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Study of Rayleigh Benard Convection by Lattice Boltzman Method

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Abstract: The lattice Boltzmann method (LBM and its schema D2Q9 and D2Q5) is applied for the numerical study of hydrodynamic and thermal instability during confinement of air or water in rectangular cavities with aspect ratios A = L/H varying in the range (0.5, 1, 2, 3, 5), The lattice Boltzmann method (LBM) was used to discretize the steady-state and transient flow equations.

The cavity is differentially heated on the horizontal walls. The study is carried out for a Prandtl number of Pr = 0.71 (air), and for Pr = 7.01 (water). The Rayleigh number values change between (6×10^3)-(130×10^3). We have discussed and analyzed the influence of Rayleigh number on the dynamic and thermal fields as well as on the average Nusselt number of the flow. In addition, critical frequencies dominating the oscillatory flow have been determined.

The results show the frequency dependence with the aspect ratio and the critical Rayleigh number. Although our study is two-dimensional, it is expected that the results of a three-dimensional numerical simulation, focusing on the possible obstacles or structure in the flow, as well as the possible presence of the instabilities due to the double diffusion, confirm the qualitative results obtained in this work.

Key words: Lattice Boltzmann method, Rayleigh-Bénard convection, rectangular cavity, instability

1. Introduction

The presence of natural convection phenomena in many industrial systems has increased the interest of the scientific community in this branch of aerothermics. The state of knowledge shows an important need to understand the thermal environment of these systems in order to arrive at a correct prediction of the circulation of fluids and heat transfer inside increasingly complex geometries.

In the shorter term, current studies tend to respond directly or indirectly to energy saving issues. The challenge becomes double when these natural convection phenomena take place near composite materials whose mechanical performance is strongly related to the thermal environment in which they are placed. This type of flow is encountered in the soft belly, the wing boxes or the air intake compartments of the aircraft. Faced with this need, Airbus, Limsi and Onera met around the MAEVA II project and the thesis of Marie-Laure Toulouse [1] and Ludovic Perrin [2], part of this unifying project, was particularly interested studying the cooling of air conditioning packs placed in the soft belly. In this case, the natural convection flows are generated by the presence of a heating obstacle inside a cavity. This work has thus made it possible to characterize the dynamic and thermal behavior of the flows in various configurations and thus to provide a database serving as a reference for the validation of research or industrial calculation codes with the future goal of optimizing these ventilation circuits. avoiding the too expensive successive modifications tested in flight.

Thus, through several numerical simulations using the lattice method of Lattice Boltzmann, we have studied the effect of confinement on the topology of the flow generated by heating the wall less than a constant temperature in a laminar regime, which represents the case of an industrial flow such as that encountered within a flat solar collector without obstacles, or in the

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convective movement manifesting in the atmospheric layer of the terrestrial globe, as well as in the flows generated by the thermo-halin movement in the oceans.

1.1 Rayleigh–Benard Convection

When a thin layer of fluid is heated from below or cooled from above, the upward heat transfer can be achieved by conduction, that is, in the absence of motion on the part of the fluid because its viscosity cannot be overcome by the buoyancy forces. However, this can occur only in the rather extreme case of a very thin and very viscous fluid. Lord Rayleigh studied this problem and obtained a straightforward criterion. For a horizontal fluid layer of thickness H in contact with a lower temperature T along its top surface and with a higher temperature T + ΔT along its bottom (Fig. 1), the threshold separating the quiet from the convective regime is expressed in terms of the Rayleigh number:

$$R_a = \frac{g\alpha\Delta T H^3}{\upsilon\kappa} = G_{rL}.P_r$$

in which α is the thermal expansion coefficient, ν the kinematic viscosity, and κ is the thermal diffusivity. Values for water and air at ambient temperatures and pressures are shown in Table 1.

No convective motion occurs at low values of the Rayleigh number, Ra < 1708, and the fluid transports heat exclusively by molecular heat diffusion. At Rayleigh numbers slightly exceeding the critical value of 1708, convection occurs in alternating patterns of upward and downward motion.

