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View the World by Equations

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Abstract: With the progress of modern science and technology, we can translate any equations we can imagine to images with the help of the power of calculate ability by computer. People in the past have been looking forward to using the equations to express the Chinese character image. However as a senior high student to be, I can express any pictures I want, whatever it's huge like star or tiny like molecular in equations with the help of computers.

In this paper, with the use of HP graphing calculator, based on using equations to write Chinese characters, then we can spread it out to use equations building reality and inspire the innovation of creation. From the point to the surface, I will lead you into the equation world in my eyes step by step. I will show you all my works drown by equations and point out my ideas about them.

Key words: equation, graphing calculator, geometry, algebraic, artwork

1. Introduction

As time goes by, people study deeper about equations. In the 19th century, people made great progress in astronomy and geography under the guidance of some equations. For example, on September 23, 1846, by the observation on the perturbation of Uranian orbit, astronomers deduced the existence and possible location of Neptune. Nowadays the applications of equations are almost everywhere. However, that might not be the whole story. Possibly there are still many mysteries buried in the equations waiting for us to discover.

In a lesson called Computation and Geometry in grade 8, we first learned the geometric properties of the circle and then deduced its equation. After that we generalized the equation to get the equation of the ellipse, which is more complex. By the study of this equation we learned more about the geometric properties of the ellipse. I was impressed by the idea that we moved ahead from the simple to the complex and from the special to the general. When the geometric object becomes more convoluted, the corresponding algebraic structure also becomes more complex. On the contrary, if we construct complex algebraic structure we will get convoluted geometric object. So in this paper we try to make graphs by algebraic expressions.

2. Make Given Graphs by Algebraic Expressions: Write Chinese Characters by Equations

In this part we try to write Chinese characters by equations, and our goal is to write 我爱北京市第二十二中学 (I love Beijing No. 22 middle school). Firstly we view each character as a geometric object. Each character

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consists of several strokes, and each of them can be assigned to a fundamental geometric object. Then we try to depict these fundamental objects so that we can write the characters. This idea can be realized by technology with the help of advanced graphing function in HP graphing calculator. Now we will give an example of the character 北.

The character 北 has 5 strokes. We set it at left bottom of the first quadrant in the coordinate plane. The first 3 strokes on the left can be simply expressed by linear equations. The fourth stroke ㇒ is a curve, which is more difficult to depict. We consider achieving it by the exponential function. With the help of HP graphing calculator, we can make a first survey on the exponential function, i.e., to see how the graph changes when we alter the parameters a and m in the expression y = max (Figures 1 & 2). By the location of the character we shall limit the domain of the exponential function in the interval [3, 5]. Input the value of a and m and we get fitting curve of the stroke. At first we try a = 2 and Vm = 1, and we find y varies too much in the interval, so we can make a and m slightly smaller to fit our requirement. After repeated attempts we get the right algebraic expression y = (6/5) x+2 and x >= 3 and x <= 5 (Figures 3 & 4). In the process of writing Chinese characters we also learn many properties of functions and equations. Our final result is shown in Figure 5.

