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A Study on Economic Growth Convergence and Human Capital: Updating an Empirical Study

Kaveh Dalvand
(School of Economics, University of Maine, USA)

Abstract: In this paper first the text book Solow model for growth has been considered. Taking the rates of saving and population growth as exogenous to correlate them with growth rates (GDP change has been considered as a proxy for growth rate). Second, an augmented Solow model is used to include the accumulation of human capital as well as physical capital and see the effect of it on the economic growth. And finally, examining the predictions of the augmented Solow model is for a behavior out of steady state condition. In this part, the assumption of decreasing returns of all capitals is left out so that there are no steady state for countries' per capita income and differences in income per capita can persist indefinitely even if countries have the same savings and population growth rates. The time span that the data is gathered is from1986-2010.

Key words: Solow model; cross-country growth convergence; production function; OLS
JEL codes: O470, E230

1. Introduction

Our world consists of all types of economies that are different in richness and growth rate. Some countries are very rich and some are very poor. Some of these countries are growing fast while some do not grow at all. Most of the countries lie between these intervals. What variables can affect the level of development (developed and developing countries) and how these parameters are linked to the level of GDP or growth rate? This question is what this paper is going to explore by examining the Solow Growth model consistency with cross country data and using the approach of Mankiw et al. (1992). Other effective variables on economic growth, like private investment, government spending, political stability and price distortion, are studied by Robert Barro (1991). A comprehensive study on the well-stablished facts in economic growth has been recently done by Jones (2015). The worldwide spread of economic growth is a debatable issue that has gotten so much attention of economic scholars since a few decades ago. During the recent two centuries of economic history, human education (Aghion et al., 2009) and technological changes (Verspagen, 20001) which are deeply interrelated, have been recognized as two of the main causes of the difference in the economic development levels of nations (Easterlin, 1981). Economic growth is also largely seen as aresult of investment (Borensztein et al., 1998). This is particularly obvious when investment considered as a source to increase future income. So many researchers have been conducted to explore and identify different types of investment made by several economic sectors (Kendrick et al., 1976).

Kaveh Dalvand, Master's Student, School of Economics, University of Maine; research areas/interests: macroeconomics, growth economics, international finance. E-mail: Kaveh.dalvand@maine.edu.

Robert Solow (1956) proposed a model of economic growth by assuming a standard production function with decreasing returns to capital. For steady state level of per capita income, he took the saving rate and the population growth rate as exogenous variables. Because different countries have different rate of saving and population growth rate then they reach to the different levels of steady state. Saving rate has a direct relationship and population growth rate has an inverse relationship to the growth rate. By using recent data (1986-2010) for a large set of countries we found out that the prediction of Solow Model for the direction of saving rate and population growth rate effects on income is consistent with real data. Although the model correctly predicts the sign of saving rate and population growth rate but it does not correctly predict the magnitudes. To better understand the relationship between income, saving rate and population growth rate it is necessary to go beyond the textbook Solow Model.

For this purpose, we should include human capital accumulation in the equation. Barro (2001), uses the role of education to measure the value of human capital. He distinguishes the quantity of school attainment from the school quality by using the results from international standard examinations. Hanushek and Dennis (2000) also discussed in detail the labor force quality and its relationship with development. In this paper, we just use the method as used by Mankiw et al. (1992). Adding human capital variable to the equation can increase the precision of the model because human capital accumulation maybe correlated with saving rate and population growth rate; hence omitting human capital accumulation would bias the estimated coefficients on saving rates and population growth rates. For augmented Solow model, a proxy for human capital accumulation is included as an additional explanatory variable. By adding this variable into the model it can account for about more than 60 percent of cross-country variation in income. Considering the inescapable imperfection of in this sort of cross-country data, the fit of this simple model is notable.

After augmented Solow model, the issue of convergence of countries in per capita income has been examined. Solow model predicts different countries reach to different steady states and one should not expect all countries to converge to the same steady state level. There are so many articles that discuss the convergence even among industrial nations appears to have weaker force than for divergence. Their finding does not support the claim that those nations that are able to embrace technological changes are converged to the same level of income and productivity (De long, 1988).

