Journal of Business and Economics, ISSN 2155-7950, USA
September 2019, Volume 10, No. 9, pp. 852-860
DOI: 10.15341/jbe(2155-7950)/09.10.2019/006

http://www.academicstar.us

Prediction of Technical Reserves Based on Grey Model — GM(1,1): Evidence from Non-life Egyptian Insurance Market

Mahmoud Elsayed, Amr Soliman
(Insurance & Actuarial Sciences Department, Cairo University, Egypt)

Abstract: Grey system theory is a mathematical technique used to predict data with known and unknown characteristics. The aim of our research is to forecast the future amount of technical reserves (outstanding claims reserve, loss ratio fluctuations reserve and unearned premiums reserve) up to 2029/2030. This study applies the Grey Model GM(1,1) using data obtained from the Egyptian Financial Supervisory Authority (EFSA) over the period from 2005/2006 to 2015/2016 for non-life Egyptian insurance market. We found that the predicted amounts of outstanding claims reserve and loss ratio fluctuations reserve are highly significant than the unearned premiums reserve according to the value of Posterior Error Ratio (PER).

Key words: grey theory; technical reserves; posterior error ratio; insurance market
JEL codes: C13, C52, C58

1. Introduction

The Egyptian economy faced several challenges since 2011, which has a big influence in the insurance industry. By the end of 2016, the Egyptian economy faced economic recession and devaluation. According to the slowdown in the Egyptian economy and the impact of devaluation the amounts of claims become insufficient to meet the insured's satisfaction. Moreover, the devaluation has indeed led to a dramatic increase in the inflation rate. Furthermore, insurance companies should thoroughly estimate technical reserves, by which they meet future obligations. Table 1 shows the amounts of technical reserves over the period from 2005/2006 to 2015/2016.

Table 1 and Figure 1 also show the trends of the three types of technical reserves. Thus, the unearned premiums reserve fall within the period 2010/2011 to 2013/2014. However, a proper model is needed to predict technical reserves to overcome current situations.

Grey models are adopted to predict future technical reserves using a recent set of key and positive data. In addition, the grey models are used to deal with systems that have insufficient information to predict the behavior of unknown distribution.

Mahmoud Elsayed, Associate Professor of Insurance and Actuarial Sciences, Cairo University; research areas/interests: insurance and actuarial sciences.Email: mah.elsayed_njk@foc.cu.edu.eg.
Amr Soliman, Assistant Professor of Insurance and Actuarial Sciences, Cairo University; research areas/interests: insurance and actuarial sciences. Email: s.amr@foc.cu.edu.eg.

Table 1 Technical Reserve from the Egyptian Insurance Market

in thousands

Year	Outstanding Claims Reserve	Loss Ratio Fluctuations Reserve	Unearned Premiums Reserve
2005/2006	2583135	1130323	870954
2006/2007	3347467	1083111	989206
2007/2008	5153205	1519680	1362077
2008/2009	5190489	1498050	1473889
2009/2010	5486311	1548887	1617696
2010/2011	5718028	1648482	1841748
2011/2012	5688413	1829163	159755
2012/2013	6031883	2117913	199436
2013/2014	5984323	2457960	2327612
2014/2015	6054039	2654169	2550817
2015/2016	5978670	3137598	2779986

Source: Egyptian Financial Supervisory Authority (EFSA).

