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Performance Dynamics of Hedge Fund Index Investing

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Abstract: This paper applies ARMA-GARCH-type modeling to shed light on the persistence of performance and volatility of daily management hedge fund index returns from April 1, 2003 to August 11, 2014. Time series data of four principal hedge fund strategy indices (Equity Hedge, Event Driven, Macro/CTA, Relative Value Arbitrage) have peculiar characteristics — that is, serially correlated and volatility clustered returns. In addition, their unconditional distributions are heavy-tailed and negatively skewed. Hedge funds are generally free to change their trading strategies as market conditions evolve. This flexibility is a distinctive feature that delivers hedge fund returns. At the same time, it is possible to say that this feature potentially amplifies market volatility. Therefore, a rolling application of ARMA-GARCH modeling can potentially capture the broad shift of performance and volatility persistence in the investment strategy, especially in the period surrounding the financial crisis of 2007-2009. The empirical results show important differences concerning persistence performance between the directional and the mispricing strategies. Moreover, the Macro/CTA strategy only indicates a negative news impact. Finally, time-varying parameter estimations reveal that Macro/CTA was strongly affected by the isolated outliers during the financial crisis.

Key words: hedge funds; rolling ARMA-GARCH modeling; serial correlation; asymmetric volatility; index investing

JEL codes: C2, G1, G2

1. Introduction

Hedge funds are often said to potentially amplify market volatility. They are generally free to change their investment positions in various asset markets. Their peculiar performance characteristics tend to generate returns less uncorrelated to those of market benchmark returns. After the IT bubble burst in 2001, the unrelenting capital inflow of institutional investors to hedge funds accelerated the institutionalization of the hedge fund industry and the growth of multi-strategy. Various hedge fund data providers started to offer investable hedge fund indices around 2003. The most common structural problems of hedge fund investing are low transparency, low liquidity and high costs, while the index-based investing in hedge funds offered transparency, liquidity, and significantly lower fee levels. The purpose of these indices is to provide hedge-fund-like returns without investing in hedge funds for institutional investors.

On the other hand, many investors have questioned whether the offered investable hedge fund indices are attractive investment products. Many previous studies (Fung & Hsieh, 2006; Hasanhodzic & Lo, 2007; Jaegar,

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2008; Amenc et al., 2010) have argued that the hedge fund investable indices (or hedge fund replication products) substantially underperformed their respective style benchmarks. In fact, the average monthly returns of the HFRX (investable) for four principal strategy indices have slightly out-performed those of the corresponding HFRI (benchmark indices) for all strategies from its inception in January 1998 until December 2002, whereas the HFRX monthly returns significantly under-performed those of the corresponding HFRI indices for all strategies during the period from January 2003 to January 2016.[1] It seems that the underperformance of the investable indices over the benchmark indices has accompanied the rapid growth of asset allocation of institutional investors to hedge funds. What is a fair reward for the risks taken by the investor? How is an optimally diversified portfolio involving hedge fund investing constructed? Before answering to these fundamental questions, it is necessary to analyze the performance characteristics of the investable hedge fund indices.

The purpose of this paper is to investigate the performance dynamics of the investable hedge fund indices, especially focusing on their persistence of volatility. I would like to construct an appropriate model for their volatility-clustered returns in order to reveal long memory and leverage effects, and illustrate the dynamic impact on volatility during the financial crisis. Many studies have argued that nonlinear processes model the volatility behavior of hedge fund strategies better (Füss et al., 2007; Blazsek S. & A. Downarowicz, 2011; Del Brio et al., 2014; Teulon et al., 2014). My primary empirical tool is GARCH-type model with which is possible to combine together more than one of the time series models, such as ARMA models. Such “hybrid” models can simultaneously account for time-varying volatility, serial correlation, skewness and kurtosis in hedge fund strategy index returns. Moreover, it is important to note that accurate appraisal of hedge fund strategy performance must recognize the freedom with which managers change their trading tactics as market conditions evolve. A rolling application of ARMA-GARCH modeling can potentially capture the broad shift of performance and volatility persistence in the investment strategy, especially in the period surrounding the financial crisis of 2007-2009. This article extends Munechika (2015) by applying a rolling regression technique to ARMA-GARCH modeling.

The remainder of this paper is organized as follows. Section 2 describes statistical properties of different hedge fund strategy index returns. Section 3 gives a brief overview of ARMA-GARCH-type modeling employed, and the motivation of applying a rolling regression to the models is introduced. Section 4 reports the empirical results. Some concluding remarks are offered in the final section.

2. Data

The author’s analysis relies on the HFRX Global Hedge Fund Index of Hedge Fund Research Inc. (hereafter HFR). HFRX Global Hedge Fund Index is designed to be representative of the overall composition of the hedge fund universe and to be investable, as well as to offer full transparency, daily pricing and consistent fund selection. It is comprised of all eligible hedge fund strategies falling within four principal strategies. In this paper, four principal strategies indices (Equity Hedge, Event Driven, Macro/CTA, and Relative Arbitrage Value) are investigated.[2] Data are daily and span the period March 31, 2003 to August 11, 2014.

[1] According to the author’s calculations, the average monthly returns of the HFRI benchmark indices for Equity Hedge, Event Driven, Macro/CTA and Relative Value Arbitrage were under-performed those of the HFRX investable indices with -0.330%, -0.017%, -0.555% and -0.028% during the period from January 1998 until December 2002, respectively. On the other hand, those of the HFRI indices were out-performed those of the HFRX indices with 0.354%, 0.336%, 0.293% and 0.381% during the period from January 2003 until January 2016, respectively.

[2] HFR (2014), http://www.hedgefundresearch.com, for descriptions of each investment strategy index.

