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Elimination of White Noise from Time Series through the Wavelet Transform

Iván F. Palacios, and Marilyn V. Cepeda

Department of Earth Sciences and Construction, University of the Armed Forces ESPE, Ecuador

Abstract: The time series is an extensive data set in which you have information about a phenomenon of nature, these data contain certain values that correspond to stochastic errors that occur at the time of obtaining the data. These series are non-stationary in nature so it is assumed that they have white noise, which, for geodetic calculations that require a lot of data accuracy, such as velocity field, gravity, geoid undulation, direct and inverse problem, impact on the final results. The random error is known as white noise, which can be eliminated by using functions that work in variable time and frequency, this new series without noise, is the result of the application of a useful tool called Wavelet, within the types of wavelet is the Haar family that analyzes series with abrupt changes. It is this study was performed automatically the elimination of white noise present in three time series of scale values of the SIRGAS-EPEC station by using the Discrete transformed Wavelet type Haar in order to obtain the clean signal of noise, its energy retained and graphs of the resulting series. It was possible to obtain the graphs of the three series with and without noise, in which a percentage of energy retained was 97.792%, 99.811%, 90.852% in series 1, 2 and 3 respectively. The application of wavelets of the Haar type allowed to obtain the three series without noise, in the case of series 1 and 2.

Key words: white noise, Haar, retained energy, random error, SIRGAS

1. Introduction

In recent years a new methodology has been introduced to represent phenomena that vary over time, due to this behavior several techniques have been implemented to analyze these changes and determine their behavior in the future. One of the most used tools in the study of signals or series, is the Fast Fourier Transform (FFT) [1], however the behavior of the signals of the series does not vary only in time also in frequency [2, 4], since the decade of the 80s has evolved in the study of time series with a new method called Wavelets.

In the study of the phenomena that present time series, the data of them contain certain values that correspond to stochasticity when they are obtained [5]. These random values correspond to the white noise present in the series. Frequently the treatment of the series has been done with FFT [6], STFT (Short Fourier Transform) [1], which process in a better way stationary signals located in time because their frequency does not vary and when passing from the frequency domain to the time domain loses information [1, 3, 7], in order to Wavelet is chosen, which is efficient for the analysis of non-stationary signals and fast transience, which avoids this loss of data [8, 9].

A wavelet is an oscillatory signal, of short duration and finite energy concentrated in a time interval around a point [1, 3, 7, 8]. Wavelets are basic functions of the Wavelet Transform generated from a mother function [10], which represents a signal in translated and dilated versions of a finite wave. The Wavelet Transform is not only local in time, but also in frequency [9], which is the main advantage compared to Fourier analysis [3].

The wavelet transform is expressed as:

Corresponding author: Iván Fernando Palacios Orejuela, Student; research areas/interests: geodesy, cartography, remote sensing, GIS, cadaster, geostatistics, environmental sciences. E-mail: ivan199632@hotmail.com.

$$WT\left(f\left(x\right)\right)=f\left(x\right)*\Psi\left(x\right)=\frac{1}{a}\int_{-\infty}^{+\infty}f\left(t\right)\Psi\left(\frac{x-t}{a}\right)dt \quad (1)$$

Where "a" is the scale factor (dilation), "t" is the time (translation) and "x" is the position. The function ψ is called "mother wavelet"; first, wavelet because it is of an oscillating nature and of finite duration (compact support) and it is called mother for serving as the basis for the generation of the remaining window functions [3]. Among the best known wavelet families are: Haar, Daubechies, Coiflets, Symlets, Biorthogonal, Meyer, Mexican hat, Shannon and Morlet [5, 11].

In the analysis of non-stationary data, there are two types of wavelets, the Wavelet Continuous Transform (CWT) and the Discrete Wavelet Transform (DWT) [1, 7]. In the numerical analysis of the DWT it must be considered that the data is discretized and not stationary, this idea was developed by Mallat in 1988, who implemented an algorithm based on sequential filters, which allow the Wavelet Transform to be obtained instantaneously represented in the Mallat tree [12]. The scale is changed through operations of interpolation upsampling and downsampling. The upsampling (↑ 2) consists of increasing the sampling rate by inserting new samples into the signal and the downsampling (↓ 2) removes samples of the signal, thus reducing the sample rate [13].

The signal passes through two complementary filters: low-pass filter related to the scaling function that gives a global view of the signal and high-pass filter associated with the details of the signal [14]; these pair of filters allow to separate the portions of the high frequency signal from those of low frequency [6], in this way two signals emerge [12], presented in Fig. 1. To avoid getting twice as many samples as the original signal, a single point of two is taken and so the final two signals will reconstruct the original signal with half the samples as the original signal [2], as shown in Fig. 2.