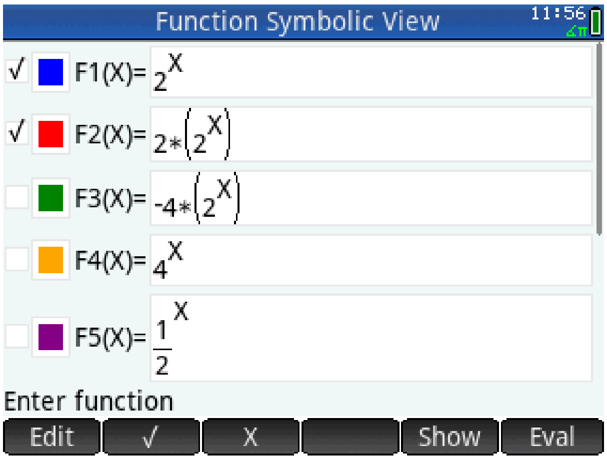

Figure 1

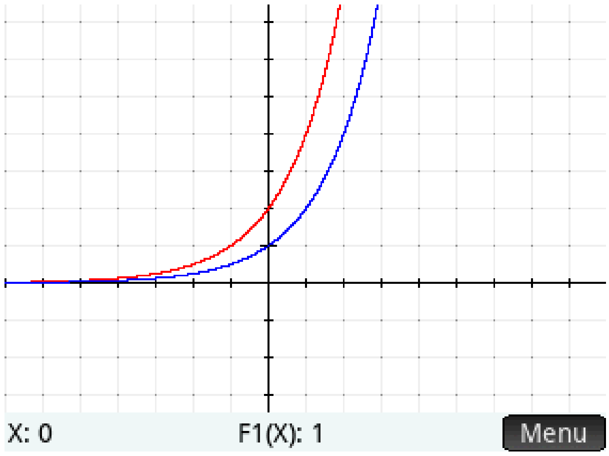

Figure 2

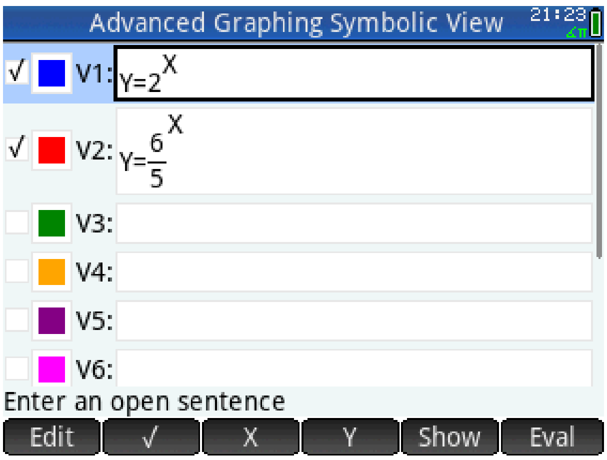

Figure 3

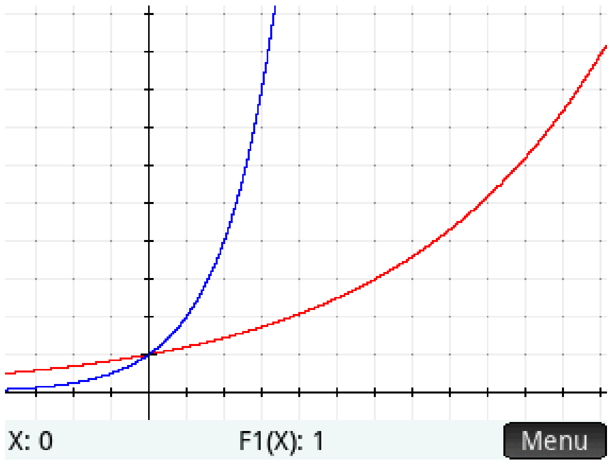

Figure 4

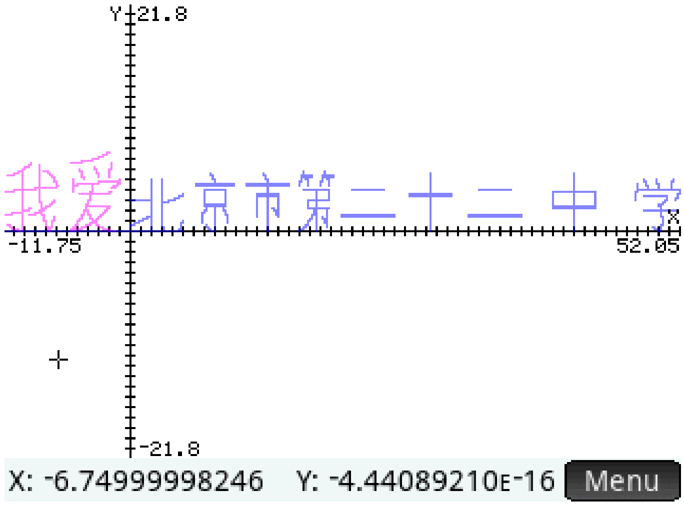

Figure 5

3. Create Certain Pattern by Algebraic Expressions: Study on the Mosaic

3.1 Construct a Mosaic in a Limited Area

Figure 6 is what is shown by the graphing calculator when the plot interface is open, possibly due to the large quantity of computation. It is called mosaic by my classmates for fun. Inspired by this idea, our next goal is to construct a true mosaic (Figure 7). A mosaic consists of several squares, while each square can be easily constructed by algebraic expressions. For example, if we want to make a square with side length 5, a vertex at the origin and two sides lie on the x-axis and y-axis respectively, we can write y >= 0, x >= 0, y <= 5, x <= 5 to remove the region outside the square.

Consequently this part is quite easy, and our result is shown in Figure 7.