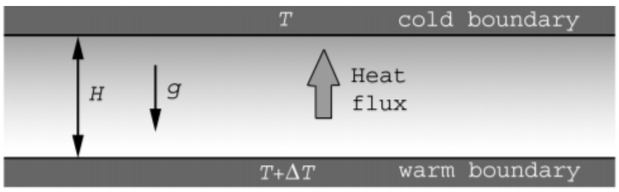

Fig. 1 The convection problem studied by Lord Rayleigh and simulated in the laboratory by Henri Benard [4].

Table 1 Values of physical properties of fresh water (at 10°C) and air (at 15°C) at atmospheric pressure.

Physical property	Notation	Water	Air
Thermal expansion coefficient	a	2.6×10^{-4}/°C	3.5×10^{-3}/°C
Kinematic viscosity	v	$1.3\times10^{-6}\ m^2/s$	$1.5\times10^{-5}\ m^2/s$
Thermal diffusivity	k	$1.4\times10^{-7}\ m^2/s$	$2.2\times10^{-5}\ m^2/s$

1.2 Lattice Boltzmann Model

The Lattice Boltzmann method [1-3] was originated from Ludwig Boltzmann's kinetic theory of gases. The fundamental idea is that gases/fluids can be imagined as consisting of a large number of small particles moving with random motions. The exchange of momentum and energy is achieved through particle streaming and billiard-like particle collision. This process can be modelled by the Boltzmann transport equation, which is

$$\frac{\partial f}{\partial t} + \vec{u}\nabla f = \Omega$$

where f($\vec{x}$, t) is the particle distribution function, $\vec{u}$ is the particle velocity, and **Ω** is the collision operator. The LBM simplifies Boltzmann's original idea of gas dynamics by reducing the number of particles and confining them to the nodes of a lattice. For a two dimensional model, a particle is restricted to stream in a possible of 9 directions, including the one staying at rest. These velocities are referred to as the microscopic velocities and denoted by $\vec{e_i}$, where i = 0, . . . , 8. This model is commonly known as the D2Q9 model as it is two dimensional and involves 9 velocity vectors. Fig. 2 shows a typical lattice node of D2Q9 model with 9 velocities $\vec{e_i}$ defined by

$$\vec{e_i} = \begin{cases} (0,0) & i = 0 \\ (1,0),(0,1),(-1,0),(0,-1) & i = 1,2,3,4 \\ (1,1),(-1,1),(-1,-1),(1,-1) & i = 5,6,7,8 \end{cases}$$

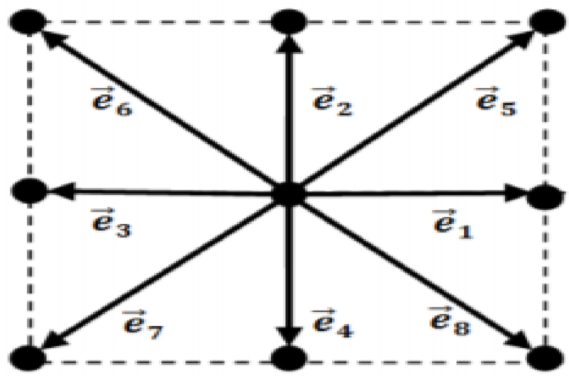

Fig. 2 Illustration of a lattice node of the D2Q9 model [5].

For each particle on the lattice, we associate a discrete probability distribution function $f_i(\vec{x}, \vec{e_i}, t)$ or simply $f_i(\vec{x}, t)$, $i = 0 \ldots 8$, which describes the probability of streaming in one particular direction [3]

2. Results and Discussion

(1) Instability of R-B, in single air cavities Pr = 0.71:

Aspect ratio = 2 and $Ra_{cr} = 72.99\times10^3$.

The isotherms of Fig. 6 show that the thermal field spread uniformly distributed, where there is dissipation of heat, the temperature distribution is almost constant.

By inspecting the lines of the contours of the magnitude of the velocity (Fig. 7) show a deviation of the velocity vectors around, the edges of the middle of a hot wall of heat, Near the adiabatic bottom wall the modules of the vectors are very weak, therefore a stagnation of the fluid. The fluid layers in this region receive heat from the convective circulation of the cells $V_{max} = 0.019$.

