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Early Trace of the Current Hypothesis Test: Graunt (1662), Arbuthnott (1710) and Sagravesande (1712)

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Abstract: The hypothesis test, as today we understand, are born in the first decades of the twentieth century, with the work of Fisher, on the one hand, and Neyman and Pearson, on the other, which were developed from antagonistic philosophical positions, which they have been maintained over time. But for two centuries before there were attempts to endorse assumptions made from reality with the available data. Thus, from the accounts mortality weekly published the City of London, John Graunt (1662) he believed prove that "the unhealthiest years are less fertile" (exists in time a negative correlation between the annual number of burials and the birth), while John Arbuthnott (paper read in 1711 and published in 1712), gave "Arguments for Divine Providence" and sGravesande a mathematical proof that "God directs what happens in the world", in both cases, from the regular rate of births of men and women. Although the theoretical foundations of these works are of dubious rigor, proposals are ingenious and nearby, in some respects, what we do today when we perform a hypothesis test. This paper analyzes in detail the early contributions of these three authors by looking at what it looks like now and pointing out the mistakes made by them.

Key words: Graunt, Arbuthnott, sGravesande, hypothesis test, regularity of births among women men
JEL codes: C000, C120

1. Introduction

John Arbuthnot (1667-1735) was a member of the Royal Society and the Royal College of Physicians. Brief reviews of his life and work can be found in Eisenhart and Birnbaum (1967). In his first professional stage, when he moved to London from his native Scotland, he devoted himself to teaching mathematics. From that period dates his translation into English of the treatise De ludo aleae de Huygens published in Latin in 1657, and which appeared in English with the title On the Laws of Chance (1692). This edition was not only a translation of Huygens' work. Arbuthnott added his own solutions to the 5 problems proposed by Huygens at the end of the treaty, and some own section on dice and card games of chance. After studying medicine, getting his degree in 1696, and becoming a man with extensive scientific knowledge, witty writer, and satirist. For a time he was the Queen's doctor. In a letter he says that he is the favorite among his doctors in common. He was a colleague close to Jonathan Swift, and the creator of the per-sonage who became famous at the time with the name of John Bull. In 1700 he published an essay on the usefulness of learning mathematics.

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In 1662 John Graunt had published his Natural and Political Observations which included a table on the number of baptized each year in London, distinguished by sex, and used as an approximation to the number of children born each year in this city. In chapter VIII, entitled "On the difference between men and women" Graunt writes that in London there are more men than women, and to support this hypothesis, the author introduces different arguments, and some of them supported by Table 1 in which we are particularly interested in the number of baptized each year, between 1629 and 1662. Among other phrases, we read in Graunt that "there are 14 men born for every 13 women".

Table 1 Graunt's Table on the Number of Burials and Baptisms in London, by Sex, between 1629 and 1660

Ann.	Burials		Baptisms	
Dom.	Males	Females	Males	Females
1629	4668	4103	5218	4683
1630	5660	4894	4858	4457
1631	4549	4013	4422	4102
1632	4932	4603	4994	4590
1633	4369	4023	5158	4839
1634	5676	5224	5035	4820
1635	5548	5103	5106	4928
1636	12377	10982	4917	4605
	47779	43945	39708	37024
1637	6392	5371	4703	4457
1638	7168	6456	5359	4952
1639	5351	4511	5366	4784
1640	6761	6010	5518	5332
Total	73451	65293	60664	56549
1641	6872	6270	5470	5200
1642	7049	6224	5460	4910
1643	6842	6360	4793	4617
1644	5659	5274	4107	3997
1645	6014	5465	4047	3919
1646	6683	6097	3768	3395
1647	7313	6746	3796	3536
1648	5145	4749	3363	3181
	51577	47185	34804	32755
1649	5454	5112	3079	2746
1650	4548	4216	2890	2722
1651	5680	5147	3231	2840
1652	6543	6026	3220	2908
1653	5416	4671	3196	2959
1654	6972	6275	3441	3179
1655	6027	5330	3655	3349
1656	7365	6556	3668	3382
	44005	41333	26380	24085
1657	6578	5856	3396	3289
1658	7936	7057	3157	3013
1659	7451	7305	9209	2781
1660	7960	7158	3724	3247
	29925	27376	13186	12330
Total	198952	181187	135034	126759

The Philosophical Transactions of the Royal Society dated October-December 1710, contains a contribution entitled "An Argument for Divine Providence, taken from the constant Regularity observed in the Births of both Sexes". The work was written by John Arbuthnot and is the reference on which we rely to elaborate this document presented here. Figure 1 collects the title with which this fragment appears object of our interest in this work.