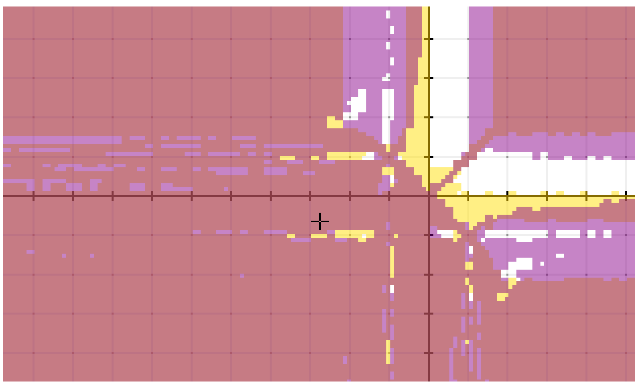

Figure 6

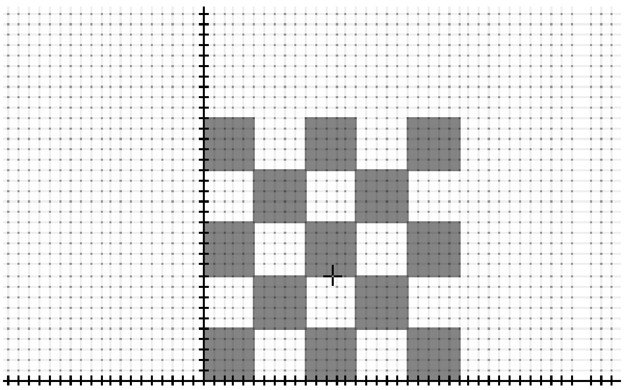

Figure 7

3.2 Unexpected Discovery: A First Survey on MOD Operation

Actually we are not so satisfied with the former result, since it constructs mosaic only in a bounded region. Next we want to find an algebraic expression that can fill the whole coordinate plane with the mosaic pattern. A first observation on the mosaic pattern shows that in a single row, the black and white (blank) squares turn up alternately (Figure 8). Thus we need a function taking two constant values periodically (like Figure 9). After hard thinking without any solutions I decided to look up information, and I found an operation that I have never seen before: MOD. With the graphing calculator we can construct some equations containing this operation to find out how it is defined. I tried x MOD 1 = y, 1 MOD x = y, x MOD y = y, y MOD x = y, etc., and took down their graphs and number charts. However, I did not find any regularity that could help me to guess the definition. Finally I turned to the references again and got the answer. MOD is the modulo operation, i.e., a MOD b gives the reminder of the division a over b.

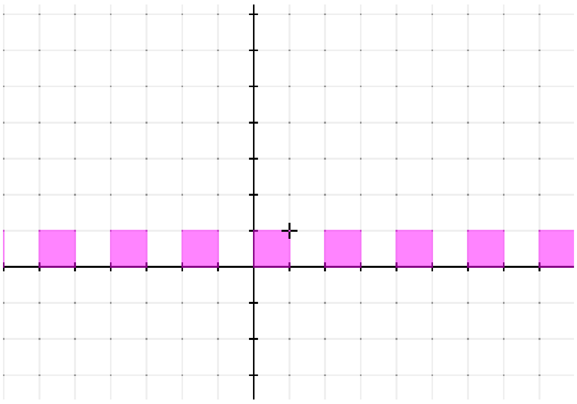

Figure 8

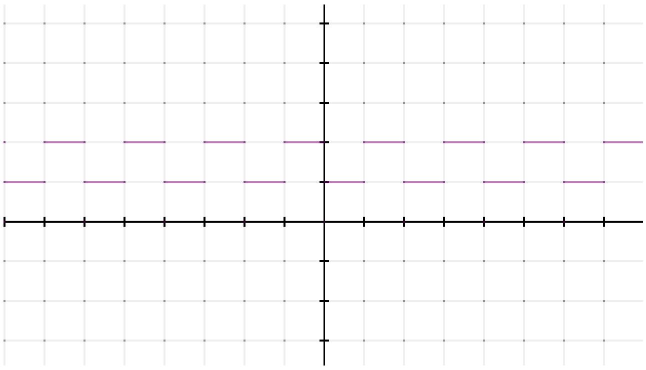

Figure 9

3.3 Think Deeper: A Survey on the Mosaic Covering Whole Plane

Equipped with the new operation, we start to analyze how to construct the mosaic pattern in the whole coordinate plane. Now the graph of x MOD 1 = y, which I tried before, called my sight (Figure 10). This is a periodic function with period 1. For simplicity we consider x MOD 2 = y, and then we take integral part to make the function have only 2 values, getting the equation CEILING (x MOD 2) = y (Figure 9). Next, to express the region, we take inequalities CEILING (x MOD 2) <= y and CEILING (x MOD 2) –1 >= y. Combined with y >= 0, we can get two rows of squares in the mosaic pattern. Similarly we can get two columns of that. However, they cannot construct the mosaic pattern in the whole plane in finite steps.

