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Continued Formation of Teachers Who Teach Mathematics: The Seizure of the Process by the Teachers

Renata Camacho Bezerra[1], *Maria Raquel Miotto Morelatti*[2]

1. State University of the West of Paraná (UNIOESTE), Campus of Foz do Iguaçu/PR, Brazil;

2. Faculty of Science and Technology (FCT), Universidade Estadual Paulista (UNESP), SP Campus, Brazil)

Abstract: This paper reports the seizures of a group of teachers at of a municipal school located in Foz do Iguaçu, in Paraná state, Brazil, who teach Mathematics in the early years of Elementary Education, about a formation process based on the Lesson Study during the year of 2016. The Lesson Study corresponds to an education process which leads the teachers to reflect about their teaching practice, through an eminently collaborative work among peers, focusing on the student learning process, whose main characteristics are reflection and collaboration. It still corresponds to a continual activity of many participants, in which the teacher can not only share his/her knowledge, but also learn with one another, with students, in addition to contribute towards the improvement of the teaching and learning process. Based on the analysis of teachers' seizure regarding the formation process and its contribution to the mathematics learning process of students, it was we concluded that the Lesson Study is an important strategy of continued formation and that it contributes to reflection upon the pedagogical practice, meaningful learning and, consequently, to the professional development of teachers.

Key words: continued formation of teachers, lesson study, collaboration, reflection, seizure

1. Introduction

The low Mathematical performance of students in Brazil is stated by national level assessments, such as the Basic Education Evaluation System (Saeb), applied to the federation units, as Prova Brasil, carried to the municipalities, and international assessments, such as the Programme for International Student Assessment (PISA)[1]. This last one revealed that in 2012 Brazil as rated at 580 in the rank regarding mathematics, reaching 391 points and remaining below the average of the Organization for Economic Cooperation and Development (OECD), which reached 494 points. According to (Libâneo, 2013),

Renata Camacho Bezerra, Ed.D, Professor, Foz do Iguaçu University (UNIOESTE), Center of Engineering and Exact Sciences, Campus de Foz do Iguaçu/PR; research areas/interests: lesson study, teacher education, trends in mathematics education, teaching methodologies, research methodology, teaching technologies, didactics, internship and teaching practice. E-mail: renatacamachobezerra@gmail.com.

Maria Raquel Miotto Morelatti, Ed.D, São Paulo University (UNESP), Faculty of Science and Technology, Campus de Presidente Prudente/SP; research areas/interests: practices and formative processes in education, mathematics teaching and learning, mathematics teacher education, new educational technologies and collaborative practices in education. E-mail: mariaraquelmm@gmail.com.

[1] For further information access the report available on the website: http://download.inep.gov.br/acoes_internacionais/pisa/ resultados/2014/ relatorio_nacional_pisa_2012_resultados_brasileiros.pdf.

> […] it is notorious the learning precariousness of children and adolescents, attested by official rates and by international organizations surveys. Within educational investigation, this problem is, most of the times, explained by external factors, such as poverty of families, inefficient educational policies, failures in the education system management, low salaries of teachers, uncertainty and ambiguities in the curricular format of professional education, a dispersed and inefficient legislation, disconnection between the educational policies and reality of schools, teachers and students. However, there are those diagnoses attempting to also identify, especially the in-school factors – organizational and didactic-pedagogical – that act to determine the school failure of children, affecting the productivity of the teaching system as a whole. One of them is the professional performance of teachers in the early years of Elementary Education. […] the failure in education ends up resulting in the students' learning failure (p. 74, our translation).

Alongside that, the teacher with a degree in Pedagogy or Normal Superior[2], or even in high school level, is qualified to teach Mathematics in the early years of Elementary Education in Brazil, however, "[…] they conclude their education course lacking knowledge of mathematical contents that they will teach, not only concerning to concepts but also about to procedures, […]" being the "knowledge "of and about" Mathematics little emphasized, even those contents expected to be taught in the early years of Elementary Education, […]" Curi (2004, p.77, our translation).