The spectral analysis of the temporal evolutions of the different signals by means of the Fast Fourier Transform (FFT) gives us the graph as shown in (Fig. 7). The latter shows the predominant frequency of the flow, which in this case is equal to $fr_{cr} = 0.0669$ Hz.

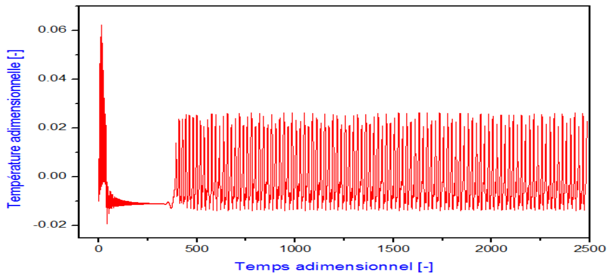

Fig. 3 The transition of the temperature near the hot wall towards the oscillatory regime.

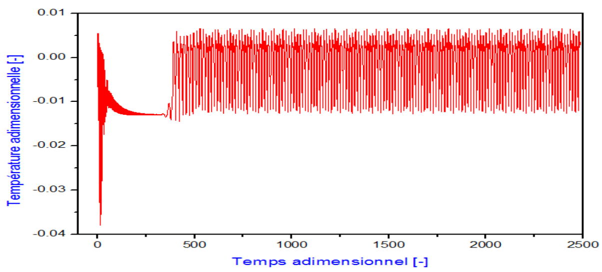

Fig. 4 The transition of the temperature near the hot wall towards the oscillatory regime.

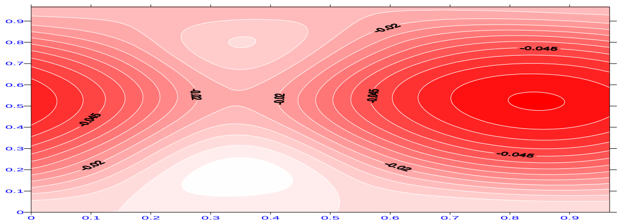

Fig. 5 the current lines having two symmetrical recirculation zones, one anti-clockwise (left part of the enclosure) and the other clockwise (right part).

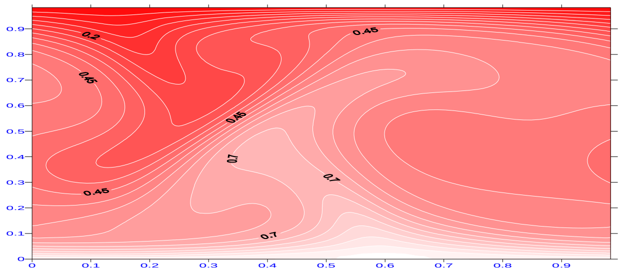

Fig. 6 Isothermal temperatures in the aspect ratio cavity 2.

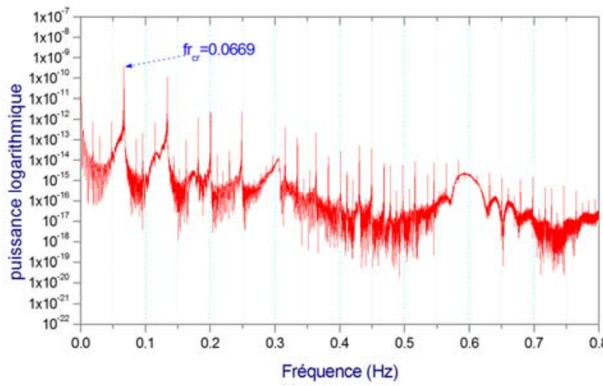

Fig. 7 Spectrum of the energy of the speed U.

(2) Instability of R-B, in single water cavities Pr = 7.01:

Aspect ratio = 2. and $Ra_{cr} = 24\times10^3$

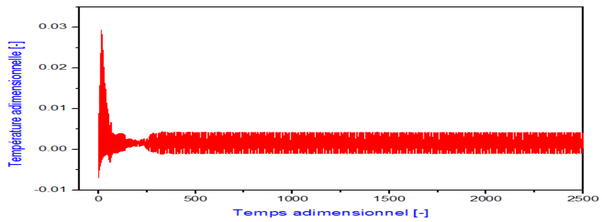

Fig. 8 Temporal evolution of the adimensional temperature within the cavity of RA = 2. in the case of water.