Consequently let's turn to Figure 11. In this figure, all the boundary lines are parallel to the y-axis, so we can conclude that variable y will not appear in its corresponding algebraic expression. Thus we may simply consider the equation CEILING (x MOD 2) = 1, and its graph is exactly Figure 11. From this observation we could construct without difficulty the mosaic pattern covering the whole plane: CEILING (x MOD 2) = 1 and CEILING (y MOD 2) = 1 or CEILING (x MOD 2) = 2 and CEILING (y MOD 2) = 2 (Figure 12, notice here and has higher priority than or).

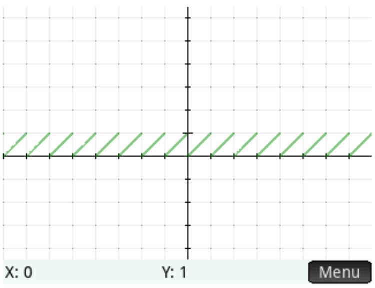

Figure 10

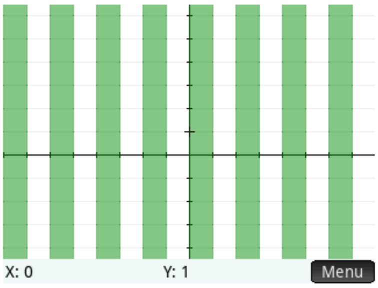

Figure 11

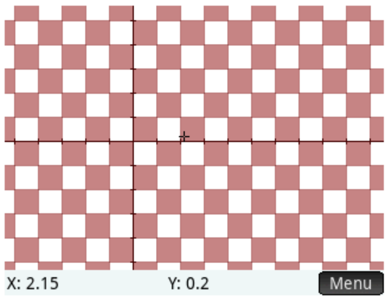

Figure 12

3.4 Reflection and Simplification

After thinking over of the operation MOD, we can find simpler expression for the mosaic pattern. Recall in 3.1 we use some inequalities to depict a region. Following the same idea, we can get rid of the CEILING operation. Therefore we construct another algebraic expression for the whole plane mosaic, which is x MOD 2 <= 1 and y MOD 2 <= 1 or x MOD 2 >= 1 and y MOD 2 >= 1. Now our survey on the mosaic comes to an end.

4. Further Creation and Reflection: The Beauty of Equations

In the process of creating certain patterns by algebraic expressions, I also tried constructing some novel expressions based on symmetry. At the very beginning of my creation I tried modifying some simple equations, with the hope of getting some elaborate expressions. Since the function y = x is quite simple, we shall first modify it. In order to avoid complexity, we shall only use at most two operations. Of course, in our new equations, variables x and y should be at equal position, in which case we shall say they satisfy symmetrical structure. We can let the two hand sides of the equation have the same operations. For example, we can write y-x = x-y, but this is still a linear equation. Now let's consider the basic operations: addition, subtraction, multiplication, and division. The first two will create equations of degree no more than one, which are quite simple, and the last two will create equations of finite degree, which are either familiar to us or of little importance to study. Therefore, after excluding these operations, I decided to try power operation. Based on y = x, I consider a series of equations: x^y = y^x, x^x = y^y, x^(y^x) = y^(x^y) (Figure 13), x^(x^x) = y^(y^y) (Figure 15), etc. With the graphing calculator we can see their corresponding graphs. The graphs are unexpectedly beautiful, but apart from that we can observe something reasonable that the graphs are all symmetrical with respect to the line y = x, due to our symmetrical algebraic construction. Thus we can see how perfectly algebra and geometry match each other in symmetry.

Besides, I attempt to combine some basic geometric objects to create some artworks (Figure 18). The building block of Figure 18 is the parabola y = x2. In order to make it rotate 90 degrees clockwise with respect to the origin, we can exchange x and y to gain x = y2. It is because the rotation of the parabola clockwise is the same thing as the rotation of the coordinate system counterclockwise. Thus we can replace x with –y and replace y with x, so we get the equation x = y2. We have another parabola (x+y)2 = x. Reflecting it by the x-axis gives (x–y)2 = x, and reflecting the new one by the y-axis gives (x+y)2 = -x. In this case we can see that the rotation and reflection of geometric objects correspond to the change of variables and their signs in the algebraic expressions.

In addition, I studied the graphs of axy = yx where a is a parameter. When a is valued by reciprocal numbers, the graphs are symmetrical with respect to the line y = x. This is easy to see if we observe the algebraic expressions. Graphed in different colors, Figure 17 shows the graphs of the equations (including the case a = 1).

