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Looking for the Best Light Transmission Model for the Earth’s Atmosphere and Natural Waters

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Abstract: Transmission of light is one of the key optical processes in the Earth’s atmosphere and natural waters, and transmittance (T) is an optical parameter showing the rate of change of irradiance with the optical depth. A knowledge of T or another optical parameter, diffuse attenuation coefficient, K_d, steady connected with the T, allows many practical tasks to be solved regarding the ocean and atmospheric optics, such as water quality, primary production, and atmospheric correction. Therefore, knowledge of the reliable relationships between T (or K_d) and such parameters as incident illumination angle, cloud coverage, diffuseness of irradiance, and inherent optical properties (such as the scattering phase function, backscattering probability, scattering asymmetry parameter, and single-scattering albedo) is crucial. We have analyzed the impact of these parameters on the T and K_d. We computed T and K_d using a synthetic dataset covering any possible values of parameters by the numerical method (MDOM) and 21 analytical models and compared results with the MDOM solutions. An analysis of individual models has shown that the best of them yield average errors for T and K_d better than 10% for the majority of real optical conditions in the Earth’s atmosphere and natural waters.

Key words: transmittance, radiative transfer, Earth’s atmosphere and natural waters

1. Introduction

Transmission of light is one of the key optical processes in the Earth’s atmosphere and natural waters, and transmittance (T) is an optical parameter showing the rate of change of irradiance with the optical depth τ. A knowledge of T or another optical parameter, diffuse attenuation coefficient, K_d, steady connected with the T, allows many practical tasks to be solved regarding the ocean and atmospheric optics, such as water quality, primary production, and atmospheric correction. Therefore, knowledge of the reliable relationships between T (or K_d) and such parameters as incident illumination angle, cloud coverage, and inherent optical properties (IOPs) is crucial.

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Today there are numerous solutions for T and K_d; however, we feel there is a lack of publications analyzing such solutions. Recently, we began a series of publications devoted to the analysis of existing analytical approximations for these parameters [1, 2]; however, these publications were limited to considering only the plane transmittance models, i.e., media transmitting only direct (for example, solar or laser) incident radiation. Now we can extend this analysis for the spherical (i.e., for diffuse sources of radiation, for example, the sky, lamps, or computer screens) and natural (i.e., combinations of plane and spherical) transmittances.

2. Background

Transmission of light is one of the key optical processes in the Earth's atmosphere and natural waters, and due to its direct relation to turbidity and the diffuse attenuation coefficient, it may be an indicator of air and water pollution. The value of reliable T and K_d models increased considerably in the era of artificial satellites equipped with optical sensors. Today, many different algorithms have been developed to estimate IOPs and components' concentrations in the Earth's atmosphere and natural waters. The next important step may be made from these results toward estimation of T and K_d. Obviously, this is not a trivial task, because these optical properties depend on many other parameters, such as incidence angle θ_i; diffuseness of irradiance $d_E = E_{d,\,dif}/(E_{d,\,dir}+E_{d,\,dif})$, where $E_{d,dif}$ and $E_{d,dir}$ are diffuse and direct incoming irradiances; scattering phase function $p(\theta)$; backscattering probability B (or forward scattering probability F = 1-B); scattering asymmetry parameter g; single-scattering albedo ω_0; and optical depth $\tau = cz$ (c and z are beam attenuation coefficient and geometric depth, respectively) of the layer.

Currently, we limited ourselves by consideration only the optically-homogeneous layers, one value of θ_i = 30.5°; one value of d_E = 0.345 (this value corresponds to the clear sky conditions with cloud coverage C < 30%, λ = 490 nm, and θ_i = 30.5° [3]; 13 values of ω_0: 0.1(0.1)0.9, 0.95, 0.99, 0.999, 0.9999; and three different $p(\theta)$ with the following sets of {g, B}: {0.00193, 0.4986}; {0.5033, 0.1559}; and {0.9583, 0.00869}. Note that the first $p(\theta)$ is an almost isotropic Rayleigh scattering phase function that describes scattering by the air and water molecules [4], the second $p(\theta)$ belongs to the medium with ~ 0.2 μm size particles, and the last $p(\theta)$ corresponds to the medium with ~ 10 μm size particles [1, 5]. Thus, selected optical parameters span practically any possible situations occurring in natural waters and Earth's atmosphere.