In Brazil, the formation of teachers who teach Mathematics to the early years of Elementary Education has had, for many years, a low priority, and still today, has been shown insufficient to the development of the students' teaching and learning process. One of the factors responsible for this current situation is the fragility of the initial formation provided to teachers, that being so recognized by MEC, supported by Cristovão (2015) and the studies of Gatti & Nunes (2009) and Gatti & Barreto (2009), suggest investments in continued formation programs.

However, it is not approached here a continued formation in the perspective of technical rationality that involves courses, seminars and events (Gatti & Barreto, 2009), it is desired a continued formation that surpasses this restricted and fragmented perspective and start to be seen in a favorable perspective to learning and professional development of the teacher.

In this process, the teacher is posed as the main protagonist of the action, regardless it is considered the formation or professional development, in which "[…] the teacher's professional development involves a constant inter-relation among knowledge changes, in practices and attitudes" Ferreira (2003, p.37, our translation), and must be centered in school, taking into consideration the individual and collective necessities of teachers and school (Faustino, 2011).

With that,

> […], the concept of "development" has a connotation of evolution and continuity that exceed the traditional juxtaposition between initial formation and teachers' improvement […] The professional development is an indispensable tool for school improvement.[3] (Marcelo García, Vaillant, 2013, p. 75, our translation).

In this perspective, it was proposed a continued formation action in the school context, through the Lesson Study, which aims the teacher's professional development. In a short and long term, it is expected "teachers capable of conducting the teaching of Mathematics adapted to the necessity and interests of each student, besides contributing to the improvement of formation institutions in which they can reach personal and professional

[2] Normal Superior – It is a graduation course created in Brazil to provide higher formation to teachers from Basic Formation, providing them with a higher education diploma.

[3] [...], el concepto "desarrollo" tiene una connotación de evolución y continuidad, que supera la tradicional yuxtaposición entre formación inicial y perfeccionamiento de los profesores […] El desarrollo profesional es una herramienta imprescindible para la mejora escolar. 3 Marcelo García; Vaillant (2013, p. 75).

achievements" (Ponte et al., 1998, p. 29, our translation). Besides this, "the formation in work context constitutes, this way, an important strategy in the attempt of surpassing the isolation of teachers and defying the professional existent culture" (Forte, Flores, 2012, p. 917, our translation).

In this article, it is analyzed the seizure of a group of teachers at Municipal school located in Foz do Iguaçu, in Paraná State, Brazil, who teach Mathematics in the early years of Elementary Education, after taking part of a formation process based on the Lesson Study, during the school year of 2016.

2. Literature Review

2.1 Lesson Study

The Lesson Study originated in Japan at the beginning of the XX century and owns different titles in different countries. In Japan it is known as "Jyugyo Kenkyu", in the United States "Lesson Study", in Portugal, the translation is "*Estudo de Aula*" or "Estudos de Lição", in Spain "*Estudio de Clases*", and in Brazil the translation found is "*Pesquisa de Aula" (Class research)* or "*Estudo de Planejamento de Lições" (Study of Lesson Planning).* The Lesson study was widely spread in the United States in the last decade and in several European countries, such as Portugal and Spain, although in Brazil it is still little known.

The Lesson Study corresponds to a formation process which leads teachers to reflect upon their own practice, through an eminently collaborative work among the peers, with focus on the student learning. Thus, its main characteristics are the reflection and the collaboration.

The modality of teacher formation centered in their own professional practice and which aims the professional development is a continuing activity of many authors, in which the teacher may not only share their knowledge, but also learn with one another, with students and contribute to the improvement of the teaching and learning process (Isolda, Arcavi & Lorca, 2012; Baptista et al., 2014).

In this learning process, there is the need of teachers reflect upon their own practice and acquire concepts, to do so they are motivated to understand their own reasoning and reflect critically about the teaching process what will result in building and giving meaning to their expertise and connecting new theoretical elements of action to it (Aragão, Prezotto & Affonso, 2015).