Although there are several types of categories for hedge fund strategies, it is possible to say that the hedge fund strategies are roughly classified into two trading styles: directional and mispricing. Agawal and Naik (2000) classify hedge fund strategies into two categories (directional and non-directional strategies) since these two categories exhibit very different risk-return trade-offs. The directional strategy is a strategy that generates returns by taking bets on market directional movements, while the non-directional (i.e., mispricing) strategy is a strategy that funds take relative values bets. The directional strategy funds are typically characterized by significant market exposures. The non-directional strategy is designed to exploit short-term market inefficiencies. The aim is to find discrepancies in prices and it converges price movements in the market. In HFRX Global Hedge Fund Strategy Index, Macro/CTA is classified as a directional strategy while Event Driven and Relative Value Arbitrage are classified as non-directional (i.e., mispricing) strategies, and they neutralize general market trends. However, Equity Hedge in the HFRX strategy index includes not only Equity Market Neutral, classified as a non-directional strategy but also Quantitative Directional and Short Bias, which are classified as directional strategies in the sub-strategy index. Consequently, Equity Hedge (equity-related strategy) has mixed characteristics of directional and non-directional strategies.

An overview of the return and risk characteristics of four hedge fund index returns are shown in Table 1. Although the performance of the particular strategies in the observed period is very heterogeneous, there are common features. First, all hedge fund index returns indicate that the unconditional probability distributions of their returns are leptokurtic. Second, the return distributions for all strategies are negatively skewed. Specifically, non-directional strategies such as Event-Driven and Relative Value Arbitrage have relatively large negative skewness and high excess kurtosis. As is evidenced by their significant JB-test statistics, all hedge fund index returns are not normally distributed. Table 2 reports the estimated autocorrelation coefficients of the hedge fund index returns for lag 1 to 20 together with the Ljung-Box (LB) statistics with five, ten and twenty autocorrelations. Although the returns of four indices excepting for Relative Value Arbitrage do not show high autocorrelation coefficients, some of them are still positively autocorrelated, and are highly significant at the 95% confidence level. One of the main implications of positive autocorrelation is that the true standard deviation is underestimated. Consequently, the risk-adjusted return, such as the Sharpe ratio, is biased upward. Figure 1 displays the full sample series of four daily index returns. This shows that the variance in the returns changes over time. This property is called volatility clustering.

Table 1 Summary Statistics of Hedge Fund Index Returns

April 1, 2003 to August 11, 2014

Daily Return	Mean	STD	Skewness	Kurtosis	Jarque-Bera		No. Obs.
HFRX Global Hedge Fund Index							
Equity Hedge	0.0052	0.4066	-0.8442	8.6599	4162.95	***	2864
Event Driven	0.0171	0.2959	-1.1558	15.0343	17919.96	***	2864
Macro/CTA	0.0039	0.4081	-1.0193	10.5510	7300.02	***	2864
Relative Value Arbitrage	0.0065	0.2712	-1.7268	41.7891	180971.40	***	2864

Notes: The Jarque-Bera normality test is asymptotically distributed as a central χ^2 with 2 degrees of freedom under the null hypothesis, with 10%, 5% and 1% critical values. *, **, *** denote significance at the 10%, 5%, and 1% levels, respectively.
Source: Author's calculations, based on data from Hedge Fund Research.

Table 2 Autocorrelations

ACF	Equity Hedge	Event Driven	Macro/CTA	Relative Value Arbitrage
Lag(1)	0.154***	0.108***	0.104***	0.195***
Lag(2)	0.027	0.060***	0.031*	0.107***
Lag(3)	0.031*	0.078***	0.034*	0.124***
Lag(4)	0.013	0.016	0.040**	0.125***
Lag(5)	-0.014	0.066***	-0.004	0.094***
Lag(10)	0.050***	0.032	0.031*	0.094***
Lag(15)	0.006	0.024	-0.002	0.140***
Lag(20)	0.018	0.024	-0.013	0.074***
LB-Q(5)	73.658***	74.326***	41.358***	255.97***
LB-Q(10)	85.902***	89.994***	47.886***	380.05***
LB-Q(20)	115.3***	135.61***	62.409***	771.68***

Note: The significance tests for the autocorrelation coefficients can be constructed by a non-rejection region for an estimated autocorrelation coefficient to determine whether it is significantly different from zero. Under the assumption that returns are normally distributed, confidence intervals for the correlations can be constructed. For a sample size of T, a correlation coefficient is defined as statistically significant at the 10%, 5% and 1% levels would be given by $\pm 1.65/\sqrt{T}$, $\pm 1.96/\sqrt{T}$ and $\pm 2.58/\sqrt{T}$, respectively. *, ** and *** denote significance at the 10%, 5%, and 1% levels, respectively.
Source: Author's calculations, based on data from Hedge Fund Research.