2. The Textbook Solow Model

In Solow model the rate of saving, technological progress and population growth are exogenous. Capital and labor are two inputs which are paid their marginal products. In the Cobb-Douglas production function the production at time t is:

$$Y(t) = K(t)^{\propto}\big(A(t)L(t)\big)^{1-\propto} \qquad 0 < \propto < 1 \tag{1}$$

Y is output, K capital, A level of technology, and L labor. A and L are assumed to grow exogenously at rates g and n:

$$L(t) = L(0)e^{nt} \tag{2}$$

$$A(t) = A(0)e^{gt} \tag{3}$$

The model assumes that a constant fraction of output, S, is invested. Defining k as the stock of capital per effective unit of labor, $k = \frac{K}{AL}$, and y as the level of output per effective unit of labor, $y = \frac{Y}{AL}$, the evolution of capital is indicated by:

$$\dot{k}(t) = Sy(t) - (n+g+\delta)k(t) = Sk(t)^{\alpha} - (n+g+\delta)k(t) \quad (4)$$

In the above equation δ is the rate of depreciation. Equation (4) implies that in steady-state k converges to k^*:

$$k^* = [\frac{S}{n+g+\delta}]^{\frac{1}{1-\alpha}} \quad (5)$$

Substituting equation (5) into the production function and taking logs, gives us steady-state income per capita:

$$Ln\left(\frac{Y(t)}{L(t)}\right) = LnA(0) + gt + \frac{\alpha}{1-\alpha}Ln(S) - \frac{\alpha}{1-\alpha}Ln(n+g+\delta) \quad (6)$$

A(0) reflects not only technology but natural resources, infrastructures, climate and so on. So it defers across countries. We assume that:

$$LnA(0)= a+\varepsilon \quad (7)$$

Where a is a constant and ε defers across countries. Thus the formula (6) at a given time, time zero for simplicity, turns into:

$$Ln\left(\frac{Y}{L}\right) = a + \frac{\alpha}{1-\alpha}Ln(S) - \frac{\alpha}{1-\alpha}Ln(n+g+\delta) + \varepsilon \quad (8)$$

Because this model assumes that the factors are paid their marginal products, it should predict both the sign and magnitude of coefficients on saving and population growth. The capital's share in income (α) is approximately one third, so the model implies that the elasticity of income per capita with respect to the saving rate is about 0.5 and the elasticity with respect to $n+g+\delta$ is -0.5. We assume that g and δ are constant across the countries because g shows the progress of knowledge which today can be traded between nations and is not country-specific and also there is no strong reason to accept that there is a considerable difference in depreciation rates across countries.

Equation 8 is the basis for our estimation and we assume that saving rate and population growth rate are independent of country-specific factors that can shift the production function. So S and n are independent of ε and this implies that we can use Ordinary Least Square (OLS).

The data set are gathered from the Penn World Table (8.1) and World Bank website and it includes real income, labor force, saving rate, and population growth rate. The data are annual and cover the period 1986-2010. n is measured as the average rate of growth of the working-age population, where working age is defined as 15 to 64. Saving rate is measured as the average share of real investment in real GDP, and Y/L as the real GDP in 2010 divided by working age population in that year. Three samples of countries are considered in the study. The largest sample is consisting of all countries for which data exist except those for which oil production is the dominant industry. This sample includes 90 countries. The second sample excludes countries whose population are low. The small countries are omitted because their level of income maybe dominated by specific factors. This sample consists of 71 countries. The third sample includes OECD countries whose population is greater than one million. This sample has the benefit of high-quality data and the variation in omitted country-specific factors is likely to be insignificant. But the small size of this sample is a disadvantage that removes much of the variation in the variables of interest.

Table 1 Estimations of Textbook Solow Model

Dependent Variable: log GDP per working person			
Sample	Non-Oil	Intermediate	OECD
Observations	90	71	22
Constant	18.06 (9.43)	13.32 (5.71)	13.67 (8.52)
Ln(S)	0.29 (0.97)	1.31 (2.93)	-0.25 (-0.70)
Ln(n+g+d)	-4.72 (-6.99)	-3.83 (-5.91)	-0.99 (-1.51)
R^2	0.38	0.53	0.13
s.e.e.	1.04	0.80	0.23
Harvey test (p-value)	0	0.01	0.13
Restricted regression			
Constant	8.92 (41.51)	7.59 (24.03)	11.08 (22.07)
Ln(S) - Ln(n+g+d)	0.77 (4.33)	2.13 (8.02)	0.03 (0.08)
R^2	0.17	0.48	0.003
s.e.e	1.19	0.84	0.24
Test of Restriction			
P-value	0	0.007	0.11
Implied α	0.43	0.23	0.02

Note: t-statistics are in parentheses. (g+δ) is assumed to be 0.05.