The Lesson Study is characterized in phases that are presented in Portugal by the authors (Baptista, Ponte, Velez, Belchior & Costa, 2012; Baptista, Ponte, Velez & Costa, 2014; Ponte, Quaresma, Baptista & Mata-Pereira, 2014), which can be summarize as follow: a) Planning: A mathematical theme/content is defined and the tasks are chosen[4] to a mathematical exploratory[5] teaching, next, it is planned the sequence of activities for the lesson and anticipate to the difficulties students will face, as well as their reasoning; b) Class observation: When the lesson is performed, it is recorded/shot/observed by peers, in order to discuss the actions and reactions (reasoning) of students; c) Post Class Reflection: in group, the class is watched, discussed and, after reflection, the teachers decide whether the class needs to be reworked or not; d) Post-reflection Procedures or Segment: When necessary, the reworked class is performed by other teacher in other group, and the cycle of discussion, observation and reflecting is repeated, aiming the improvement of the lesson until the whole group consider it satisfactory.

[4] According to (Ponte, 2005) when one is involved in an activity, a task is developed, and therefore, the task is the objective of the activity, when formulating suitable tasks, the teacher can awake the activity of the student.

[5] According to (Oliveira, Menezes, Canavarro, 2013), the exploratory teaching is a space in which the learning occurs simultaneously in an individual and collective way, as a result of student's interaction with the mathematical knowledge, through mathematical activities, and in the interaction with peers and teacher, using processes of negotiation of the meanings.

It is important to highlight that changes happen in the Lesson Study due to the reality of the education system of each country. However, according to Murata (2011) some characteristics of the Lesson Study process must not be modified, such as:

> (1) Lesson study is centered around teachers' interests: Teachers interests are central to their professional development. Lesson study goals should be something teachers feel is important to investigate and relevant to their own classroom practice.
>
> (2) Lesson study is student focused: lesson study is about student learning. At any part of the lesson study cycle, the activities should focus teacher's attention to student learning and its connections to lessons/teaching.
>
> (3) Lesson study has a research lesson: Teachers have shared physical observation experiences (in some special cases, video may be used in place of the live lessons, but this is not recommended), that provide opportunities for teachers to be researchers.
>
> (4) Lesson Study is a reflective process: Lesson study provides plenty of time and opportunities for teachers to reflect on their teaching practice and student learning, and the knowledge gained from and for the reflective practice should be shared.
>
> (5) Lesson study is collaborative: teachers work interdependently and collaboratively in lesson study (Murata, 2011, p. 10).

The Lesson Study suffers and/or suffered adaptation caused by the own characteristics of the educational system in each country, even though, corroborating (Murata, 2011) we highlight two features we consider fundamental, which are the collaboration among peers and reflection during the process.

3. Research Method

3.1 Participants

The group was constituted of sixteen teachers, fifteen women and one man, of these two pedagogical coordinators and one director. During investigation, all of them chose fictional names to be identified.

The participation in the formation process was voluntary, although a certificate will be provided as a way to valorize them in their continued formation, once this certificate is accepted to add points in order to reach a promotion in the municipal teaching career, Career Plan of Foz do Iguaçu Municipality (Law no 4.362, of August 17 of 2015).

According to Huberman (1995) the teacher's professional life is divided in phases, the beginning of career is considered from the first to the third years of work, the establishment from the fourth to the sixth year, diversification and experimentation from the seventh to the twenty-fifth year, serenity and affective detachment from the twenty-fifth to the thirty-fifth year, and preparation for retirement from the thirty-fifth to the fortieth year of career.

Taking these phases described by Huberman (1995) into consideration, and adapting them to Brazilian context, there are not teachers in the group who can be considered beginners. Only one teacher is found in the establishment phase, being the youngest one and who presents less teaching experience. Many teachers (11) are in the diversification and experimentation phase, and four teachers are in the affective detachment and preparation for retirement.

The group can be considered heterogeneous from the teaching experience perspective, but it is cohesive towards the difficulties in the teaching and learning process of Mathematics, in their beliefs towards deficiencies in their initial formation and that the classroom was responsible for the need of a continued formation to fulfill these gaps.