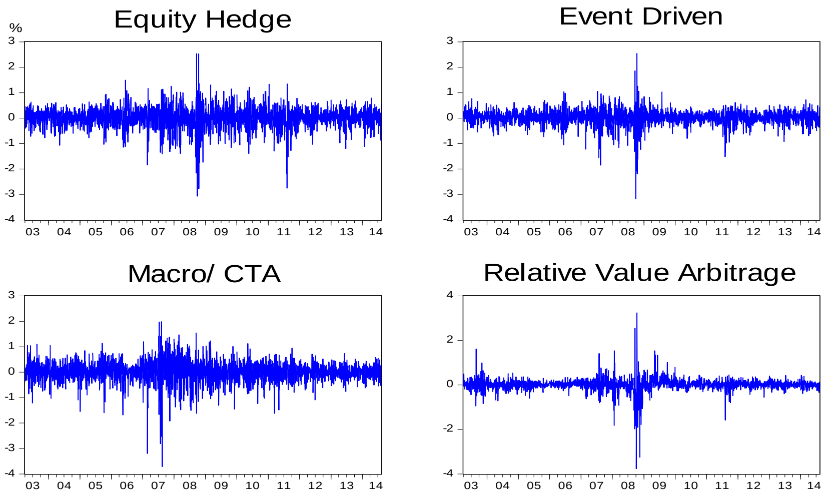

Figure 1 Four Hedge Fund Index Returns

To sum up, time-series analyses of hedge fund index returns have some of the common features: autocorrelation, time-varying variance (volatility clustering), and heavy-tailed, negatively skewed distribution. Volatility clustering implies that volatility shock today will influence the expectation of volatility many periods in the future. This means persistence of volatility to the shocks. This phenomenon requires researchers to describe returns and volatility that are nonlinear. It seems to be appropriate to combine an ARMA model for the level of the returns with a GARCH model for the variance for modeling these data. The combined models can capture the level of persistence in performance and volatility.

3. Model and Methodology

My central challenge is to examine whether and to what extent hedge fund indices in different strategies react similarly or differently to the period surrounding the financial crisis of 2007-2009. In order to estimate the persistence of performance and volatility of hedge fund strategy indices, Munechika (2015) has employed ARMA-GARCH-type models. However, hedge funds tend to shift their investment positions and trading tactics in response to changing market conditions. To account for frequent shifts in strategy, I apply rolling fixed-window regressions to ARMA-GARCH models, which yields time-varying parameter estimates. This section reviews my empirical methodology.

First, I will establish notation. Let p_t be the hedge fund strategy index value at time t and $r_t = \log(p_t/p_{t-1}) * 100$ be the continuous compounded return on the index over the period t-1 to t. In most financial time series, prices are non-stationary while the returns are stationary. It is confirmed that hedge fund index returns r_t of four investment strategies come from a stationary process based on the unit root tests.

In general, the return on any asset r_t can be divided into two parts: the expected parts of the return $\mathrm{E}[r_t]$ and the unexpected part of the return ε_t. The expected parts of the return is what can be predicted using the knowledge from the past, which is denoted by Ω_{t-1} the information set of all available information up to and including time t-1. This expected part of the return is the conditional mean $\mathrm{E}[r_t|\Omega_{t-1}]$, which is the mean at time t conditional on the information set taken by the series in previous periods and defined as

$$r_t = \mathrm{E}[r_t|\Omega_{t-1}] + \varepsilon_{t.} \tag{1}$$

$$r_t = \mu_t + \varepsilon_t. \tag{2}$$

Where ε_t, is known as the disturbance, or error term. The conditional mean is defined by

$$\mu_t = \mathrm{E}[r_t|\Omega_{t-1}]. \tag{3}$$

According to equation (1), the forecast error is considered as

$$\varepsilon_t = r_t - \mathrm{E}[r_t|\Omega_{t-1}] = r_t - \mu_t. \tag{4}$$

The process ε_t corresponds to the unpredictable movements in r_t, which is also called the innovation process.

In the context of financial analysis, the errors ε_t are often considered as “shocks” or “news” since they represent unexpected factors. Then, equation (1) implies that an observed time series r_t is related to an underlying sequence of shocks ε_t. Clearly the distribution of ε_t is central in this definition. Sometime a model will assume that the error term ε_t has the following properties:

$$\mathrm{E}[\varepsilon_t] = 0. \tag{5}$$

$$\mathrm{E}[\varepsilon_t^2] = \sigma^2. \tag{6}$$

$$\mathrm{E}[\varepsilon_t \varepsilon_s] = 0 \qquad for\ s \neq t. \tag{7}$$

The disturbance term is a random variable that has probabilistic properties with zero mean, constant variance (i.e., homoskedasiticity) and is serially uncorrelated, also known as a white noise error term.

Next, the conditional variance is naturally defined as

$$h_t = E[(r_t - \mu_t)^2|\Omega_{t-1}] = E[\varepsilon_t^2|\Omega_{t-1}] = \sigma_t^2. \tag{8}$$

The conditional variance can estimate the variance of a series at a particular point in time t. The conditional variance is $\sigma_t^2 = \mathrm{var}(r_t|\Omega_{t-1})$, where $\Omega_{t-1} = \{r_{t-1}, r_{t-2,} \cdots\}$ is the available information set at time t-1.

3.1 ARMA-GARCH Modeling: Combined Models for Level and Variance

In this paper, an ARMA model for the level r_t and a GARCH model for the variance of the innovations $\varepsilon_t = r_t - \mathrm{E}[r_t|\mathrm{R}_{t-1}]$ are combined. To account for the serial correlation in the hedge fund index returns, the mean equation is modeled by an ARMA process:

$$r_t = \mu + \sum_{i=1}^{p} \emptyset_i\, r_{t-p} + \varepsilon_t + \varepsilon_t \sum_{j=1}^{q} \theta_j\, \varepsilon_{t-j} \tag{9}$$

Equation (9) states that the current value of returns series r_t depends linearly on its own pervious values plus a combination of current and previous values of a white noise error terms.