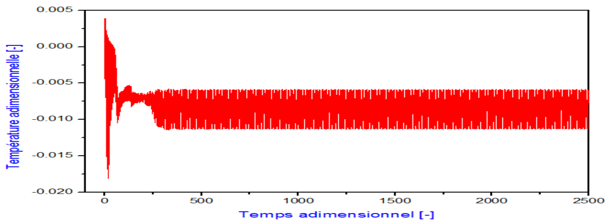

Fig. 9 Temporal evolution of the adimensional velocity u within the cavity of RA = 2. And Pr = 7.01 for the second sampling point.

Figs. 10 and 11 show that the deformation of the isothermal lines is caused by the appearance of two recirculation zones that are almost similar (symmetrical) but in opposite directions, hence a recirculation mass flow rate and a convective transport important.

Fig. 12 shows a deviation of the velocity vectors around, the edges and in the middle of the hot wall, close to the adiabatic bottom wall the modules of the vectors are very weak, thus a stagnation of the fluid. The fluid layers of this region receive heat from the convective circulation of cells where V_{max} = 0.38.

Fig. 13 shows a deviation of the velocity vectors around, the edges and in the middle of the hot wall, close to the adiabatic bottom wall the modules of the

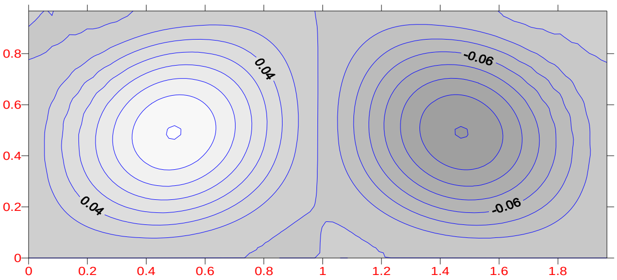

Fig. 10 Current lines for the R-B convection case considered Ψ_{max} = 0.137.

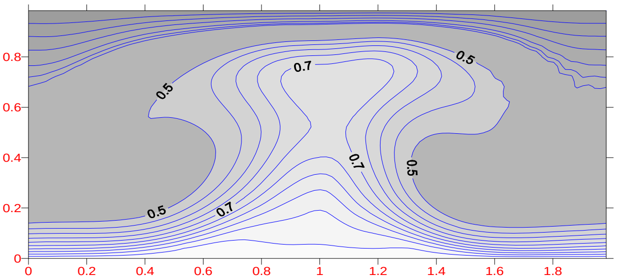

Fig. 11 The isothermal lines for the convection case considered.

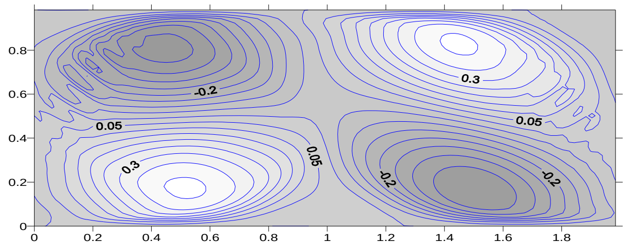

Fig. 12 Iso-values of the adimensional speed for the case of RA = 2 And Pr = 7.01.

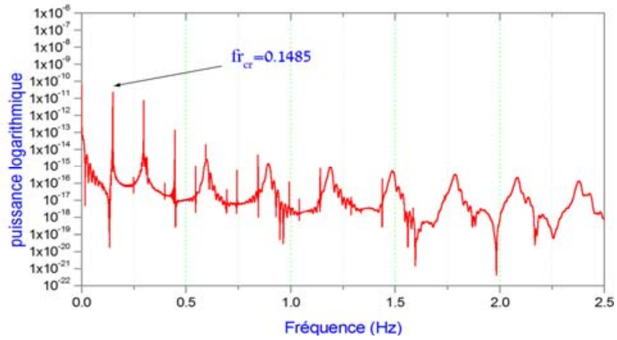

Fig. 13 Spectrum of the energy of the speed U.

vectors are very weak, thus a stagnation of the fluid. The fluid layers of this region receive heat from the convective circulation of cells where V_{max}= 0.38.