Besides that, all the sixteen teachers, including the principal and the pedagogical coordinators, own a university degree, and just one of them do not have a specialization course, while all the others have accomplished more than one specialization course.

In the group, fifteen teachers have a forty-hour work journey and just one has a twenty-hour work journey. In addition, fifteen teachers teach at the school where the formation process took place, and just one teaches at a nearby school.

In the next section, it is presented a formation process lived by teachers in the Lesson Study context.

3.2 The Formation Process

During the school year of 2016, nineteen formation meetings, of approximately two hours long, were carried out. They were held at the school, on Thursdays, every fifteen days, when the classes were over, according to the teachers' decision.

In the first semester, the teachers chose living the formation cycle of the Lesson Study approaching the content "division". This cycle involved a total of ten meetings. In the second semester, it was chosen the content "multiplication", involving a total of nine meetings.

The meetings, according to the Lesson Study dynamic, involved theoretical studies, the choosing of a theme/content by the group, activities elaboration to didactic sequence which aimed to work on the exploratory teaching of Mathematics and anticipate possible difficulties and to the reasoning of students, the performance of the lesson by one of the teachers from the group, which was recorded/photographed by other teachers, the post-class reflection based on the film, reflection and analysis of teachers towards the work developed and future planning. At the end of each meeting in all phases of the formation cycle, the teachers were invited to register by narratives their seizures of the work accomplished, as well as their criticism and suggestions.

Hereinafter, it was summarized the meetings of the first formation cycle lived by the teachers who taught mathematics in the early years of Elementary Education, approaching the division content.

Table 1 Development of the First Class, about Division, in the Lesson Study Context.

Lesson Study Phases	Formation Process Phases	Description of performed activities
Beginning	1st meeting (Constitution of the group) Presentation of the Lesson Study, establishment of rules, Reading and adaptation between researcher and group of teachers.	a) Presentation of the Lesson Study approach to teachers; b) Definition of sessions time; c) Presentation of the sessions' schedule; d) Creation of a closed group on facebook; e) Collective Reading of the text "Task in teaching and learning of Mathematics" (Ponte, 2014, our translation); f) Discussion and reflection upon the text.
Planning	2nd meeting Beginning of planning according to the steps suggested by the Lesson Study.	a) Taking up the objectives of sessions due to the fact of having new participants in the group; b) Teacher's decision of creating a group on *whatsapp* to streamline the communication; c) Taking up the discussion upon the text read in the previous session; d) Definition of the specific content to be approached (Division); e) Beginning of collective Reading of the text "Curricular

		Management in Mathematics" (Ponte, 2005, our translation).
Planning	3rd meeting Continuing the process of planning according to the steps suggested by the Lesson Study.	a) End of the text reading and discussion/reflection of the group; b) Establishment of the grade and group where the lesson will be performed; c) Collective summary of the main ideas presented in both texts.
Planning	4th meeting Continuing the process of planning according to the steps suggested by the Lesson Study.	a) Discussion of the content Division aided by *slideplayer* "Algorithm and concepts: What to do with division?" (Ferreira, 2006, our translation) and the table (Annex B) with different meanings of division operation adapted by (Alcobia, 2014); b) Doubts of teacher towards the content and concepts performed in Division (concept of measure, inverse operation, sharing and ratio).
Planning	5th meeting Continuing the process of planning according to the steps suggested by the Lesson Study. At this moment it was needed an adaptation due to the big size of the group.	a) Planning a didactic sequence to work the content of division. The teachers were divided in three smaller groups.
Planning	6th meeting Continuing the process of planning according to the steps suggested by the Lesson Study	a) Planning a didactic sequence in collective, in possession of the three sequences prepared, the teachers came to an agreement of objectives and activities that should be kept in the collective sequence.
Planning	7th meeting Continuing the process of planning according to the steps suggested by the Lesson Study	a) Solving of the didactic sequence by the teachers in small groups.
Planning	8th meeting Continuing the process of planning according to the steps suggested by the Lesson Study	a) Discussion of each activity linked to the objectives previously established in the bigger group; b) Performance of the lesson by the researcher with the teachers; c) Redefinition of teacher who will perform the lesson and the group.
Planning	Meeting with the smaller group that will perform the lesson.	a) Meeting in the school with the teachers who will take part in the performance of the lesson prepared in collective by the teachers.
Observation	The lesson was performed by a teacher who takes part in the education process and works as a pedagogical coordinator in the school. She was observed by the principal, the titular teacher of the group and by two more teachers of the group.	a) Lesson performance.
Reflection	9th meeting In collective, the group watched the lesson and analyzed if the planned objectives were reached, as well as reflecting upon the elaboration process of the lesson.	a) Watch the video of the lesson performed; b) Reflection upon the lesson performed by the teachers who took part and by the teacher who watched the video; c) Analysis of the activities performed by students.
Reflection	10th meeting In collective, the group watched the lesson and analyzed if the planned objectives were reached, as well as reflecting upon the elaboration process of the lesson.	a) Conclusion of analyses of the activities performed by the students; b) Discussion and reflections of teacher upon the work accomplished before, during and after lesson; c) Completion of the preparation cycle of the didactic sequence, performance of the lesson and evaluation.