Figure 1 shows that the variance in the returns changes over time. It is a sign of positive serial correlation in squared returns. If the variance depends on the past, the series is described as conditionally heteroscedastic. In addition, if this dependence on the past can be serially correlated, which can be expressed by autoregression, then this gives the so-called ARCH (autoregressive conditional heteroscedasticity) process. To capture this serial correlation of volatility, Engle (1982) developed the autoregressive conditional heteroscedasticity (ARCH) model. The key idea of the ARCH model is that the variance of ε at time *t*, that is, σ_t^2 depends on the size of the squared error term at the previous time t-1, that is, on ε_{t-1}^2. For instance, the ARCH (1) model is expressed as

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2. \tag{10}$$

Where the conditional variance depends on only one lagged squared error. The conditions $\alpha_0 > 0$ and $\alpha_1 > 0$ are imposed since variances σ_t^2 is non-negative. In the case of $\alpha_1 > 0$, the conditional variances are positively related, as σ_t^2 is larger for larger values of the previous innovation ε_{t-1}. In general, an ARCH(q) model with q lags is given by

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 \cdots + \alpha_q \varepsilon_{t-q}^2. \tag{11}$$

The ARCH(q) process follows an AR(q) process in the squared innovations ε_t^2.

One of the shortcomings of an ARCH(q) model is that there are $q+1$ parameters to estimate. The accuracy of model estimation might be lost as q becomes a large number. The generalized ARCH (or GARCH) model by Bollerslev (1986) is an alternative method for capturing long-lagged effects with a highly parsimonious lag shape by using ARMA modeling for the series ε_t^2. For instance, the GARCH(1,1) model is defined as

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{12}$$

Where $\omega = (\alpha_0 - \beta_1 \alpha_0)$. As variances σ_t^2 is non-negative, all three parameters ω, α_1, β_1 are non-negative. It is well known that the GARCH(1,1) model has been empirically successful in the vast majority of cases.

An ARMA(1,1)-GARCH (1,1) model is given by the conditional mean equation

$$r_t = \mu + \emptyset_1 r_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} \text{ for the level,} \tag{13}$$

Where $\varepsilon_t | \Omega_{t-1} \sim N(0, \sigma_t^2)$ with the conditional variance equation given by equation (12).

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2. \tag{12}$$

The conditional variance equation (12) states that the current fitted variance, σ_t^2 is interpreted as a weighted function of a long-term average value (dependent on ω) and information about volatility during the previous period ($\alpha_1 \varepsilon_{t-1}^2$) and fitted variance from the model during the previous period ($\beta_1 \sigma_{t-1}^2$). Large coefficient α_1 means that volatility reacts quite intensely to market movements of the previous period (i.e., the ARCH term is a reaction coefficient). Large coefficient β_1 indicates that shocks to conditional variance in the previous period are persistent and take a long time to die out, so volatility is persistent (i.e., the GARCH term is a persistence coefficient). If α_1 is relatively high and β_1 is relatively low then volatility tend to be more "spiky" (large reaction and low persistence). The sum of α and β is referred to as the persistence of the conditional variance process.[3] This GARCH(1,1) model is a special case of the more general GARCH(p,q) model, where p is the number of lagged h terms and q is the number of lagged ε^2 terms. It is worth noting that GARCH(p,q) modeling of the

[3] Bauwens et al. (2015) state that, for financial return series, empirical estimates of α and β are often in the ranges [0.02, 0.25] and [0.75, 0.98], respectively, with α often in the lower part of the interval and β in the upper part for daily series, p. 5.

conditional variance is analogous to ARMA(p,q) modeling of the conditional mean.

3.2 Asymmetric GARCH Model

It is well known that positive and negative news are often treated asymmetrically in financial markets. It has been argued that negative news about stock returns is likely to cause volatility to rise by more than positive news of the same magnitudes. Such asymmetries are called leverage effects.

The threshold ARCH model (i.e., T-ARCH) is a simple extension of GARCH with an additional term added to account for possible asymmetries. The T-GARCH model is also referred to the GJR model, named after the authors Glosten, Jagannathan and Runkle (1993). In the GJR version of the model, the specification of the conditional variance is

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \gamma d_{t-1} \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2. \tag{14}$$

$$d_t = \begin{cases} 1 & \varepsilon_t < 0 \ (bad\ news) \\ 0 & \varepsilon_t \geq 0 \ (good\ news \end{cases} \tag{15}$$

Where γ is known as the asymmetry term. When γ is 0, the GJR model converges to the standard GARCH form. On the other hand, when the shock is positive (i.e., good news) the effect on volatility is α_1 but when the news is negative (i.e., bad news) the effect on volatility is $\alpha_1+\gamma$. Thus, so long as γ is significant and positive, negative shocks have a larger effect on σ_t^2 than positive shocks. This phenomenon is well known as the leverage effect. It is important to clarify the difference between asymmetry and leverage. Asymmetry is a feature that positive and negative shocks of equal magnitudes have different impacts on volatility. The leverage effect means that negative shocks increase volatility while positive shocks decrease volatility (Caporin & McAleer, 2012).

3.3 Rolling Regression

A rolling window analysis is often used to examine whether the coefficients of the parameters are time-invariant. In fact, the hedge fund managers can introduce different strategies into the portfolio mix to diversify their income stream. This flexibility to change trading strategies is the distinctive feature that delivers hedge fund returns. These strategy shifts can be seen in response to changing market conditions, especially the structural shifts in the way diversified portfolios of hedge funds tend to correspond to major extreme market events such as the financial crisis of 2007-2009. However, the strategy classification of hedge funds in the strategy index rarely changes to reflect subsequent shifts in the fund's investment style or strategy blending (McGuire et al., 2005). Tracking the time-varying parameter can help in identifying changes in trading strategies in the index. To this end, I use a technique of rolling regression that is basically running multiple regressions with different overlapping window of values at a time. A rolling application of ARMA-GARCH modeling can potentially capture broad shift of volatility persistence in investment strategy. The estimated coefficients from these rolling regressions enable us to inspect the time-varying properties of the sensitivity to market conditions through time. The estimation of time-varying sensitivity parameters is carried out using rolling regressions of a 3-year window on daily returns, in which 756 observations are included. As the data window is rolled forward day by day, the parameter estimates were recorded.