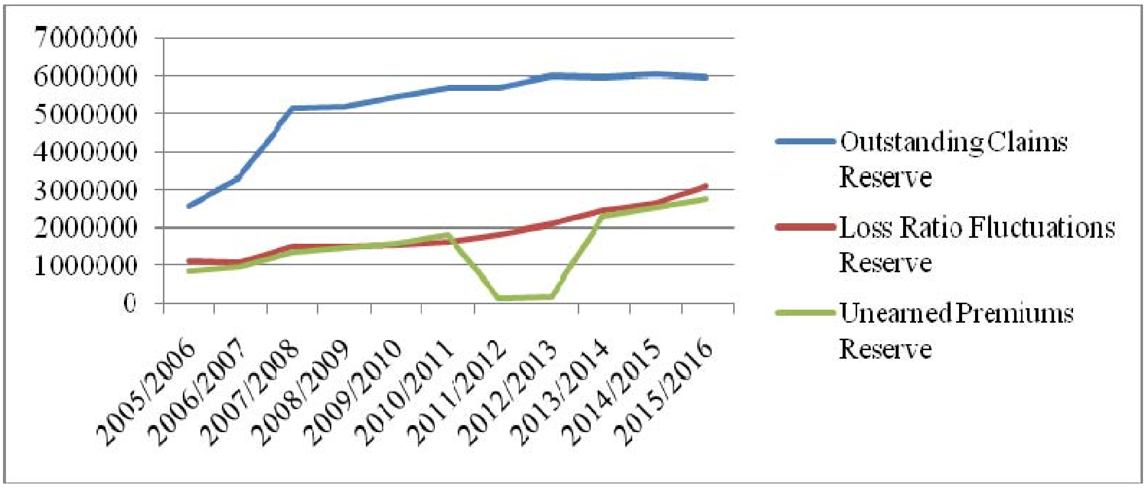

Figure 1 Technical Reserve from the Egyptian Insurance Market

Any set of data can be classified with colors according to the amount of clear information about the data. A set of data is identified as a black box if the characteristics of the data are completely unknown. However, a set of data is called white if the characteristics of the data are completely known. Moreover, the grey model is defined as a set of data with known and unknown characteristics (Lin & Liu, 2004). For Grey theory, GM(n,m) is defined as a grey model where n is the order of differential equations and m is the number of variables (Kayacan et al., 2010). In our research we consider the GM(1,1) model.

The aim of this paper is to introduce a new approach for a small sample with incomplete information, in order to perform accurate prediction for technical reserves.

The remainder of our research is structured as follows: Section 2 an overview of previous studies. Section 3 presents the theoretical framework of the Grey model GM(1,1). Section 4 is the modeling findings, and Section 5 concludes with implications.

2. Literature Review

Tianxiang et al. (2009) illustrated the explanation of grey system theory by considering geometric sequences. They determined as the number of samples increase the relative error also increase in case of non-negative increasing monotonous exponential sequence. However, the non-negative decreasing monotonous exponential sequence there exist the sample that has the least average relative error.

In another study Kayacan et al. (2010) discussed different methods of grey system analysis (i.e., GM(1,1), Grey Verhulst model and modified grey model), for a high noisy data obtained from the USA dollar to euro data. Modified Grey model obtained the best fit and forecasting results. They also examined the GM(1,1) model that produce an accurate prediction in case of monotonous data.

Chang and Wang (2013) analyzed data from social media channels by used GM(1,1) model and Verhulst model. This article concludes that GM(1,1) model produce better accuracy for prediction than Verhulst model. They also suggest a GM(1,N) model for forecasting a multi-variables in a Grey system.

Chen and Huang (2013) discussed the necessary and sufficient conditions to apply the GM(1,1) model. According to the predictive model of GM(1,1) they focused on the value of the parameter a, they concluded that if the value of a is zero then the model is meaningless and that is the substantial and adequate condition for the GM(1,1) predictive model.

Sifeng and Yang (2017) explained the terms and the definition of the grey forecasting models; they also present some practical implications on the grey theory.

Wang et al. (2010) and Mahdi and Norizan (2017) presented GM(1,1) model and discovered that this model may yield a large forecasting error, so they adopted the minimum sum square error to improve some initial conditions for the GM(1,1) model.

Tianxiang et al. (2012) established a Generalized Discrete Grey Model (GDGM) that can solve non-equidistance data, they conclude that (GDGM) perfectly simulate non-equidistance exponential series. Furthermore, they found out that (GDGM) fit decimal non-equidistance data not only integer non-equidistance data. In addition, (GDGM) is useful with highly speed growth and low speed growth geometric sequences.