3. Approach

The following approach has been applied in order to establish the best approximations:

(1) Computation of T and K_d using a highly accurate numerical method: the Modified Discrete Ordinates Method (MDOM) [6] for selected parameters.

(2) A selection of theoretical models for T and K_d from the literature.

(3) Testing of selected models to satisfy the physical constrains and boundary conditions.

(4) Derivation of the total $\bar{\mu}(\tau)$ and downwelling $\bar{\mu}_d(\tau)$ average cosines for direct and diffuse light from the MDOM calculations.

(5) Estimation of T and K_d by the selected models with the $\bar{\mu}(\tau)$ and $\bar{\mu}_d(\tau)$ found in step 4. Such an approach allows finding the best models in the sense of their independence on the light field.

(6) Development of the relevant analytical model for the average cosines.

(7) Estimation of T and K_d by the selected models with the $\bar{\mu}(\tau)$ and $\bar{\mu}_d(\tau)$, which are found in step 6.

(8) Comparison of T and K_d derived from the selected analytical models (in the both steps 5 and 7) with the MDOM results.

We suggest the following set of the physical constrains and boundary conditions (thereby extending the similar list suggested by Sokoletsky et al. (2014) [1] for the first time):

T1) $T(\tau = 0) = 1$;

T2) $0 < T(0\rightarrow \tau) < 1$ for any $0 < \tau < \infty$;

T3) $T(0\rightarrow\infty) \rightarrow 0$ excepting the case of $\omega_0 = F = 1$;

T4) $T(0\rightarrow \tau) = 1$ at $\omega_0 = F = 1$ and any τ;

T5) $\partial T(0\rightarrow \tau)/\partial\tau < 0$ for any τ;

T6) $T(0\rightarrow \tau) = \exp(-\tau_{ef})$ at $\omega_0 = 0$ and any τ [$\tau_{ef} = \tau/\bar{\mu}_d(\tau)$ is the effective optical depth];

T7) $T(0\rightarrow \tau) = \exp[-(1-\omega_0)\tau_{ef}]$ at $F = 1$ and any τ;

T8) $\partial T(0\rightarrow \tau)/\partial\omega_0 > 0$ for any τ;

T9) $\partial T(0\rightarrow \tau)/\partial F > 0$ for any τ;

T10) $\partial T(0\rightarrow \tau)/\partial\bar{\mu}_d > 0$ for any τ;

K1) $\bar{K}_d(0 \rightarrow \tau) \geq 0$ for any $0 \leq \tau < \infty$;

K2) $\bar{K}_d(0 \rightarrow \tau) = 0$ at $\omega_0 = F = 1$ and any τ;

K3) $\bar{K}_d(0 \rightarrow \tau) = a/\bar{\mu}_d(\tau)$ at $\omega_0 = 0$ and any τ;

K4) $\bar{K}_d(0 \rightarrow \tau) = a/\bar{\mu}_d(\tau)$ at $F = 1$ and any τ;

K5) $\left[\partial\bar{K}_d(0 \rightarrow \tau)/\partial\omega_0\right]/c < 0$ for any τ;

K6) $\partial\overline{K}_d(0\to\tau)/\partial F<0$ for any τ;

K7) $\partial\overline{K}_d(0\to\tau)/\partial\overline{\mu}_d<0$ for any τ.

Here $\overline{K}_d(0\to\tau)$ is the average (from 0 to τ) diffuse attenuation coefficient related to $K_d(0\to\tau)$ and $T(0\to\tau)$ by equation:

$$\overline{K}_d(0\to\tau)=\frac{1}{\tau}\int_0^\tau K_d(\tau)d\tau=-\frac{c\ln T(0\to\tau)}{\tau} \quad (1)$$

We computed first transmittances separately for direct and diffuse illumination, and then found the results for natural illumination [2]:

$$T(0\to\tau)=(1-d_E)T_p(0\to\tau)+d_E t(0\to\tau), \quad (2)$$

where T_p and t are the plane and spherical transmittances, respectively.