Note. Source: Organized by the authors.

At this first formation cycle it was not performed the last step "Post-reflection Procedures or Segment", since the teachers considered unnecessary redoing the lesson once the objectives were reached.

Next, it is presented the second formation cycle lived by the teachers who taught mathematics in the early years of Elementary education with the content of multiplication.

Table 2 Development of the Second Lesson about Multiplication, in the Context of the Lesson Study

Lesson Study Phases	Formation Process Phases	Description of Performed Activities
Planning	1st meeting Definition of the content and, through the reading of texts, the teachers could get to know other realities in which the Lesson Study was performed with the teacher group.	a) Definition of the content to be presented "Multiplication"; b) Definition of the grade, group and teacher that would perform the lesson; c) Division of the teachers in two groups. One group read the text "Lesson Study in the Formation of Teacher of the 1st Cycle of Elementary Education" by Mônica Baptista, João Pedro da Ponte, Estela Costa, Isabel Velez and Margarida Belchior. The other group read "The Lesson Study as a Strategy of Formation of Teachers through the Professional Practice" by Mônica Baptista, Jão Pedro da Ponte, Isabel Velez, Margarida Belchior and Estela Costa.
Planning	2nd meeting Adaptation of the Lesson Study. Teachers reflected upon the process lived in the first formation cycle and about the two texts read in the previous meeting.	a) It was presented and discussed the two texts read in the previous meeting; b) Teacher discussed the main characteristics of the texts read comparing them to the first lesson performed.
Planning	3rd meeting Continuation the planning according to the steps suggested by the Lesson Study.	a) The teachers reported class situation when Multiplication was worked, where they were successful/had difficult in the teaching and learning process; b) Definition of using a diagnoses questionnaire with students; c)Discussion about the concepts used in Multiplication; d) Beginning of the diagnoses questionnaire elaboration.
Planning	4th meeting Continuation the planning according the steps suggested by the Lesson Study. Teacher agreed to perform the diagnoses questionnaire after the reading of the texts in the first meeting of the second formation cycle.	a) Ending of the diagnoses questionnaire and discussion about the activities.
Planning	The questionnaire was elaborated by a group of teachers and performed the titular teacher.	a) Accomplishment of the diagnoses questionnaire with students of 3rd year.
Planning	5th meeting Continuation of the planning according to the steps suggested by the Lesson Study.	a) Analysis of the diagnoses questionnaire performed by the students; b) Definition of the lesson objectives to be prepared by the group.
Planning	6th meeting Continuation of the planning according to the steps suggested by the Lesson Study. At this moment an adaptation was needed due to the big size of the group.	a) The teacher, divided in three groups, began to prepare the sequence of activities to the lesson performance.
Planning	7th meeting Continuation of the planning according to the steps suggested by the Lesson Study.	a) In collective, the three lessons were discussed until the bigger group come to an agreement on the activities which should integrate the activity sequence of the lesson to be performed.
Observation	The lesson was performed by the titular teacher and observed/shot/photographed by two other teachers of the group.	a) Lesson performance.
Reflection	8th meeting In collective, the group watched the lesson and evaluated if the planned objectives were reached, as well as,	a) Teachers watched the video of the lesson performed. b) Analysis of the activities performed by students. c) It was presented the main observations performed by the teachers who took part in the lesson.