Xiao and Peng (2011) introduced the generalized non-equidistance GM(1,1) model based on matrix analysis and using the generalized accumulated generating operation theory. They concluded that the new model produced accurate results with the raw sequence for generalized non-equidistance GM(1,1) model.

Changium et al. (2011) suggested the grey error of the GM(1,1) model to predict cultivated land in Yiyang city. This research indicated that grey error model yields a high prediction accuracy than the traditional GM(1,1) model.

Zhoa et al. (2016) considered the forecasting of annual electricity consumption by GM(1,1) model, they introduced a hybrid method by combining GM(1,1) and moth-flame optimization (MFO) algorithm with rolling mechanism to improve the prediction accuracy.

Mohammadi et al. (2011) considered the traffic accidents forecasting with unknown and known information and used the grey analysis system. They used the GM(1,1), Verhulst model and direct grey model DGM(2,1). They proved that GM(1,1) is more accurate that Verhulst model and DGM(2,1).

In insurance market, technical reserves are highly nonlinear, stochastic and highly non-stationary financial time series. Furthermore, traditional linear statistical techniques are un-useful to predict significant reserves. However, the grey system theory provides a helpful tool in doing so.

3. Grey System

3.1 GM(1,1) Grey Model

Assume $X^{(0)}$ is a sequence of n observations obtained from the actual data (AD), then $X^{(0)}$ can be expressed as:

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \ldots x^{(0)}(n)\right), n \geq 4 \tag{1}$$

where $x^{(0)}(i)$ is a non-negative real number.

Assume $X^{(1)}$ is the Accumulated Generating Operator (AGO) which is a sequence of the accumulated values of $X^{(0)}$, then $X^{(1)}$ can be expressed as:

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), \ldots x^{(1)}(n)\right), n \geq 4 \tag{2}$$

where

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, 3 \ldots n \tag{3}$$

let $Z^{(1)}$ be a sequence of the generated means (GM) of $x^{(0)}(i)$, where $Z^{(1)}$ can be expressed as:

$$Z^{(1)} = \left(z^{(1)}(1), z^{(1)}(2), \ldots z^{(1)}(n)\right) \tag{4}$$

$$Z^{(1)}(i) = \frac{1}{2} x^{(1)}(i) + \frac{1}{2} x^{(1)}(i-1), i = 2, 3, 4, \ldots n \tag{5}$$

Deng (1990) defined the least square estimate sequence of the grey difference equation of GM(1,1) as follows:

$$\beta = x^{(0)}(k) + \alpha z^{(1)} \tag{6}$$

The following differential equation (7) is used to whitening the difference equation (6):

$$\frac{dx^{(1)}(t)}{dt} = \alpha x^{(1)}(t) + \beta \tag{7}$$

The parameters (α, β) are estimated using the GM(1,1) as shown in equation (8).

$$[\alpha, \beta]^T = (M^T M)^{-1} M^T V \tag{8}$$

where V is a vector of $x^{(0)}(i)$, i =2,3,4,…,n

$$V = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \bullet \\ \bullet \\ \bullet \\ x^{(0)}(\mathrm{n}) \end{bmatrix} \tag{9}$$

and M is a data array of $-z^{(1)}(i)$, i =2,3,4,…,n.

$$M = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \bullet & \bullet \\ \bullet & \bullet \\ \bullet & \bullet \\ -z^{(1)}(n) & 1 \end{bmatrix} \tag{10}$$

According to equation (7), the solution of $x^{(1)}(t)$ at time i is expressed in equations (11), (12) and (13).

$$\hat{x}^{(1)}(i+1) = \left[x^{(0)}(1) - \frac{\beta}{\alpha} \right] e^{-\alpha i} + \frac{\beta}{\alpha} \tag{11}$$

$$\hat{x}^{(1)}(i+1) = \left[x^{(0)}(1) - \frac{\beta}{\alpha} \right] e^{-\alpha i}(1 - e^{\alpha}) \tag{12}$$

$$\hat{x}^{(0)}(i+h) = \left[x^{(0)}(1) - \frac{\beta}{\alpha} \right] e^{-\alpha(i+h-1)}(1 - e^{\alpha}) \tag{13}$$

Where $\hat{x}^{(1)}(i)$ is the predicted data (PD).