The total cosines of the light field were estimated from MDOM-derived transmittances using Gershun's law [7, 8] as follows:

$$\overline{\mu}_{dir}(\tau)=-\frac{(1-\omega_0)\tau}{\ln T_p(0\to\tau)},\quad \overline{\mu}_{dif}(\tau)=-\frac{(1-\omega_0)\tau}{\ln t(0\to\tau)}, \quad (3)$$

then the downwelling cosines were estimated as [9, 10].

$$\overline{\mu}_{d,dir}(\tau)=\frac{1+2R_p}{1-R_p}\overline{\mu}_{dir}(\tau),\quad \overline{\mu}_{d,dif}(\tau)=\frac{1+2r}{1-r}\overline{\mu}_{dif}(\tau), \quad (4)$$

where R_p and r are the plane and spherical albedos of the infinite layer, respectively, determined by the numerical invariant imbedding method for the same μ_i, $p(\theta)$, and ω_0 as T_p and t [5].

An alternate method of the $\overline{\mu}_d(\tau)$ computation was based on using the first two terms of the infinite chain fraction [10, 11].

$$\overline{\mu}_d(\tau)=\overline{\mu}_{d,\infty}+(\overline{\mu}_{d,0}-\overline{\mu}_{d,\infty})\exp\left[-\frac{q}{\overline{\mu}_{d,\infty}+(\overline{\mu}_{d,0}-\overline{\mu}_{d,\infty})\exp(-q)}\right], \quad (5)$$

$$q=(1-g)\omega_0\tau,$$

where $\overline{\mu}_{d,0}$ and $\overline{\mu}_{d,\infty}$ are the downwelling average cosines for the top and bottom of the layer, respectively.

The parameters for Eq. (5) were defined as follows [5, 9, 11-14]:

$$\overline{\mu}_{d,0}=\begin{cases}\mu_i \text{ for direct light}\\ \cos(48^o+14.12s-22.77s^2+19.24s^3), \textit{otherwise}\end{cases}$$

$$s=\sqrt{\frac{1-\omega_0}{1-g\omega_0}}, \quad (6)$$

$$\overline{\mu}_\infty=\sqrt{\frac{1+2G-\sqrt{G(4+5G)}}{1+G}},\ G=\frac{\omega_0 B}{1-\omega_0 F}, \quad (7)$$

$$\overline{\mu}_{d,\infty}=\begin{cases}\frac{1+2R_p}{1-R_p}\overline{\mu}_\infty \text{ for direct light}\\ \frac{1+2r}{1-r}\overline{\mu}_\infty \text{ for diffuse light}\end{cases} \quad (8)$$

The r values for this method were estimated by Eqs. (28)-(29) and R_p-by Eqs. (30)-(32) by Sokoletsky and Shen (2014) [2]. Note that Eqs. (3-8) were not applied to the MDOM, Haltrin (1998) [14], Nechad and Ruddick (2010) [15], and Gege (2012) [16] models.

4. Results and Discussion

Table 1 provides the main information about the selected models including the violation of the above physical constrains and boundary conditions. Note that three models, #15, 16, and 22 are mathematically identical, and we will abbreviate them as "Sobolev", the name of the first scientist to develop this model.

Figs. 1 and 2 illustrate the results derived by MDOM and several selected approximations. Fig. 1 was obtained with the average cosines equal to the corresponding average cosines derived from the MDOM algorithm (Eqs. (3) and (4)), while Eqs. (5)-(8) were used for Fig. 2.

The key criteria for the model selection were the minimal errors for the remotely sensed values of diffuse attenuation coefficient computed similar to the remotely sensed values of chlorophyll *a* concentration [17] and reliable vertical distribution of T and K_d. From a comparison between Fig. 1 and Fig. 2, it seems that the lists of the best T and K_d models are very similar, and we can recommend them (along with analytical method for the average cosines) for reliable retrievals of transmittances and diffuse attenuation coefficients in the Earth's atmosphere and natural waters.

Table 1 The short information about the selected models: the authors, model equations, and violations of physical constrains and boundary conditions (T1-K7). Z_m and $\tau_m = cZ_m$ are the middle depth and the middle optical depth of the euphotic zone, respectively; $\tau_{ef} = \tau/\bar{\mu}_d(\tau)$ is the effective optical depth. The T means T_p or t, R—R_p or r; $\bar{\mu}$—$\bar{\mu}_{dir}$ or $\bar{\mu}_{dif}$, and $\bar{\mu}_d$—$\bar{\mu}_{d,dir}$ or $\bar{\mu}_{d,dif}$ for direct or diffuse illumination, respectively.