In this paper, I use a three-stage estimation of rolling ARMA-GARCH models. In the first stage, the ARMA method is used to select the appropriate AR and MA orders for modeling the serial correlation in the lagged dependent variable (i.e., index return) and in the disturbance. The procedures follow the so-called Box-Jenkins approach: identification, estimation and diagnostic checking. The appropriate model orders are selected by using the Schwarz's (Bayesian) information criterion. In the second stage, the conditional mean equations of the GARCH(1,1) and GJR(1,1) models are estimated by using the specified AR and MA orders of ARMA models from the first stage.[4] The third stage applies the method of rolling regression to the estimated ARMA-GARCH models. The estimated coefficients from these rolling regressions reveal the time-varying properties of the sensitivity to each parameter through time.

4. Empirical Results

4.1 Persistence and Asymmetric Impact of the Shock on Volatility

The ARMA-GARCH & GJR results are given in Table 3. The conditional variance equation is specified in a GARCH(1,1) model, whose function consists of three terms: a constant term ω, the ARCH term α_1 and the GARCH term β_1. The parameter restrictions are fulfilled for all hedge fund indices. The coefficients on both the lagged squared residual and lagged conditional variance terms in the conditional variance equation are highly statistically significant for all hedge fund index returns. The persistence of the volatility is measured as the sum of $\hat{\alpha}$ and $\hat{\beta}$. The results indicate that the volatility of hedge fund returns is quite persistent. Especially, the sum of $\hat{\alpha}$ and $\hat{\beta}$ for Macro/CTA and Relative Value Arbitrage is very close to unity (approximately 0.99). This implies that shocks to the conditional variance will be highly persistent and a large positive and a large negative return will lead future forecasts of the variance to be high for a subsequent period. A volatility of half-life (i.e., the half-life period: HLP) takes 22.757 days for the Equity Hedge and 29.921 days for the Event Driven, whereby the HLP of 69.668 and 147.131 days for Macro/CTA and Relative Value Arbitrage are much higher.[5] Therefore, the return volatilities of four hedge fund indices have quite long memories. In addition, the sum of $\hat{\alpha}$ and $\hat{\beta}$ is significantly less than one, which implies the volatility process does return to its mean (Engle & Patton, 2001), in other words, it exhibits so-called mean reverting behavior.

The ARCH LM(1) test determines whether any ARCH effects remain in the residuals. The null hypothesis that no ARCH effects remain in the residuals is not rejected for all hedge fund indices in ARMA-GARCH(1,1) modeling. The ARCH-LM(1) tests confirm the null hypothesis of no first-order ARCH effects in the squared residuals of the models for four hedge fund index return-series. This result means that the ARMA-GARCH modeling takes the heteroscedasticity and the changing unconditional and conditional variance in the return-series into account.

Next, an examination of asymmetric effects on the conditional variance is conducted through assessment of an ARMA-GJR(1,1) model. The coefficient $\hat{\gamma}$ in Table 3 is positive for Equity Hedge, Event Driven, and Relative Value Arbitrage, and statistically significant. The coefficient α implies an impact of good news, while the sum of the $\hat{\alpha}+\hat{\gamma}$ implies an impact of bad news. There is the largest leverage effect for Equity Hedge since the coefficient $\hat{\gamma}$ is 0.1723. However, the coefficient $\hat{\gamma}$ is negative for Macro/CTA, provided that $\hat{\alpha}+\hat{\gamma}$ is 0.0295 ≥ 0. The specification of the GJR model is still admissible. All hedge fund index return series seem to prefer the GJR model to the GARCH model since all values of SIC decrease and ones of log likelihood function increase in the ARMA-GJR(1,1) modeling from the ARMA-GARCH(1,1) modeling. However, the Jarque-Berra statistics of ARMA-GARCH and ARMA-GJR estimations suggest that skewness and kurotosis in the standardized residuals are not completely eliminated. Munechika (2015) states that the distributions of the standardized residuals were close to Student-t distributions. It implies that the extreme downside risk cannot be captured by these models.

[4] See Munechika (2015) for AMRA-GARCH modeling and diagnostic checking in more details.

[5] Füss et al. (2007) compute the half-life period (HLP) of a shock on the process, that is, the length until half of the volatility generated by a price innovation as $\mathrm{HLP} = \log(0.5)/[\log(\hat{\alpha}+\hat{\beta})]$.