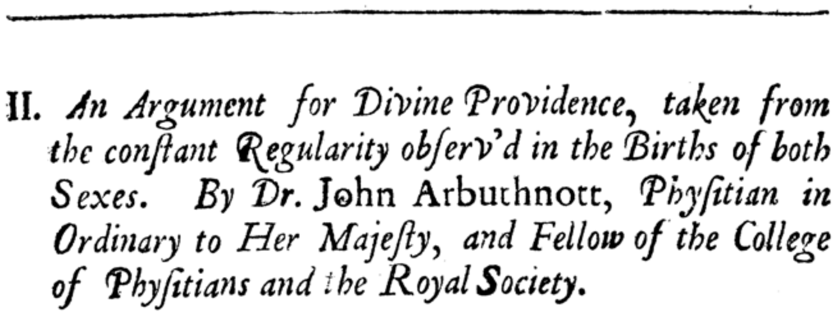
II. *An Argument for Divine Providence, taken from the conſtant Regularity obſerv'd in the Births of both Sexes. By Dr.* John Arbuthnott, *Phyſitian in Ordinary to Her Majeſty, and Fellow of the College of Phyſitians and the Royal Society.*

Figure 1 Header of Arbuthnott's Work

The fundamental thesis of Arbuthnot in this work is the existence of a being di-vino, a superior being who, under his designs, establishes birth quotas of men and women, which are produced year after year with some numbers, for both sexes, which remain in an almost constant relationship. This is how the author presents his idea:

> Among the innumerable traces of Divine Providence that are found in the works of nature, there is a very remarkable one that must be observed in the exact balance that is maintained between the number of men and women; for by this it is provided that the species can neither fail nor disappear, since every man can have his wife, and of a proportional age. This equality of men and women is not the effect of chance, but of Divine Providence, working for a good purpose, as I show.

The basis on which he based his argument was the previous table of Graunt, but extended until the year 1710. At no time, Arbuthnott makes reference to Graunt. He is assuming that the number of baptisms of each year is a good estimate of the number of births, which was not always the case throughout the years analyzed[1]. The totals do not appear in the table, as in the case of Graunt, and in the text there is no figure between the number of births of men and women, such as that of 14 to 13 of Graunt himself. Next, we present the Arbuthnott table.

During the 82 years that Arbuthnott analyzed the number of children baptized, it was always, each year, higher than that of girls, although not much higher. The highest ratio (number of children)/(number of girls) was 1661, with a value of 1.1561, which equals proportions (boys, girls) equal (53.6%, 46.4%). The lowest one was found in the year 1703, with a ratio (number of children) (number of girls) equal to 1.011, or a proportions (50.3%, 49.7%). Of course, anyone who observes both Graunt's and Arbuthnott's data would conclude that, if christenings represent births well, the probability of being born a child in that period is greater than the probability of being born a girl. Our author introduces the following argument justifying the difference in favor of the male sex:

> …in almost a constant proportion for each of the 82 years. To judge of the wisdom of the Contrivance, we must observe that the external Accidents to which Males are subject (who must seek their Food with danger) do make a great havock of them, and that this loss exceeds far that of the other Sex, occasioned by Diseases incident to is, as Experience convinces us. To repair that Loss, provident Mature, by the Disposal of its wise Creator, brings forth more Males than Females, and that in almost a constant proportion. This appears in the attached tables, which contain 82-year observations of the London births.