3.2 Testing the Accuracy of GM(1,1)

In this section we use three different tools (Residual Error (RE), Relative Percentage Error (RPE) and Posterior Error Ratio (PER)) that evaluate the accuracy of the prediction to decide whether the predicted values are reasonable or not (Deng, 1990). The equations of tests are:

Residual Error (RE) test:

$$\Delta^{(0)}(i) = \left| X^{(0)}(i) - \hat{X}^{(0)}(i) \right|, i = 1, 2, 3, \ldots, n \tag{14}$$

Relative Percentage Error (RPE) test:

$$\phi(i) = \frac{\Delta^{(0)}(i)}{X^{(0)}(i)} \times 100\%, i = 1, 2, 3, \ldots, n \tag{15}$$

Posterior Error Ratio (PER) test:

$$C = \frac{S_2}{S_1} = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(\Delta^{(0)}(i) - \overline{\Delta})^2}}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(X^{(0)}(i) - \overline{X})^2}} \tag{16}$$

Table 2 Posterior Error Ratio and Corresponding Precision

	High Accuracy	Good	Reasonable	Inaccurate
Posterior Error Ratio (PER) C	*C≤0.35*	*0.35<C≤0.50*	*0.50<C≤0.65*	*0.65< C≤0.80*

Source: J. Deng (1990), Grey System Theory, Huazhong University of Science and Technology Press.

Table 2 presents the Posterior Error Ratio (PER) where the lower the PER, the higher the accuracy of the prediction. Moreover, if PER is below 35% the model is highly significant, if PER is between 35% and 50% then the model is good with acceptable accuracy. Otherwise the model is not significant.

4. Modeling Estimations

The data adopted in this research is collected from a non-life Egyptian insurance market over the period from 2005/2006 to 2015/2016, this data describe aggregate three technical reserves (these are outstanding claims reserve, loss ratio fluctuations reserve and unearned premiums reserve) as time series data (AD). Accordingly, we apply the (AGO) in order to reduce the randomization of the (AD) see equations (2) and (3). Afterwards, we calculated the generated means (GM) equation (5). Then we used equation (13) to predict the future amount of reserve (PD). Moreover, we used three different tests to check whether the predicted results are reasonable or not these tests are (RE), (RPE) and (PER) according to equation (14), (15) and (16) respectively.

Table 3 presents the analysis of the grey system GM(1,1) for outstanding claims reserve where the estimated values of the grey model parameters are $\alpha = 0.03720903$ and $\beta = 4416913$.

Table 3 GM(1,1) for Outstanding Claims Reserve

Year	(AD)	(AGO)	(GM)	(PD)	(RE)	(RPE)	(PER)
(i)	$X^{(0)}(i)$	$X^{(1)}(i)$	$Z^{(1)}(i)$	$\hat{X}^{(0)}(i)$	$\Delta^{(0)}(i)$	$\phi(i)$	C
1	2583135	2583135	2583135	2583135	0	0%	0.28
2	3347467	5930602	4256869	4569076.57	1221610	36.49%	
3	5153205	11083807	8507205	4742290.04	410915	7.97%	
4	5190489	16274296	13679052	4922070.03	268419	5.17%	
5	5486311	21760607	19017452	5108665.47	377645.5	6.88%	
6	5718028	27478635	24619621	5302334.74	415693.3	7.27%	
7	5688413	33167048	30322842	5503346	185067	3.25%	
8	6031883	39198931	36182990	5711977.58	319905.4	5.30%	
9	5984323	45183254	42191093	5928518.38	55804.62	0.93%	
10	6054039	51237293	48210274	6153268.23	99229.23	1.64%	
11	5978670	57215963	54226628	6386538.34	407868.3	6.82%	

Source: Authors' Calculations.