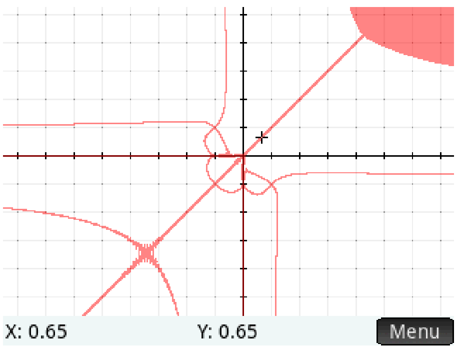

Figure 13

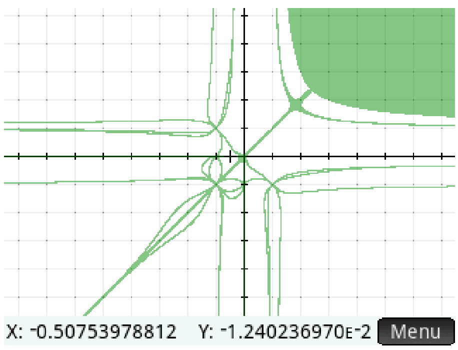

Figure 14

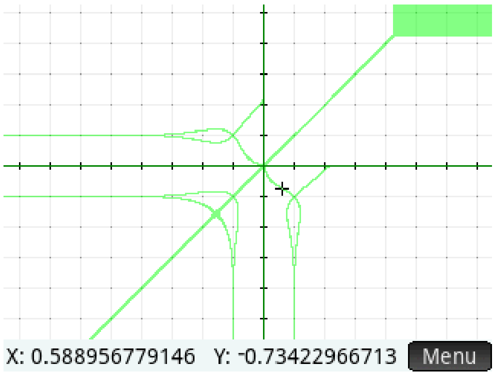

Figure 15

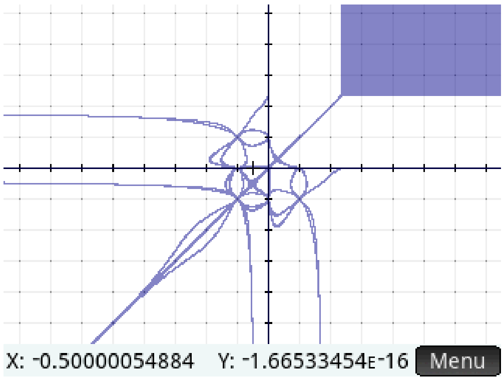

Figure 16

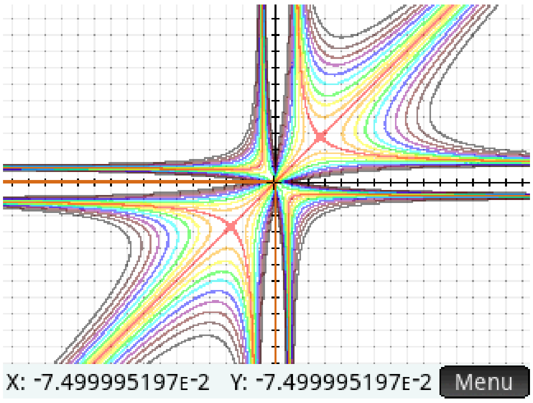

Figure 17

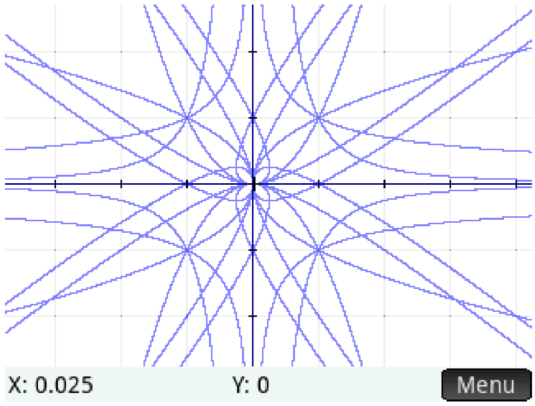

Figure 18

In the previous parts we already know we could express many objects algebraically. With the above creations we see the beauty of the equations, namely how the beauty of algebraic structures corresponds to that of geometric objects. Possibly this relation can help create new artworks.

5. Further Study Ideas

5.1 Find Expressions for Three or Higher Dimensional Objects

With the technology of higher dimensional graphing, we can construct equations of more than two variables to depict higher dimensional objects.

5.2 Informationize the Process of Logically Combining Several Equations

For instance, in HP graphing calculator, we can input several scattered points and the type of fitting function (e.g., linear function, quadratic function, etc.) to get a fitting function. We hope that in a similar way, we could input an image that we ask for algebraic expression and choose types of fitting functions, how the expressions are logically combined, and the computer will produce the right algebraic expressions.

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