Table 3 ARMA-GARCH & GJR Modeling

	ARMA-GARCH(1,1) modeling				ARMA-GJR(1,1) modeling			
	Equity Hedge	Event Driven	Macro /CTA	Relative Value Arbitrage	Equity Hedge	Event Driven	Macro /CTA	Relative Value Arbitrage
	AR(1)	ARMA(1,2)	AR(1)	ARMA(1,2)	AR(1)	ARMA(1,2)	AR(1)	ARMA(1,2)
Mean equation								
$\hat{\mu}$	0.0288***	0.0319***	-0.0007	0.0239***	0.0162	0.0275***	0.0051	0.0156*
	(0.0071)	(0.0055)	(0.0067)	(0.0058)	(0.0076)	(0.0054)	(0.0064)	(0.0081)
$\hat{\emptyset}_1$	0.1791***	0.3063	0.0699***	0.9574***	0.1878***	0.3809*	0.0551***	0.9715***
	(0.0198)	(0.2503)	(0.0209)	(0.0131)	(0.0197)	(0.2210)	(0.0200)	(0.0106)
$\hat{\theta}_1$	—	-0.2034	—	-0.9009***	—	-0.2745		-0.9109***
		(0.2510)		(0.0249)		(0.2219)		(0.0246)
$\hat{\theta}_2$	—	0.0390	—	-0.0107	—	0.0337		-0.0080
		(0.0366)		(0.0221)		(0.0351)		(0.0224)
Variance equation								
$\hat{\omega}$	0.0045***	0.0018***	0.0020***	0.0006***	0.0077***	0.0027***	0.0011**	0.0006***
	(0.0010)	(0.0004)	(0.0005)	(0.0002)	(0.0014)	(0.0006)	(0.0005)	(0.0002)
$\hat{\alpha}_1$	0.1080***	0.0998***	0.0851***	0.1241***	0.0119	0.0448***	0.0961***	0.0737***
	(0.0187)	(0.0160)	(0.0112)	(0.0224)	(0.0193)	(0.0168)	(0.0147)	(0.0263)
$\hat{\gamma}$	—	—	—	—	0.1723***	0.0925***	-0.0666***	0.0861**
					(0.0284)	(0.0263)	(0.0160)	(0.0421)
$\hat{\alpha}+\hat{\gamma}$	—	—	—	—	0.1842	0.1373	0.0295	0.1598
$\hat{\beta}_1$	0.8620***	0.8773***	0.9051***	0.8712***	0.8392***	0.8668***	0.9344***	0.8778***
	(0.0183)	(0.0151)	(0.0122)	(0.0187)	(0.0182)	(0.0162)	(0.0123)	(0.0153)
$\hat{\alpha}_1+\hat{\beta}_1$	0.9700	0.9771	0.9901	0.9953				
HLP	22.757	29.921	69.668	147.131				
SIC	0.7472	0.0422	0.7750	-0.5957	0.7266	0.0367	0.7672	-0.6023
LogL	-1049.69	-32.5273	-1089.5	880.6709	-1016.22	-20.6415	-1074.31	894.0463
ARCH effect: $\hat{\varepsilon}^2$								
ARCH LM(1) test	1.9547	0.7942	0.0003	0.4168	3.9839**	1.1978	2.3625	1.4231
Standarized Residuals: $\hat{z}_t = \hat{\varepsilon}_t / \hat{\sigma}_t$								
Mean	-0.0429	-0.0271	0.0178	-0.0275	-0.0126	-0.0105	0.0000	-0.0051
Std. Dev.	0.9985	0.9993	0.9996	0.9990	0.9995	0.9998	0.9997	0.9993
Skewness	-0.4978	-0.4286	-0.5018	-0.1246	-0.4515	-0.4309	-0.4624	-0.0361
Kurtosis	4.8586	5.1456	6.5193	5.8927	4.9037	5.2338	5.8252	6.1179
Jarque-Bera	530.33***	636.844***	1597.628***	1005.583***	529.625***	683.873***	1054.157***	1160.280***
Ljung-Box statistic	*Ho*: no-autocorrelation							
$\hat{z}_t$: Q(12)	6.968	20.180**	3.754	12.672	7.144	19.107**	4.004	8.516
$\hat{z}_t^2$: Q(12)	19.385**	16.435*	3.118	16.047*	15.456	16.182**	11.550	14.330

Notes: Based on daily continuously compounded returns for 2864 observations 04/01/2003 to 08/11/2014; standard errors are presented in parenthesis; The statistical significance is determined by using Bollerslev-Wooldridge robust standard errors; ***, **, * denote significance at 99%, 95% and 90% confidence levels, respectively.

4.2 Volatility Dynamics and the Financial Crisis

Rolling ARMA-GARCH modeling is conducted using a three-year rolling window on daily data over the full sample period. Figure 2 summarizes the rolling ω coefficients of four strategies. The vertical axis represents rolling ω coefficients, that is, the long-term volatility and the horizontal axis represents the starting date of the sample period. Panel (a), (b) and (c) plot the rolling GARCH omegas, the rolling GJR omegas excluding

Macro/CTA) and the rolling GJR omegas including Macro/CTA, respectively. The shaded area represents the data period including the financial crisis. In panel (a), the long-term level of volatility had been slightly drifting downwards prior to the financial crisis, with the possible exception of Macro/CTA. Moreover, the difference of the rolling GJR omegas across strategies decreased gradually during the period. The convergence in long-term volatility is generally consistent with the growth of the industry that has led to greater institutionalization of the hedge fund sector.[6] The GARCH ω constant for Equity Hedge increased during the financial crisis and declined significantly thereafter. That of Macro/CTA suddenly jumped and fluctuated widely during the financial crisis. It is possible to say that the financial crisis of 2007-2009 strongly affected the directional strategies. On the contrary, the GARCHω constant for Event Driven and Relative Value Arbitrage were relatively stable for the whole sample period. Panels (b) and (c) show the estimates of rolling GJR model. The development of the rolling GJR ω coefficients for Equity Hedge, Event Driven and Relative Value Arbitrage were not so different from the results of rolling GARCH model. However, the rolling GJR ω coefficient revealed isolated outliers of Macro/CTA during the latter part of the financial crisis.