Table 1 reports the results of the estimation of equation 8 both with and without imposing the constraint that the coefficients of Ln(S) and Ln(n+g+δ) are equal in magnitude and opposite in sign. There are correct signs in three groups of countries except for slope coefficient in OECD countries which it is not statistically significant. The restriction that the coefficients on Ln(S) and Ln(n+g+d) are equal in magnitude and opposite in signs is just accepted for OECD countries but it has been rejected for two other samples. The empirical level of α is about $\frac{1}{3}$. In the first two samples the calculated α level is fairly close but in the third sample (OECD) has been tremendously underestimated. In the third sample because of the small size of sample results are not satisfactory but in two other samples, the model can account for about half of variation in income per capita among nations. This model is not completely successful because the estimated impacts of saving and labor force growth are much larger than the model predicts.

3. Adding Human-capital Accumulation to the Solow Model (Augmented Model)

In this section, the Solow model is expanded to include the human capital. Let the production function for this case be:

$$Y(t) = K(t)^{\propto} H(t)^{\beta} \big(A(t)L(t)\big)^{1-\propto-\beta} \tag{9}$$

Where H is the stock of human capital and all other variables are the same as before. If S_k be the fraction of income invested in physical capital and S_h be the fraction of income invested in human capital the change in physical capital and human capital over time is denoted by:

$$\dot{k}(t) = Sk \times y(t) - (n + g + \delta)k(t) \tag{10}$$

$$\dot{h}(t) = Sh \times y(t) - (n + g + \delta)h(t) \tag{11}$$

Where y = Y/AL and k = K/AL, and h = H/AL are quantities per effective unit of labor. We assume that α+β < 1. This implies that there are decreasing returns to all capitals and we have a steady state. Otherwise, there is no steady state for this model (α+β =1, α+β > 1, this case will be discussed in the next session).

$$k^* = [\frac{{S_k}^{1-\beta}{S_h}^{\beta}}{n+g+\delta}]^{\frac{1}{1-\propto-\beta}}$$
$$h^* = [\frac{{S_k}^{\alpha}{S_h}^{1-\alpha}}{n+g+\delta}]^{\frac{1}{1-\propto-\beta}} \qquad (12)$$

Substituting equation (12) in the production function and taking logs results an equation for income per capita as follows:

$$Ln\left(\frac{Y(t)}{L(t)}\right) = LnA(0) + gt - \frac{\propto+\beta}{1-\propto-\beta}Ln(n+g+\delta) + \frac{\propto}{1-\propto-\beta}Ln(S_k) + \frac{\beta}{1-\propto-\beta}Ln(S_h) \qquad (13)$$

Equation 13 shows that income per capita depends on population growth and accumulation of physical and human capital. Likewise the former section, α is the physical capital share of income, and β in this equation is human capital share of income. This factor, β, empirically is expected to be between one-third and one-half. Equation (13) makes two predictions about the model when human capital is introduced to the model. First, the coefficient on Ln(s) would be greater than the previous section where human capital had not been introduced. Hence the presence of human capital accumulation increases the impact of physical capital accumulation on income. Second, the coefficient of Ln(n+g+δ) is larger in absolute value than the coefficient on Ln(S_k). So the high population growth lowers income per capita because the physical and human capital must be devoted more lightly over the population.

To use this model official education is used as a proxy for human capital thus the investment in healthcare or any informal education will be ignored. The rate of human capital accumulation (S_h) measures roughly the percentage of the working-age population that is in tertiary school (Rate of human capital accumulation = % tertiary school enrolment × % labor force participation rate for age 15-24, in 1985).