Currently there are several applications for this mathematical analysis tool, as for example in the analysis of electroencephalogram signals [15], for the separation of ocean waves [16], elimination of noise in

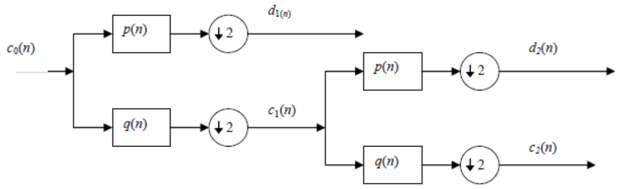

Fig. 1 Scheme of signal decomposition with Mallat tree.

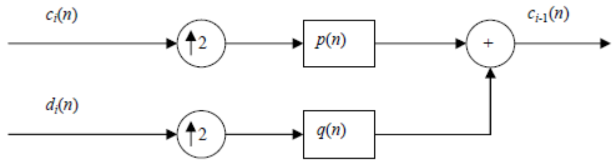

Fig. 2 Signal reconstruction scheme.

signals [5], time series [8], noise in radar signals [17], seismic movements [18], fractals [19], among others.

In this paper, it was proposed to analyze the temporal series of scale values of the GPS week 1945 obtained from the EPEC continuous monitoring station, located in the GIS, Remote Sensing and Photogrammetry Laboratory of the University of the Armed Forces ESPE, which belongs to the SIRGAS network. In these time series, due to their nature, they are non-stationary, so it will be assumed that they have white noise, which, for geodetic calculations that require a lot of data accuracy (velocity field, gravity, geoid ripple, direct and inverse problem) could cause not so precise results [20, 21].

The objective of this work was to eliminate the white noise present in the time series of scale values of the SIRGAS-EPEC station through the use of the discrete wavelet transform, Haar type, in order to obtain the clean signal of noise, its energy retained and graphs of the resulting series.

2. Methodology

It was based on the Wavelet type selection, which eliminated the white noise of the temporary series. White noise is understood as random errors in which values of a signal in two different times do not have statistical correlation, that is, it has a mean null, constant variance, zero covariance and has a normal distribution [22].

For the treatment of signals, one of the most used in this field is the Haar family, which visualizes better the DWT, therefore Haar works well on signals with abrupt changes [5]. This wavelet separates the signal into two matrices defined in Eq. (2):

$$An = \frac{S_{2n-1}+S_{2n}}{\sqrt{2}} \quad y \quad Dn = \frac{S_{2n-1}-S_{2n}}{\sqrt{2}} \qquad (2)$$

By subdividing the signal (A and D) two values are obtained (An and Dn) that represent the coefficients of the decomposition of the signal [12]. The number of times the signal is filtered is given by the level of decomposition [3]. It is not recommended to use a very low level of decomposition as DB_3, because it takes certain steps to achieve a good filtering of the signal, likewise it is not recommended a very high level, like the DB_{20}, because it can eliminate part of the information; it is best to consider a level according to the nature of the signal to study, so a good level of decomposition for this type of series would be a DB_5, with which an effective noise filtering is achieved [23].

The elimination of the noise was done by the technique wavelet shrinkage, which consists of designating a threshold and excluding the components obtained from the Wavelet Transform that are under a threshold, or instead, applying the multiplication with a weighted value before performing the inverse transformation [9]. Because the signal shows remarkable changes, Haar is used [5]. The threshold method that was used is non-linear, since the noise is in each coefficient and is distributed over all the scales [24]. The threshold used was soft-threshold (soft threshold) in which only the coefficients of the transform that are under the value of said threshold were eliminated.

The threshold is calculated using statistics with Eq. (3)

$$\delta = \sigma\sqrt{2\log(N)} \qquad (3)$$

Where N is the data number and σ is the standard deviation of the coefficients of the transform, the deviation is given by:

$$\sigma = \frac{media\,|C(i,j)|}{0.6745} \qquad (4)$$

Where C (i, j) is the mean absolute deviation of the wavelet coefficients. Once the coefficients that are below the established threshold have been eliminated, the inverse transformation was carried out following the same reasoning in the opposite direction, starting with the coefficients and then going through the filtering process, which must be correctly considered as the number of iterations for the reconstruction of the original signal as can be seen in the Fig. 2.