Table 4 presents the analysis of the grey system GM(1,1) for loss ratio fluctuations reserve where the estimated values of the grey model parameters are $\alpha = 0.1057271$ and $\beta = 974499.5$.

Table 5 presents the analysis of the grey system GM(1,1) for unearned premiums reserve where the estimated values of the grey model parameters are $\alpha = 0.1094553$ and $\beta = 722052$

Table 4 GM(1,1) for Loss Ratio Fluctuations Reserve

Year	(AD)	(AGO)	(GM)	(PD)	(RE)	(RPE)	(PER)
(i)	$X^{(0)}(i)$	$X^{(1)}(i)$	$Z^{(1)}(i)$	$\hat{X}^{(0)}(i)$	$\Delta^{(0)}(i)$	$\phi(i)$	C
1	1130323	1130323	1130323	1130323	0	0%	0.22
2	1083111	2213434	1671879	1002398	80712.91	7.45%	
3	1519680	3733114	2973274	1114184	405496	26.68%	
4	1498050	5231164	4482139	1238436	259613.8	17.33%	
5	1548887	6780051	6005608	1376545	172342.3	11.13%	
6	1648482	8428533	7604292	1530055	118427.1	7.18%	
7	1829163	10257696	9343115	1700684	128478.7	7.02%	
8	2117913	12375609	11316653	1890342	227570.9	10.75%	
9	2457960	14833569	13604589	2101150	356809.8	14.52%	
10	2654169	17487738	16160654	2335467	318701.7	12.01%	
11	3137598	20625336	19056537	2595915	541682.8	17.26%	

Source: Authors' Calculations.

Table 5 GM(1,1) for Unearned Premiums Reserve

Year	(AD)	(AGO)	(GM)	(PD)	(RE)	(RPE)	(PER)
(i)	$X^{(0)}(i)$	$X^{(1)}(i)$	$Z^{(1)}(i)$	$\hat{X}^{(0)}(i)$	$\Delta^{(0)}(i)$	$\phi(i)$	C
1	870954	870954	870954	870954	0	0%	0.32
2	989206	1860160	1365557	738915.8	250290.2	25.30%	
3	1362077	3222237	2541199	824386.3	537690.7	39.48%	
4	1473889	4696126	3959182	919743.2	554145.8	37.60%	
5	1617696	6313822	5504974	1026130	591565.9	36.57%	
6	1841748	8155570	7234696	1144823	696925.2	37.84%	
7	159755	8315325	8235448	1277245	1117490	699.50%	
8	199436	8514761	8415043	1424984	1225548	614.51%	
9	2327612	10842373	9678567	1589812	737800	31.70%	
10	2550817	13393190	12117782	1773706	777111.1	30.47%	
11	2779986	16173176	14783183	1978871	801115.2	28.82%	

Source: Authors' Calculations.