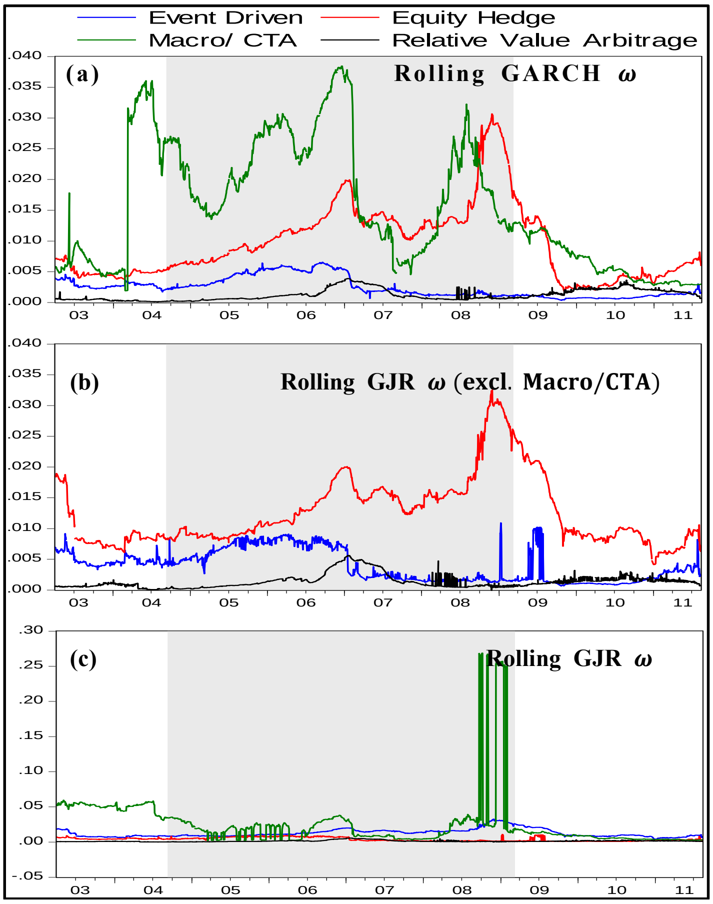

Figure 2 Long-term Volatility Levels

[6] McGuire and Tsatsaronis (2008) states that hedge funds have been increasingly forced to adopt more stable investment profiles, and aim to deliver more predictable returns, even at the expense of the absolute level of those returns, p. 9.

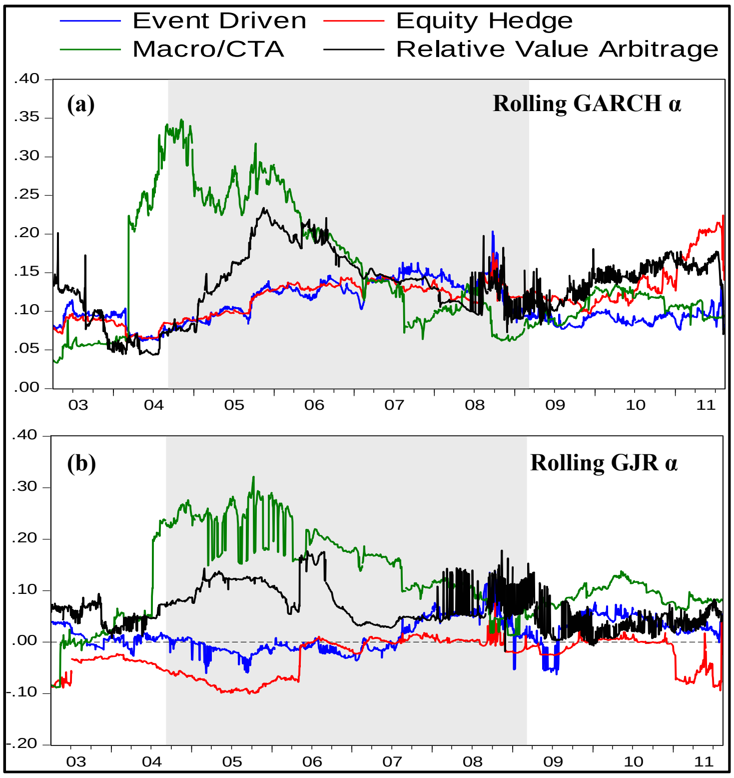

Figure 3 Reaction Coefficients

Figure 3 presents the reaction coefficients α for the different fund styles based on the rolling regressions. In panel (a), the variation in the rolling GARCH α follows a similar pattern for Equity Hedge and Event Driven during the period of 2003 and 2008. Following the financial crisis, their estimated rolling GARCH α increased gradually. This pattern is particularly clear for Relative Value Arbitrage, and the estimated rolling GARCH α showed larger fluctuations during the crises, after which appeared a quite noisy pattern. Macro/CTA displayed the largest fluctuation among four strategies during the period of the early part of the financial crisis, subsequently the reaction coefficient diminished greatly. Interestingly, the rolling GJR α for Event Driven and Equity Hedge in panel (b) displays quite different patterns from panel (a). The estimated rolling GJR α coefficients for the two strategies frequently indicate the negative values, over the sample period, especially during the financial crisis. The negative values of the estimated coefficients are not allowed under the non-negativity conditions of the GARCH and GJR models. In Figure 4, the pattern of the rolling GARCH and GJR β (persistence) coefficients for all strategies follows the almost reverse movements to ones of the rolling GARCH and GJR α. In particular, the rolling GJR β coefficient for Macro/CTA displays the isolated outliers in the latter part of the financial crisis. Figure 5 represents the rolling GJR γ coefficient, which exhibits asymmetric impact to the shocks. Quite interestingly, Macro/CTA showed a negative asymmetric impact between 2005-2008.

Over all, these results allow for some tentative but broad conclusions. First, hedge fund strategies that supposedly follow different investment styles appear to have, to some degree, similar volatility movements. The rolling GARCH and GJR ω and α coefficients considerably increased during the financial crisis. The similarity in the pattern of parameter shifts of the equity-related funds (Equity Hedge), excepting for Macro/CTA, and the non-directonal funds (Event Driven and Relative Value Arbitrage) over the sample period is particularly striking. Second, the volatility characteristic of Macro/CTA was extremely dynamic which indicated over-reactive, low persistence, and the negative asymmetric effect of volatility. The GJR model is still admissivle for Macro/CTA because the sum of α and γ is larger than zero. The strategy of Macro/CTA is called "trend following". The strategy tends to apply mechanical rules, such as moving averages of asset prices, to capture "trends" in markets. Fung and Hsieh (2007) point out that, while it may be easy to identify a trend ex-post, it is difficult to do so ex-ante.