	reflecting about the elaboration process and performance of the lesson.	
Reflection	9th meeting In collective, the group watched the lesson and evaluated if the planned objectives were reached, as well as, reflecting about the elaboration process and performance of the lesson.	a) Discussion of the lesson performed and appointments of possible changes, closing the evaluation cycle.

Note: Source: Organized by the authors.

In this second formation cycle, the teachers again did not consider necessary accomplish the last step of the Lesson Study "Post-reflection Procedures and Segment" and added the diagnoses questionnaire before the performance of the lesson to verify the main difficulties of students. In addition, in the second cycle there was one meeting less than in the first cycle, and the teachers participated more actively during discussions.

During all the experience in this formation process, the teachers were invited to present, through narratives, those points they considered more important and consequently more relevant in their analysis, highlighting the learning and the seizures of change in the teaching process caused by the formation process experienced in the context of the Lesson Study.

4. Results

4.1 Teachers' Seizures in the Formation Process

As the project took place in collectiveness, it was possible to realize the participant's sensibility progress towards how the students not only realize, but also try to respond to the cognitive stimulations presented, through activities proposed in class, after being elaborated with the collaboration of each one of the teachers.

Besides this, each one was able to contribute through their own experience in class, their contact with students and by their own observation regarding the way their students undertook a resolution to the problems presented.

Teachers highlighted in their talks the seizure they had about several points they consider important, among them, we detach some categories which emerged from the seizure of change itself and/or teachers' learning during the formation process. They are:

4.1.1 About the Pedagogical Practice

The authors Gatti & Barreto (2009), point that teachers highlight the opportunity to deepen their knowledge, the access to new concepts, the interaction among peers and the contact with trainers as positive aspects of the continued formation. Among the negative aspects, they highlight that the continued formation programs do not provide follow-up and support to the pedagogical practice of teachers, what makes it difficult to them to understand the relation between the formation programs developed and their action in the school daily routine. The Lesson Study comes to supply this deficit as it is pointed by teachers in their talks.

> Teacher Guadalupe – The meetings helped me to redirect part of my attention to the needs of each kid. The class emphasized the importance of adapting the pedagogical practice in such form it meets the specificities of everyone, what improved my work in class. Following this line of reasoning, my work with the collective becomes much more productive, once I deal with each of the pieces from the all with close attention, leading to a positive result to the majority and not to just a small part of students. […] the sessions offered me new perspectives and gave me opportunity to aggregate new concepts and practices to my regular school days.

Through the Lesson study, the teacher's professional development is possible to occur, once "we see the teacher's professional development as an intersection of paths, as the glue that allows to combine educative, pedagogical and schooling practices and teaching" (Marcelo García, 1999, p 139, our translation), and that is fundamentally based on the change of thinking and practice in teachers (Days, 2001).

Teachers highlighted the importance of concrete activities performed in group.

> Teacher Brigitte – I learned that working in groups makes kids exchange knowledge among them.
>
> Teacher Flor – […] see that with concrete activities and group work make it easier to be developed […]. The collective work of planning and punt into practice makes everything easier […].

Group work was something teachers avoided to use, it seems it was due to the fact of the formation was, basically, accomplished in group. When they finally experienced the collaborative work and the cooperation, they showed, in their talks, they took this experience into their classrooms.