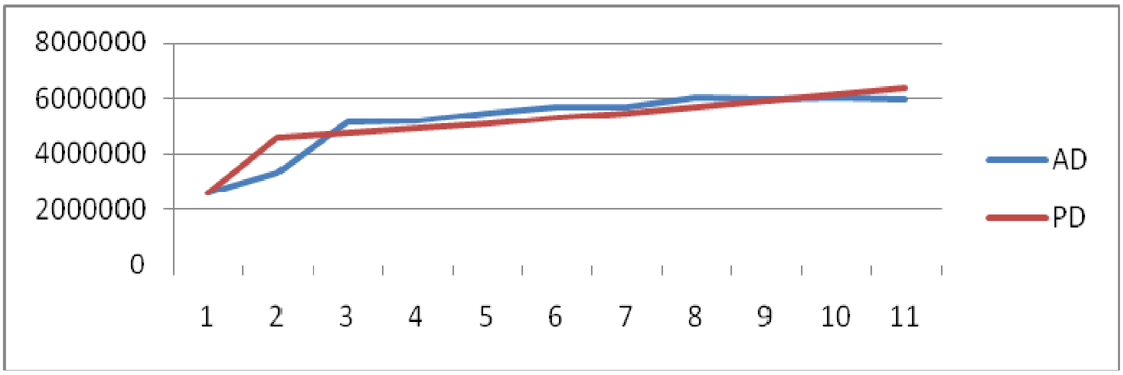

Figure 2 GM(1,1) for Outstanding Claims Reserve

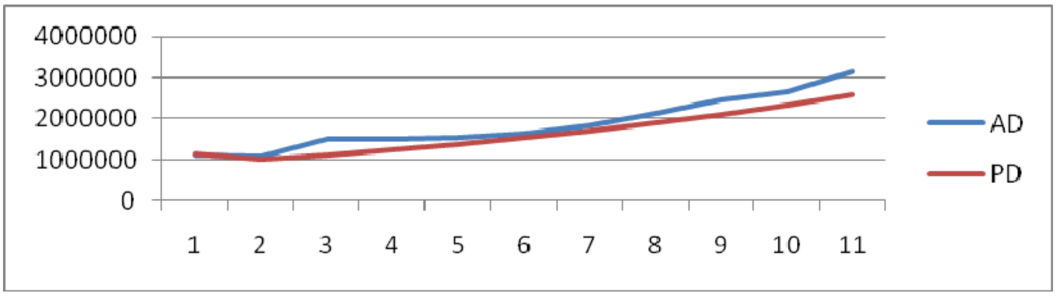

Figure 3 GM(1,1) for Loss Ratio Fluctuations Reserve

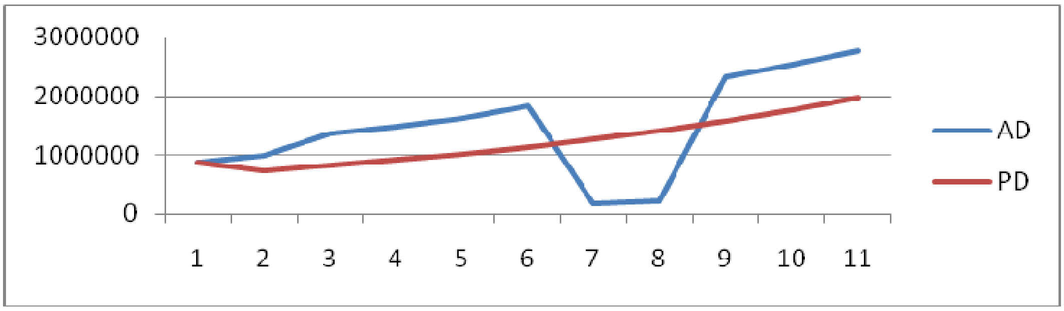

Figure 4 GM(1,1) for Unearned Premiums Reserve

From Tables 3-5 we can conclude that the values of posterior error ratios (0.28, 0.22 and 0.32) are below 0.35, so the results are highly accurate for the three technical reserves. Furthermore, Figures 2-4 present the accuracy of fitting between the actual data and predicted data from the GM(1,1) models for the three types of technical reserves. In addition, the predicted amounts of Outstanding Claims Reserve and Loss Ratio Fluctuations Reserve are highly accurate than the unearned premiums reserve according to results obtained from Posterior Error Ratio test.

5. Conclusion

This research demonstrates a new methodology used for incomplete information data so-called grey system. We introduced a GM(1,1) model to predict future amounts of the three technical reserves Outstanding Claims Reserve, Loss Ratio Fluctuations Reserve and Unearned Premiums Reserve used in the non-life Egyptian insurance market over the period 2005/2006 to 2015/2016. In this paper we applied GM(1,1) by the Accumulated Generating Operator (AGO) that removes the randomization in the actual data then find the mean generated values to forecast the reserves. In the same vein, no large amount of data is required to predict future reserve when applying GM(1,1) model. Furthermore, we applied several tests to check the accuracy of the predicted values. On the other hand, if the original data have the same trend, the PER is small, and the error increase as the data have opposite direction. Finally, we estimated the values of reserves over the period 2016/2017 to 2029/2030 are shown in Table 6.