[1] In the period 1639-1648, due to religious dissension, there was a significant decrease in the number of baptized persons, with

Table 2 Arbuthnott's Data on the Annual Number of Men and Women Baptized in London, from 1629 to 1710

Baptisms				Baptisms		
Year	Males	Females		Year	Males	Females
1629	5218	4683		1648	3363	3181
30	4858	4457		49	3079	2746
31	4422	4102		50	2890	2722
32	4994	4590		51	3231	2840
33	5158	4839		52	3220	2908
34	5035	4820		53	3196	2959
35	5106	4928		54	3441	3179
36	4917	4605		55	3655	3349
37	4703	4457		56	3668	3382
38	5359	4952		57	3396	3289
39	5366	4784		58	3157	3013
40	5518	5332		59	3209	2781
41	5470	5200		60	3724	3247
42	5460	4910		61	4748	4107
43	4793	4617		62	5216	4803
44	4107	3997		63	5411	4881
45	4047	3919		64	6041	5681
46	3768	3395		65	5114	4858
47	3796	3536		66	4678	4319
Baptisms				Baptisms		
Year	Males	Females		Year	Males	Females
1667	5616	5322		1689	7604	7167
68	6073	5560		90	7909	7302
69	6506	5829		91	7662	7392
70	6278	5719		92	7602	7316
71	6449	6061		93	7676	7483
72	6443	6120		94	6985	6647
73	6073	5822		95	7263	6713
74	6113	5738		96	7632	7229
75	6058	5717		97	8062	7767
76	6552	5847		98	8426	7626
77	6423	6203		99	7911	7452
78	6568	6033		1700	7578	7061
79	6247	6041		1701	8102	7514
80	6548	6299		1702	8031	7656
81	6822	6533		1703	7765	7683
82	6909	6744		1704	6113	5738
83	7577	7158		1705	8366	7779
84	7575	7127		1706	7952	7417
85	7484	7246		1707	8379	7687
86	7575	7119		1708	8239	7623
87	7737	7214		1709	7840	7680
88	7487	7101		1710	7640	7288

From this paragraph, Arbuthnott introduces a probabilistic type demonstration of the previous statement, "the probability of being born a child is greater than that of being born a girl". That is the part that surprises us about this text, and to which we dedicate our research from our humble situation of people who start studying in these fields. Of course, important researchers of the history of statistics have entered into the analysis of this work. The work of Bellhouse (1989), Hacking (1991, although the original version in English was published in 1961), Hald

respect to the number of children born (Camúñez & Basulto, 2009, p. 59).

(1990) and Shoesmith (1987) has called our attention.

The work of Arbuthnot was read in the meeting of the Royal Society of April 19, 1711 (Shoesmith, 1987), although it was printed in the Philosophical Transactions, in the number corresponding to October-December 1710, which was published in 1712.

We complete this introduction commenting that this fact of the formulation of hypotheses based on statistical observations, as Arbuthnott does, Graunt had already done in his 1662 text. Specifically, Graunt states this hypothesis: "The unhealthy years are the less fertile", that is, Graunt maintained the existence of an inverse relationship between the number of deaths in each year and the number born in that same year. For this, he looks at some of the years of his historical series in which, when a local maximum occurred, in the series of deaths, at the same time, there was a local minimum in the number of baptized, and vice versa.

2. The Arbuthnot Statistical Demonstration

The author compares each birth of a child or, rather, each baptism with the release of a die that has two faces that he calls *M* and *F*. Then if there are *n* births, we can carry out the binomial development of $(M+F)^n$ and the coefficients of this development will give us the chances of the number of boys and girls that will be born. Thus, writes Arbuthnott, in four such dice we have $M^4+4M^3F+6M^2F^2+4MF^3+F^4$, that is, a chance for quadruple *M*, one for *F* quadruple, four for a triple *M* and a simple *F*, four for a simple *M* and a triple *F*, and six for a double *M* and double *F*. And so, in a pair of dice, only the central term of this development gives us the chances that the number of children matches that of girls. In all other terms, either the number of children is greater than that of girls, or vice versa. Therefore, if someone commits to get, by launching an even number of dice, that they leave as many *M* as *F*, he will have all the terms of the development $M^n + \frac{n}{1}\times M^{n-1}F + \frac{n}{1}\times\frac{n-1}{2}\times M^{n-2}F^2 + \frac{n}{1}\times\frac{n-1}{2}\times\frac{n-2}{3}\times M^{n-3}F^3 + \cdots$ against him, except the central term. And he adds, his chance (his probability) is how the coefficient of the middle term is to the power of 2 raised to an exponent equal to the number of dice: thus, with two dice his chance is $\frac{2}{4}$ or $\frac{1}{2}$, with three dice, $\frac{6}{16}$ or $\frac{3}{8}$, with six dice, $\frac{20}{64}$ or $\frac{5}{16}$, with eight, $\frac{70}{256}$ o $\frac{35}{128}$, & so on.

