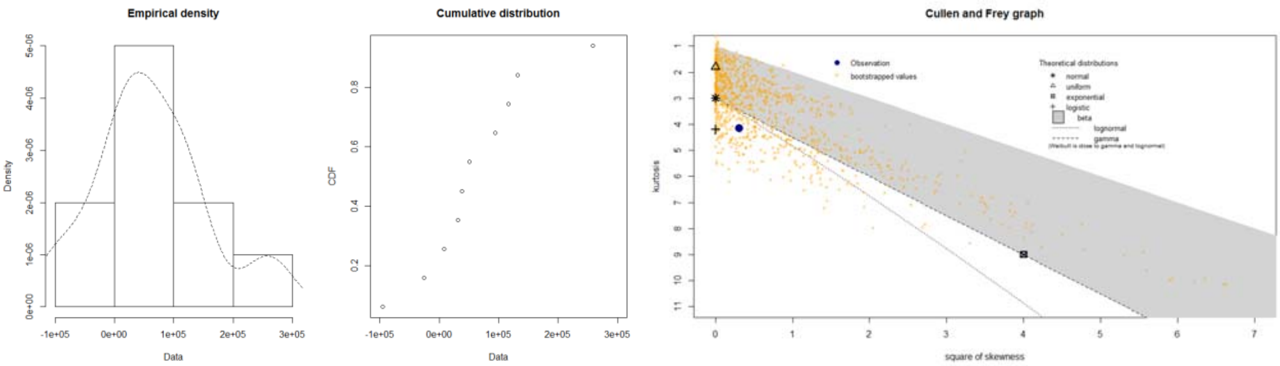

Figure 6 Pdf, CDF and Cullen and Frey Graph for Oil Claims

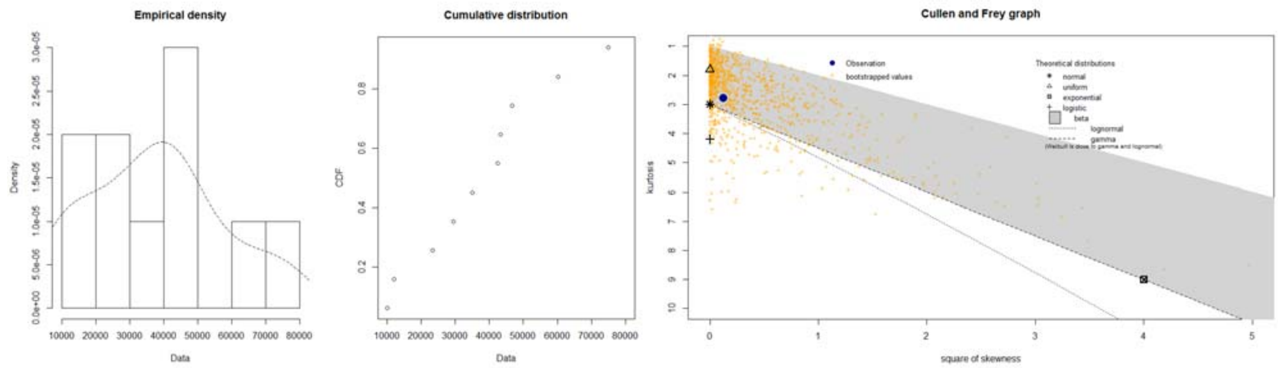

Figure 7 Pdf, CDF and Cullen and Frey Graph for Engineering Claims

Figures 2 to 7 present the density function and the distribution function of incurred claims for each branch, also it shows Cullen and Frey graph that gives the best fit for each distribution.