Specifying the bases of the transformation, we proceeded to program in MATLAB for the elimination of white noise, for this we worked separately with each of the series, first changing the format .xls to .txt, to facilitate the importation of the data in the program. We placed the type of wavelet with which we will work the transform (Haar ("h")), the number of non-zero coefficients that was obtained in the mother wavelet is of coefficient 5 ("db5"), due to the discrete nature of the signal so it is enough with DB_5. Then the level of discretization of the transformation was chosen, for this case a level of 5 was used. For the filtering process, we proceeded to calculate the values of the standard deviation of the coefficients of the transformation and making it possible to obtain the value of the threshold with which the noise was eliminated in the signal. In the DWT it is necessary to specify the low pass and high pass filters in separate variables, this will facilitate the reconstruction of the original series. MATLAB has commands in which the process is automated for the calculation of inverse transformation, the command "waverec" was used in the reconstruction of the original signal. The signal sought is the difference between the original signal and the inverse of the signal. Finally, the results were presented through the graphs of the series.

3. Results

The graphs of the three time series could be obtained through the programming in the MATLAB software in which the difference between the series with noise and without noise was clearly noticed. In Figs. 3-5, the resulting signal is seen in blue, the original in red and the black noise in series 1, 2 and 3 respectively.

The percentage of retained energy of the three series without white noise was obtained with a recovery of information of 97.792%, 99.811% and 90.852% in time series 1, 2 and 3 respectively.

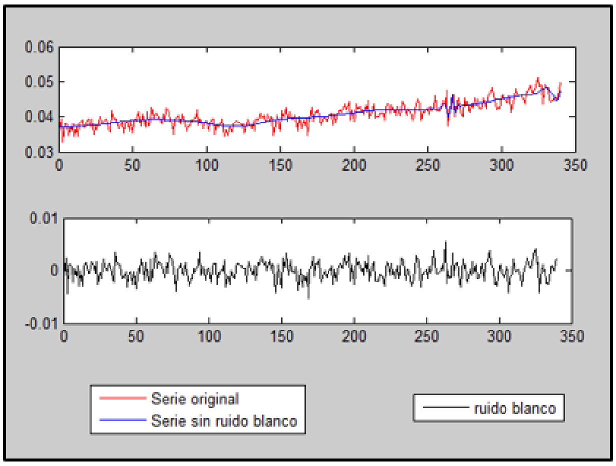

Fig. 3 Graph of time series 1: original (red), without white noise (blue) and white noise (black).

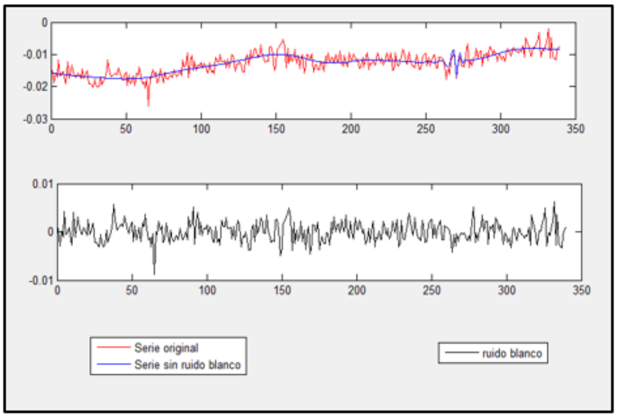

Fig. 4 Graph of time series 2: original (red), without white noise (blue) and white noise (black).

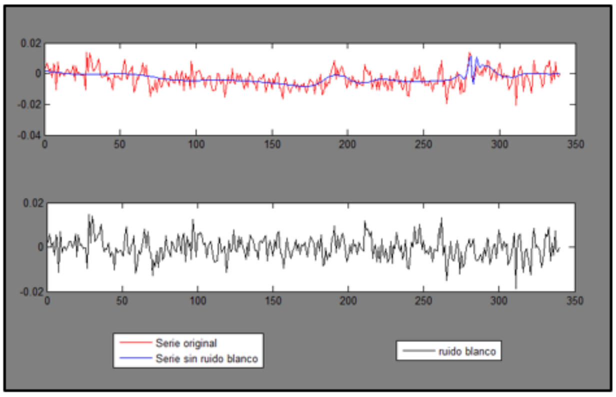

Fig. 5 Graph of time series 3: original (red), without white noise (blue) and white noise (black).

4. Conclusions

The application of wavelets of the Haar type allowed to obtain the three series without noise, in the case of series 1 and 2, the energy retained was very good, while in series 3 it was good, probably because the series presented less errors for this reason not only noise was eliminated but also information.

It is important to consider the level of discretization to be used, because if a very high level is placed it can eliminate information that is not contaminated by noise and therefore lose data that harm the result of the phenomenon under study, but if a very low level is used, a correct filtering of the noise is not achieved.

The wavelet transform works better in series that vary in time and frequency, as in the case of time series, unlike the FFT that only works in time, so it is advisable to use wavelets for these cases.

The use of Haar is recommended for the analysis of series that present abrupt changes in time due to a better analysis of the peaks of the series.

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