Then Arbuthnott informs us about how to construct the coefficient of the middle term of any binomial development of the type we are considering: $\frac{n}{1}\times\frac{n-1}{2}\times\frac{n-2}{3}$ & so on, until the number of terms is equal to $\frac{1}{2}n$. For example, the coefficient of the middle term of the tenth power is $\frac{10}{1}\times\frac{9}{2}\times\frac{8}{3}\times\frac{7}{4}\times\frac{6}{5}=252$. The tenth power of 2 is 1024. If then one tries to throw with ten dice in a pitch an equal number of *Ms* and *Fs*, he has 252 chances over 1024 to get it, this is his chance is $\frac{252}{1024}$ or $\frac{63}{256}$, which is less than $\frac{1}{4}$. The generalization to very large numbers is easy with the help of logarithms, Arbuthnott informs us, although he adds that this is not his goal. Of course, it is clear to the author that the fate of those who bet that the number of *M* will be equal to that of *F* by throwing a very large number will be very small. Therefore, for any age, the probability of birth of the same number of boys as of girls is very small (tends to zero when *n* tends to infinity, writes Hald (1990)).

Now, the author clarifies, this is not a mathematical question, but a physical one. The idea of equality in the number of births of boys and girls should not be understood in a strict sense and, therefore, when calculating this chance, the terms close to the center should be considered, and when considering them the chance turns will go in one direction or the opposite. But "it is very unlikely" that it will go as far as reaching the extremes of this development. If the chances are those that govern, that is, if the probability of being born male is equal to $\frac{1}{2}$, then in a given year, the probability of more male births than females is less than or equal to $\frac{1}{2}$.

Then he refers to the data in the table (data corresponding to 82 years) and writes, "Now, to reduce the All to a calculation, I propose this

Problem. A lays against B, that every year there shall be born more Males than Females: To find A's Lot, or the Value of his Expectation.

Since the probability of more boys than girls is less than or equal to $\frac{1}{2}$, to solve the problem Arbuthnott assumes equality and, thus, instead of the exact value calculates a higher level than the fate of A. It follows that the probability of more boys than girls each of those 82 years is $\left(\frac{1}{2}\right)^{82}$, which is equal to 1 divided by 4.836×10^{24}, very small probability. If the probability of being born male was $p=\frac{1}{2}$, Arbuthnott is showing us that for any number of births $P\left[M>F \middle| p=\frac{1}{2}\right]\leq\frac{1}{2}$. Then, if we wanted to contrast the hypothesis $P[M>F]\leq\frac{1}{2}$, as opposed to the alternative $P[M>F]>\frac{1}{2}$, for the annual number of births, the previous result supports the null hypothesis. However, the observation of each of the 82 years would lead us to support the alternative hypothesis.

Arbuthnott reinforces his argument with what follows:

> But if A wager with B, not only that the Number of Males shall exceed that of Females, every Year, but that this Excess shall happen in a constant Proportion, and the Difference lie within fix'd limits; and this not only for 82 Years, but for Ages of Ages, and not only at London, but all over the World; which it is highly probable is the Fact, and designed that every Male may have a Female of the same Country and suitable Age; then A's Chance will be near an infinitely small Quantity, at least less than any assignable fraction. From whence it flows, that it is Art, not Chance, that governs.

Arbuthnott ends his text with an apostille against polygamy, in the style that Graunt had already introduced in his Observations:

> From hence it follows, that Polygamy is contrary to the Law of Nature and Justice, and to the Propagation of the Human Race; for where Males and Females are in equal number, if one Man take Twenty Wives, Nineteen Men must live in Celibacy, which is repugnant to the Design of nature; nor is it probable that Twenty Women will be so well impregnated by one Man as by Twenty.