Table 2 Estimations of the Augmented Solow Model

Dependent Variable: log GDP per working person			
Sample	Non-Oil	Intermediate	OECD
Observations	90	71	22
Constant	13.64 (9.43)	9.18 (4.33)	12.92 (8.73)
Ln(S)	0.074 (0.33)	1.25 (2.96)	-0.22 (-0.48)
Ln(n+g+d)	-2.48 (-4.32)	-1.96 (-3.63)	-0.94 (-2.19)
Ln(hc)	0.49 (9.11)	0.43 (8.29)	0.19 (2.28)
R^2	0.62	0.71	0.27
s.e.e.	0.82	0.63	0.22
Harvey test (p-value)	0.01	0.04	0.58
Restricted regression			
Constant	9.86 (54.15)	8.57 (31.40)	10.9 (15.64)
Ln(S) - Ln(n+g+d)	0.19 (1.39)	1.32 (5.81)	-0.01 (-0.02)
Ln(hc) - Ln(n+g+d)	0.57 (9.41)	0.44 (7.34)	0.22 (2.31)
R^2	0.59	0.71	0.19
s.e.e	0.85	0.63	0.23
Test of Restriction			
P-value	0.01	0.73	0.19
Implied α	0.04	0.47	-0.22
Implied β	0.31	0.16	0.19

Note: t-statistics are in parentheses. (g+δ) is assumed to be 0.05.

In Table 2, by introducing human capital, the fit of regression has been greatly improved compared to Table 1. Harvey test is performed for three samples, for Non-oil and intermediate samples found out that the residuals are heteroskedastic. Thus to estimate std. errors and p-values of these samples robust estimation have been used to have a consistent error variance-covariance matrix. In first two samples, signs of coefficients are correct with respect to theory and they are statistically significant. In the third sample, the sign of saving share of GDP, Ln(S), is not consistent with theory and it is not statistically significant. The bottom half of Table 2 shows that the restriction that the coefficients on Ln(S), Ln(n+g+d) and Ln(hc) sum to zero is rejected in the first sample. The values of implied α and β here are less precise than Table 1. All in all, adding human capital to the Solow model improves its performance even by using an imprecise proxy for human capital.

4. No Steady-state and Convergence

As King and Rebelo (1993) suggest, by the assumption of diminishing returns to the reproducible factors of production it is difficult to explain sustained differences in growth rates. If the assumption of decreasing returns to capital in the previous section could be excluded and instead take the value of $\propto + \beta = 1$, then our model would turn into an endogenous growth model. One of the implications of this assumption is that countries that save more grow faster forever and that countries need not to converge in income per capita, even if they have the same productivity and technology level. In this section, first the correlation between per capita growth rates with the starting level of per capita income has been examined. In the Solow model this correlation tends to be negative. It means that poor countries with lower level of per capita income are more likely to have a higher growth rate in income per capita than rich countries.

The Solow model does not predict that all countries will converge to a specific level of income or welfare. It states that each country has its own specific level of convergence. Furthermore, the Solow model makes quantitative predictions on the speed of convergence to steady state. If y* be the steady state level of income per effective worker and y(t) be the actual value at time t, then the speed of convergence is given by:

$$\frac{dLn(y(t))}{dt} = \lambda(Ln(y^*) - Ln(y(t))) \quad (14)$$

Where $\lambda = (n + g + \delta)(1 - \propto - \beta)$

The model suggests a natural regression to study the rate of convergence. Then,

$$Ln(y(t)) - Ln(y(0)) = (1 - e^{-\lambda t})Ln(y*) + e^{-\lambda t}Ln(y(0)) \quad (15)$$

Substituting for y*:

$$Ln(y(t)) - Ln(y(0)) = (1 - e^{-\lambda t})\frac{\propto}{1-\propto-\beta}Ln(S_k) + (1 - e^{-\lambda t})\frac{\beta}{1-\propto-\beta}Ln(S_h) - (1 - e^{-\lambda t})\frac{\propto+\beta}{1-\propto-\beta}Ln(n + g + \delta) - (1 - e^{-\lambda t})Ln(y(0)) \quad (16)$$

Thus in the Solow model, the growth of income is a function of the final steady state level and initial level of income. In endogenous growth models, there is no steady state level of income. Specifically, Single sector model predicts no convergence of any kind. Because of this, they predict a coefficient of zero on Ln(y(0)) in equation (16).

In Table 3, the log of income per capita at the beginning of period appears alone on the right-hand side of the equation. This table shows the failure of incomes to converge for the first two groups. For non-oil and intermediate samples the $R^2$is almost zero and slope coefficients are not statistically significant which in both cases we accept the null hypothesis, means that explanatory variable have no systematic effect on the dependent variable and poor countries have no tendency to grow faster than rich countries regarding their initial GDP level.