4.1.2 About the Learning and the Teaching

It is fact that like the knowledge, the formation, initial or continuing, "[…] is not built as a building, in which it is necessary to start by the base and end by the ceiling […]" Barth (1993, p. 65, our translation). The teacher is always learning. This is of pivotal importance, once researches developed by several authors in different lines, such as Blanco & Contreras (2002), Curi (2005), Nacarato (2013) show that when teachers lack knowledge about the content they will teach, they present difficulties to perform didactic situations, they avoid themes they do not domain, reinforce conceptual errors, present uncertainty in class, try avoiding innovation and rely on textbooks and memorization of information to teach. A variety of researches show that teachers who teach Mathematics in the early years of Elementary Education "[…] conclude their formation courses without the knowledge of mathematical contents which they will work with, not only regarding to concepts but also to procedures, […]" Once the "knowledge "of and about" Mathematics is little emphasized, even towards the contents expected to be taught to students in the early years of Elementary Education, […]" (Curi, 2004, p. 77, our translation).

Is this context, the continued formation assumes an important role of provoking the teacher, but it is him/her who oversees the transformative role of establishing relations between the knowledge produced and the practice.

> Teacher Isadora – I learned to deal with the novelty, with the differences […]. I realized we can start, fearless, from the knowledge the students already bring with them and, together, build the best path to follow in this learning process the way they realize the different forms of building their own learning.
>
> Teacher Maria Rita – I learned deal with the novelty, the different and the differences […], by being more flexible with new possibilities and experiences of other peers and even from people of other countries. It was possible to apply the content prepared by an active team in classroom, experts in the academic reality of students.
>
> Teacher Maria – It is very nice to sudden realize that it is possible to be different […] and that there is nothing ready and finished.

The teacher also highlighted the opportunity to learn with peers and tackle doubts.

> Teacher Ileon – […] what attracted me was the possibility of talking about my doubts when working with mathematics and how I used to work. I learned, in the collective meetings, we really need each other, listen to and know what each one is working on, and that everyone, like me, had doubts and even skipped the contents that we felt not sure enough to teach. The importance of elicit what students know and what I want them to learn at first place. I need to have a target, so I avoid getting lost and consequently affect the students learning.

> Teacher Karl – […] the opportunity to "put myself in students' shoes", see things from their perspective towards the problems that are worked on in the classroom.
>
> Teacher Mazdha – […] by learning with the experiences, not just lived in my classroom, but also in my colleagues' classrooms. When listening to the differentiated talks, it is always possible see mathematics with new eyes, not only teachers', but also student's eyes, and learning with these.

4.1.3 About the Formation Process

In this category, the teachers especially highlight the exploratory teaching of Mathematics and the collaboration as two important aspects of their learning and as factors of changes in their practices.

In the Exploratory Teaching of Mathematics, it is necessary to emphasize two important points that differ from the teaching that teachers are used to.

First, it is the importance that communication assumes not just between teachers and students, but also the communication among students. It is through the dialog that concepts are built, and students can start necessary generalizations in Mathematical teaching. Thus, it is pivotal the teacher motivates the students to verbalize their reasoning with peers in small or bigger groups, as well as, register it. The communication in Mathematics classes is an advance, since we are used to traditional Mathematics classes, with few dialogs. The second point that deserves to be highlighted because it differs from what we are used to see in the classroom is the difference in the teacher's and student's role. In general, in the traditional expositive teaching the teacher oversees exposing the content to students and they are supposed to listen to the explanation and "learn". The teacher is the authority in the classroom.

This change in the approach was highlighted by teachers.

> Teacher Helena – […] I realized the importance of reflecting upon our students learning and about our pedagogical practices […]. The methodology of investigation, from the observation and reflection about the learning results of my students, was something innovative and has provided me more clearness and certainty in the teaching process.
>
> Teacher Rosy – […] a wide view on how to work, introduce new content and lead the student to a sequence that provides a better quality in learning.

In the exploratory teaching, the teacher proposes tasks which the students will need to solve more than just the problem. They will need to explain the solution and justify their reasoning. This way, the students is co-responsible for the building of knowledge. It is the teacher's responsibility conducting the class in such a form to query the student and, when possible, help him to generalize possible mathematical results, doing so, the authority is shared (Quaresma, Ponte, Baptista & Mata-Pereira, 2014; Menezes, Oliveira & Canavarro, 2013).