Despite its brevity, the Arbuthnott text produced an important impact among contemporary and later scientists and theologians. One of them, 'sGravesande was one of those who valued it earlier and "improved" it in a work published in 1715.

3. The Analysis of 'sGravesande

The Dutch scientist W. J. S. Gravesande (1688-1742), was professor of mathematics, astronomy and philosophy in Leiden. He analyzed the previous work and presented an improvement of the Arbuthnott test in 1712 in the work Démonstration Mathématique du soin that Dieu prend de direiger ce qui se passe dans ce monde, tirée du name des Garcons et des Filles qui naissent journellement (Mathematical proof that God is responsible for directing what happens in this world, made from the number of children born daily), which circulated among his friends. The main result of his analyzes was published in 1715 by B. Nieuwentyt in Het regt gebruik der wereldbeschouwingen, translated into English as The Religious Philosopher: Or, the Right Use of Contemplating the Works of the Creator, see Pearson (1978) for a detailed description of this work. The article by sGravesande was published in his Works (1774, Vol. 2, pp. 221-236), which also contains a small exposition on elementary theory of probability (pp. 82-97).

In 1712 Nicholas Bernoulli meets with sGravesande in The Hague on his trip through the Netherlands, England and France. They argue about the work of Arbuthnott, and this results in a correspondence between the two that was published in the Works of 'sGravesande (1774, Vol. 2, pp. 236-248).

In his test Arbuthnott uses only the fact that each of the 82 years, the number of children is greater than the number of girls. Well, sGravesande also makes use of the fact that the relative number of child birth varies between $\frac{7765}{15448} = 0.5027$ in 1703 and $\frac{4748}{8855} = 0.5362$ in 1661.

Due to the different annual number of births, the numbers of children are not directly comparable, and sGravesande then transforms the previous observations by multiplying the relative frequencies by the average number of births for the 82 years, which he finds in 11429. gives a maximum and minimum fictitious number of males born: 5745 and 6128. Then consider the data as 82 observations of the same binomial distribution with n = 11429 and all the observations contained in the interval [5745, 6128].

To find the probability of this event under the Arbuthnott hypothesis, he calculates the terms of the binomial for $p = \frac{1}{2}$ and n = 11.429 and adds the 384 terms corresponding to the interval in question.

Actually, it uses recurrence $\binom{n}{x+1} = \binom{n}{x}\frac{n-x}{x+1}$, and tabulates $10^5\binom{n}{x}\Big/\binom{n}{5715}$ from x = 5715 to 5973, after which the tabulated values become smaller than $\frac{1}{2}$. Find that

$$\Pr\left\{5745 \le x \le 6128 \middle| p = \frac{1}{2}\right\} = \frac{3849150}{13196800},$$

and comments that he has added a small amount to the numerator to make sure that the probability is not underestimated because the terms of the tail less than $\frac{1}{2}\times 10^{-5}$ have not been considered. (The probability of sGravesande equals 0.292, and the normal approximation gives 0.287.)

Under the hypothesis, the probability of the observed event becomes the power of the previous probability, which gives 1 divided by 7.5598×10^{43} (sGravesande gives all 44 digits), which is only a small fraction of the probability found by Arbuthnott. That is, under this methodology, the probability that each of those 82 years is born more boys than girls is even smaller than the one calculated by Arbuthnott, under the null hypothesis

$P[M > F] \leq \frac{1}{2}$, and yet, during each of those 82 years, they were born more children than girls.

4. Conclusion

The idea of contrasting hypotheses from statistical data was already in the minds of the scientists of the second half of the seventeenth century (Graunt) and the first half of the eighteenth century, as we have seen with Arbuthnott and 'sGravesande. Their methodological proposals do not differ much from what is currently done when a contrast of significance is carried out. What differs from the present is the interpretation of the natural laws that are discovered as evidences of the Divine Design and the existence of God. Social scientists also appear, in many cases represented by theologians, who used the regularities observed in population statistics for the same purpose. Some avoided entering into this matter, as for example Nicholas Bernoulli, who also approaches the work of Arbuthnott but does it only from a mathematical and statistical point of view, without any type of theological evaluation.

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