(3) Limits and stability diagram:

Conclude our results with a final summary table (Table 2), showing the different critical Rayleigh number values obtained from our numerical simulations, using the Boltzmann Method (Boltzmann Method). The translation of these numerical values into graphs gives us a stability diagram for each of the two substances studied numerically, namely air and water (Fig. 14).

The regions below the curve connecting the different points of the critical Rayleigh number are the region where the flow will remain stable, while the region above the curve considered represents the region of instabilities (or transition to turbulence). The values of different critical frequencies corresponding to the different critical Rayleigh numbers are shown on the stability diagram.

Table 2 Critical values of the Rayleigh number for air and water.

Aspect ratio	0.5	1	2	3	5
Ra_{cr}(air)×10^3	130	99	72.99	42.4	14.3
Ra_{cr}(water)×10^3	109.69	30.6	24	17.5	6

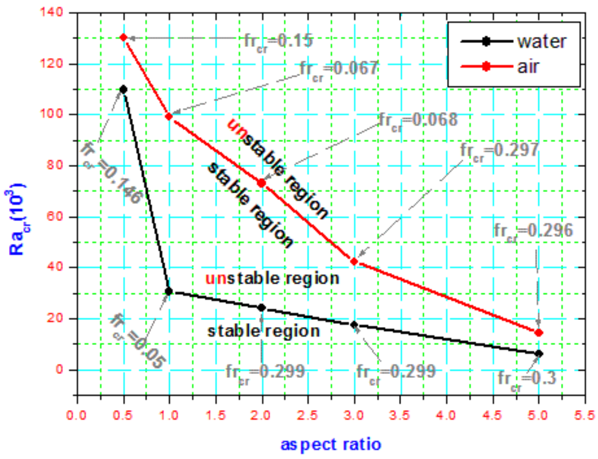

Fig. 14 Ra_{cr} **stability diagram vs cavity aspect ratio.**

3. Conclusion

Numerical study of the hydrodynamic and thermal instability during the confinement of air or water in rectangular cavities with aspect ratios A = L/H variables (0.5, 1, 2, 3, 5), was made. The lattice Boltzmann method (LBM) was used to discretize the steady-state and transient flow equations.

Validation of the calculation code was done with experimental work found in the literature of stationary natural convection in a differentially heated chamber. A good agreement was obtained between our simulations and the experimental data.

The evolution of flow and heat transfer during instability has been examined, the results of which show that the structure of the current function and the isotherms gradually deforms to a structure with oscillatory character (periodically) during instability.

Using the Fast Fourier Transform (FFT), the predominant frequencies of the oscillations were determined in all simulated cases.

The results show the frequency dependence with the aspect ratio and the critical Rayleigh number. Although our study is two-dimensional, it is expected that the results of a three-dimensional numerical simulation, with a concentration on the possible obstacles or structure in the flow, as well as the possible presence of the instabilities due to the double diffusion, confirm the qualitative results obtained in this work.

References

[1] L. Toulouse. Analysis and characterization of natural convection and mixed convection in confined Enclosures, Ph.D. thesis, National school of Aeronautics and Space, 2004.

[2] C. Tropea, A. L. Yarin and J. F. Foss, *Handbook of Experimental Fluid Mechanics*, Springer, 2007.

[3] Z. Guo, B. Shi and N. Wang, Lattice BGK Model for Incompressible Navier-Stokes Equation, *J. Comput. Phys.* 165 (2000) 288-306.

[4] E. L. Koschmieder, *Benard Cells and Taylor Vortices*, Cambridge University Press, 1993.

[5] R. Begum and M. A. Basit, Lattice Boltzmann method and its applications to fluid flow problems, *Euro. J. Sci. Research* 22 (2008) 216-231.