> Teacher Amanda – […] expecting the possible difficulties of students, I started to think more about the student, try to think as he does and find, in the learning process, his difficulties.

With this, it is possible to observe,

> […] The real pleasure kids seem to feel when are aware they understood something, after we have searched together, realize. […] In the rare moments when they produce these active exchanges, it is astonishing observe to what extent <<they get out of trouble>> in different ways with such knowledge: They know how to use it to debate, ask questions to convince and to refuse […] I became aware of the close bond between the knowledge and the way we use it. The knowledge does not exist out of the way as it is <<understood>>. [Underlined by the author] (Barth, 1993, pp. 36–37).

The real pleasure in learning pointed by the author (Barth, 193) was not felt only by the students, but by the teachers as well.

> Teacher Bia — Everything improved, once we were able to expose our problems, distress and aspirations towards Mathematics.

In the collaboration among teachers, experience exchanging and collective reflection in the group, the teachers highlighted the importance of collaboration, that is the moment when it occurs group discussion, dialogs among peers, discussion about the practice and ideas, they talk and listen to each other, in other words, a "[…] collective practice centered on the study, investigation and reflection upon the practice […]" Gama (2007, p. 146, our translation), in which the learning is understood as a" […] transactional process which is built within the dialog and negotiation […]" (p. 217).

> Teacher Anita – Currently, I try to meet colleagues more frequently to discuss learning goals, to prepare the classes, and we collaborate more with one another in the search of a better way to teach and reach better results.
>
> Teacher Ana – I learned to plan in collaboration with each other and I realized how it makes the difference.

For Day (2004), "even though this process has not always been explicit" (p. 163, our translation), in fact we can relate the collaboration concept to the reflection practice.

> The teaching act has its individual dimension when the teacher is found alone in front of the class approaching a content. What makes him to take a specific action instead of another is the collaborative dimension that makes him to walk across a track and not in another the cooperation among teachers, inside an exchange process of experience, marks, significantly, the path of teaching and learning in the school. Teodoro (2016, p. 127, our translation)

The changing seizures were highlighted by the teachers based on their reference in class and their formation. These changes realized by the teachers, are important once they can be considered trigger factors of the teachers' professional development.

> Teacher Estrela – I believed, until this moment, in have a good formation. But during the sessions it was possible to realize that I had so much to learn, through the exchange of ideas and experiences, new concepts appeared. […]. During the class planning which were a little random, today, I try to work with activities of daily basis of kids and I check and recheck different ways to reach the goals.

5. Conclusion

Through this formation process, it was possible to see "that the learning is the "engine" of professional development and changing" and, in this case, "the changing depends on the desire and activity of the future teacher and/or teacher, associated to favorable conditions (aid, intellectual support, space and time)" Ferreira (2006) (a), p. 167, our translation).

> The most important challenge of teacher formation and trainers is undoubtedly being able to trigger a conceptual change in their relationship with the knowledge and its elaboration […] he needs analysis tools to modify and widen an intuitive perception. That is all about a true conceptual transformation, thus a new comprehension about what we accomplish when we learn and teach. This is where the challenge of education relies (Barth, 1993, p. 13).

It is necessary "a fundamental changing in the pedagogical practice, in the education system as a whole and in each teacher in particular, […]" (Maldaner, 2001, p. 27, our translation), the teacher must stop being an object and assumes a subject role of his/her formation (Ponte, et. al, 1998) and (Ponte, 2012), and there must be an inversion, what was before "an in-ward oriented movement, being the teacher in charge of absorbing the knowledge" changes to "an outward movement, being the teacher responsible for the fundamental decisions" (Ponte et al., 1998, p. 28, our translation).

The teachers highlighted several seizures of the formation process and it was possible to group them in three categories (pedagogical practice, learning of the teaching, and formation). We hope the changes mentioned by the teachers may have truly contributed not only to their continued formation, aiming their professional development, but also and consequently to the improvement of the Mathematics teaching process to students of early years of Elementary Education.

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