But for OECD countries the estimation of coefficients are statistically significant and this result is consistent with the findings of literature.

Table 3 Tests for Unconditional Convergence

Dependent Variable: log difference GDP per working person 1986-2010			
Sample	Non-Oil	Intermediate	OECD
Observations	90	71	22
Constant	0.008 (0.69)	0.013 (1.02)	0.13 (3.05)
Ln(Y86)	0.0003 (0.26)	9.95E-06 (0.007)	-0.01 (-2.73)
R^2	0.0008	0.000001	0.27
s.e.e.	0.01	0.012	0.005

Note: t-statistics are in parentheses. (g+δ) is assumed to be 0.05.

Table 4 Tests for Conditional Convergence

Dependent Variable: log difference GDP per working person 1986-2010			
Sample	Non-Oil	Intermediate	OECD
Observations	90	71	22
Constant	0.07 (1.97)	0.02 (0.73)	0.13 (1.89)
Ln(Y86)	-0.002 (-1.14)	-0.004 (-3.13)	-0.009 (-2.18)
Ln(S)	0.002 (0.58)	0.02 (4.34)	-0.009 (-1.23)
Ln(n+g+d)	-0.026 (-2.34)	-0.019 (-2.08)	0.009 (0.61)
R^2	0.068	0.28	0.34
s.e.e.	0.014	0.01	0.005
Harvey test (p-value)	0	0.02	0.50

Note: t-statistics are in parentheses. (g+δ) is assumed to be 0.05.

In Table 4, the saving rate and population growth rate are included on the right-hand side of the equation. Harvey test recognizes that for Non-oil and intermediate samples the error term is heteroskedastic. Therefore, in order to estimate std. errors and p-values of these samples robust estimation have been used to have a consistent error variance-covariance matrix. First two samples signs of slope coefficients are consistent with the theory. In the third sample, OECD, the sign of the slope coefficient for saving share is not consistent with the literature and it is not statistically significant. In all three samples, even though the coefficients on the initial level of income have very small values but compared to other coefficients they are considerable and show convergence. For the second and third groups, this convergence is statistically significant while for the first group it is not. Furthermore, the addition of saving rate and population growth rates improve substantially the fit of the regression compared to the Table 3.

In Table 5, the human capital accumulation is added as an explanatory variable to the right-hand side of the regression in Table 4. The addition of human capital further improves the fit of regression. In the first sample, Non-oil, all of the signs are consistent with the theory but only the constant and human population growth rate coefficient are statistically significant. In the second sample, intermediate, the sign of coefficients are correct with regard to theory but the coefficient of human capital is not statistically significant. In the third sample, OECD, second and fourth coefficients do not have correct sign and none of them are statistically significant and this is most likely because of small sample size.

Table 5 Tests for Conditional Convergence

Dependent Variable: log difference GDP per working person 1986-2010			
Sample	Non-Oil	Intermediate	OECD
Observations	90	71	22
Constant	0.08 (2.12)	0.028 (0.78)	0.099 (1.30)
Ln(Y86)	-0.003 (-1.53)	-0.005 (-2.61)	-0.006 (-1.18)
Ln(S)	0.002 (0.49)	0.023 (4.53)	-0.010 (-1.28)
Ln(n+g+d)	-0.020 (-2.23)	-0.017 (-1.91)	0.014 (0.84)
Ln(hc)	0.001 (1.04)	0.001 (1.07)	-0.003 (-0.98)
R^2	0.08	0.29	0.38
s.e.e.	0.014	0.01	0.005
Harvey test (p-value)	0	0.06	0.93

Note: t-statistics are in parentheses. (g+δ) is assumed to be 0.05.

5. Conclusion

The results show that higher population growth rates would lower income per capita because the available capital must be spread more thinly over the population. Using the augmented Solow model indicates that incorporation of human capital would increase the model ability to capture international differences in income per capita across countries. In this augmented model, the human capital must be spread more thinly which implies higher population growth rates would lower the total factor of productivity. Augmented Solow model shows that differences in saving rates, human capitals, and population growth rates explain about 70 percent of cross-country differences in the income per capita. Unlike endogenous growth models, this model predicts that countries with similar technology levels, rates of accumulation in thehuman capital and physical capital, and population growth rates would converge in income per capita and differences in income among countries could not last indefinitely.

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