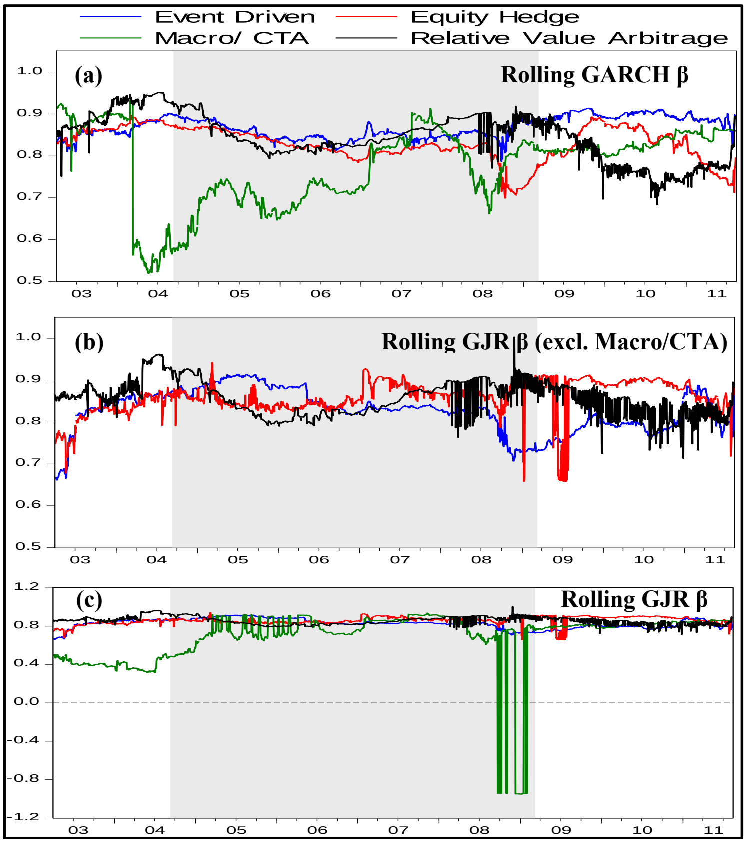

Figure 4 Persistence Coefficients

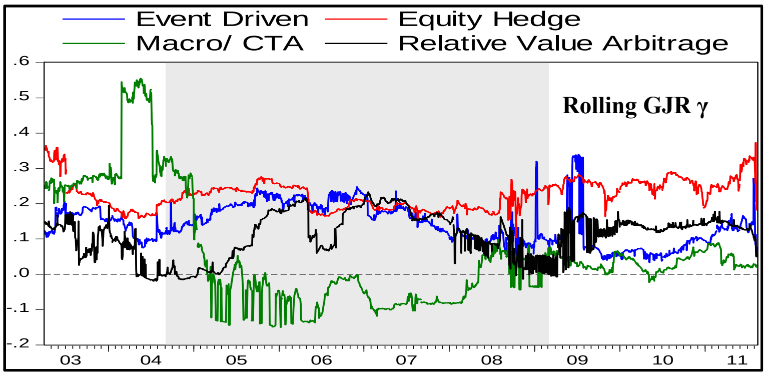

Figure 5 Asymmetric Impact

To sum up, rolling regressions have revealed time-varying characteristic of parameter estimations for hedge fund strategies. The results have demonstrated that the parameter estimation has been strongly affected by the financial crisis of 2007-2009, especially for Macro/CTA.

5. Concluding Remarks

In this paper, a rolling application of ARMA-GARCH type modeling has been employed to investigate the persistence and volatility of hedge fund index returns, and compared with each other. The empirical results shows significant differences concerning performance persistence, and asymmeric impacts of the shocks on volatility among the strategies concerned.

First, as for model selection, the ARMA-GJR(1,1) models were preferred to the ARMA-GARCH(1,1) models based on the information criterion and log likelihood function for all hedge fund strategy index returns. However, the distributions of the standardized residuals of the ARMA-GARCH and GJR models for all strategies exhibit leptokurutosis. These characteristics have significant consequences on downside risk evaluation in the case of VaR measurement including time-varying conditional volatility. Second, through the ARMA modeling for the conditional mean equation, the estimated AR(1) term of Relative Value Arbitrage exhibits high serially correlation, whereas Equity Hedge (equity-based strategy) and Macro/CTA (trend following strategy) show relatively low serial correlation. It implies that illiquid hedge fund strategies tend to exhibit high levels of performance persistence, while more liquid strategies have low levels of performance persistence. Third, the ARMA-GJR(1,1) models reveal that Equity Hedge, Event Driven, and Relative Value Arbitrage have the leverage effect and only Macro/CTA shows the negative asymmetric impact of the shock on the volatility. Moreover, a rolling regression revealed time-varying characteristics of parameter estimation for shifts in the volatility process. In particular, the volatility characteristic of Macro/CTA was extremely ‘spiky’ and negative asymmetric impact during the period of the financial crisis, in which the shifts in the volatility process to the market regime might have occurred. Macro/CTA exhibits striking differences in time-varying parameter estimations of ARMA-GJR modeling among hedge fund strategies. The outliers might have an excessive impact on its parameter estimates, and thus, potentially amplify market volatility.

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