Table 6 Predicted Reserves for Non-Life Insurance Market

in thousands

Year	Outstanding Claims Reserve	Loss Ratio Fluctuations Reserve	Unearned Premiums Reserve
2016/2017	6628652	2885408	2207767
2017/2018	6879944	3207184	2463140
2018/2019	7140762	3564844	2748052
2019/2020	7411468	3962391	3065920
2020/2021	7692436	4404270	3420555
2021/2022	7984056	4895428	3816211
2022/2023	8286731	5441359	4257633
2023/2024	8600881	6048171	4750115
2024/2025	8926940	6722654	5299561
2025/2026	9265360	7472354	5912563
2026/2027	9616609	8305660	6596470
2027/2028	9981174	9231895	7359485
2028/2029	10359560	10261422	8210758
2029/2030	10752291	11405760	9160498

Source: authors' calculations.

References

Chang. H and W. Chen, (2013). "Grey forecasting models for social media adoptions of transnational enterprises", in: *2nd International Symposium on Computer, Communication, Control and Automation* (3CA 2013).

Changjun. H, T. Zhengqia, Z. Qingshana and C. Yuanzhia, (2011). "Application of improved error GM (1, 1) model on predicting of cultivated land in Yiyang", *Energy Procedia*, Vol. 5, pp. 1172-1176.

Chen C. and S. Huang (2013). "The necessary and sufficient condition for GM(1,1) grey prediction model", *Applied Mathematics and Computation*, Vol. 219, pp. 6152-6162.

Deng J. (1990). *Grey System Theory*, Huazhong University of Science and Technology Press.

Kayacan E., U. Barus and K. Okyay (2010). "Grey system theory-based models in time series prediction", *Expert Systems with Applications*, Vol. 37, pp, 1784-1789.

Lin Y. and S. Liu (2004). "A historical introduction to grey systems theory". In *Proceedings of IEEE International Conference on Systems, Man and Cybernetics*, The Netherlands, Vol. 1, pp. 2403-2408.

Madhi M. and M. Norizan, (2017). "An initial condition optimization approach for improving the prediction precision of a GM(1,1) model", *Mathematical Computational Applications*, Vol. 22, No. 21, pp.1-8.

Mohammadi A. and Z. Sara (2011). "Appling grey forecasting method to forecast the portfolio's rate return in stock market of Iran", *Australian Journal of Business and Management Research*, Vol. 1, No. 7, pp. 1-16.

Tianxiang Y., L. Sifeng and X. Naiming (2009). "On the properties of small sample of GM(1,1) model", *Applied Mathematical Modelling*, Vol. 33, pp. 1894-1903.

Tianxiang Y., F. Jeffery and G. Zaiwu (2012)." Generalized discrete GM (1,1) model", Grey Systems: Theory and application, Vol. 2, No.1, pp. 4-12.

Wang Y, D. Yaoguo, L. Yueqing, and L. Sifeng. (2010), "An approach to increase prediction precision of GM(1,1) model based on optimization of the initial condition", *Expert Systems with Applications*, Vol. 37, pp. 5640-5644.

Xiao X. and K. Peng (2011). "Research on generalized non-equidistance GM(1,1) model based on matrix analysis", *Grey Systems: Theory and Application*, Vol. 1, No. 1, pp.87-96.

Zhoa M., Z. Chen, Y. Luling, L. Guangming, H. Juwen, Z. Haochen and H. Wenzhi (2016). "Prediction and analysis of WEEE in China based on the gray model", *Procedia Environmental Sciences*, Vol. 31, pp. 925-934.