#	Authors	Model equations and abbreviations	Violations
1	Budak and Korkin (2008) [6]	Modified discrete ordinates method (MDOM)	T2, T8, K1, K5
2	Bouguer (1729) [18], Lambert (1760) [19], and Beer (1852) [20]	Bouguer-Lambert-Beer approximation (BLB) $\frac{\bar{K}_d(0\to\tau)}{c}=\frac{1-\omega_0}{\bar{\mu}_d(\tau)}$	No
3	Gershun (1936) [7]	$\frac{\bar{K}_d(0\to\tau)}{c}=\frac{1-\omega_0}{\bar{\mu}(\tau)}$	No
4	Gordon et al. (1975) [21]	Quasi-single-scattering approximation (QSSA) $\frac{\bar{K}_d(0\to\tau)}{c}=\frac{1-\omega_0 F}{\bar{\mu}_d(\tau)}$	No
5	Aas (1987) [9]	$\frac{\bar{K}_d(0\to\tau)}{c}=\frac{1-0.978\,\omega_0}{\bar{\mu}_d(\tau)}$	T4, T7, K2, K4
6	Kirk (1984) [22]	$\frac{\bar{K}_d(0\to\tau)}{c}=\frac{(1-\omega_0)\sqrt{1+(A\bar{\mu}_d(\tau)-B)\omega_0/(1-\omega_0)}}{\bar{\mu}_d(\tau)}$, where $A = 0.473$, $B = 0.218$ at $z \le Z_m$; $A = 0.425$, $B = 0.190$, otherwise	T4, T7, K2, K4
7	Kubelka (1948) [23], King and Harshvardhan (1986) [24], Kokhanovsky (2007) [25]	$T(0\to\tau)=\frac{(1-R_\infty^2)\exp(-k\tau_{ef})}{1-R_\infty^2\exp(-2k\tau_{ef})}, R_\infty=1+\frac{K}{S}-\sqrt{\frac{K}{S}\left(\frac{K}{S}+2\right)},$ $\frac{K}{S}=\frac{1-\omega_0}{\omega_0 B},\quad k=\left[s\sqrt{3}-\frac{(0.985-0.253\,s)s^2}{6.464-5.464\,s}\right](1-\omega_0 g)$	T6, T7, K3, K4
8	Gordon (1989) [13]	$\frac{\bar{K}_d(0\to\tau)}{c}=\frac{1.04\,(1-\omega_0 F)}{\bar{\mu}_d(\tau)}$	T6, T7, K3, K4
9	Kirk (1991) [26]	$\frac{\bar{K}_d(0\to\tau)}{c}=\frac{(1-\omega_0)\sqrt{1+(A\bar{\mu}_d(\tau)-B)\omega_0/(1-\omega_0)}}{\bar{\mu}_d(\tau)}$, where $A = 2.636/g-2.447$, $B = 0.849/g-0.739$	T4, T7, T8, K2, K4, K5, K6
10	Cornet et al. (1992) [27]	$T(0\to\tau)=\frac{4\alpha}{(1+\alpha)^2\exp(\delta)-(1-\alpha)^2\exp(-\delta)}$, $\alpha=\sqrt{1-\omega_0},\quad \delta=\alpha\tau_{ef}$	T4, T7, K2, K4
11	Mobley (1994) [28]	$\frac{\bar{K}_d(0\to\tau)}{c}=\frac{1-\omega_0 F-2\omega_0 BR}{\bar{\mu}_d(\tau)}$	T4, K2
12	van de Hulst (1980) [12] and Ben-David (1997) [29]	**HBD** $:T(0\to\tau)=\frac{z}{x\sinh(z\tau_{ef})+z\cosh(z\tau_{ef})}$, $x=1-0.5(1+g)\omega_0,\ z=\sqrt{(1-\omega_0)(1-g\omega_0)}$	T7, K4
13	Haltrin (1998) [14]	$\frac{\bar{K}_d(0\to\tau)}{c}=\frac{\bar{K}_d(\infty)}{c}\frac{1-\eta R_0\xi}{1-R_0\xi},\ \frac{\bar{K}_d(\infty)}{c}=\frac{1-\omega_0}{\bar{\mu}_\infty},$	T6, T7, K3-K5

		$\bar{\mu}_\infty = \sqrt{\frac{1+2G-\sqrt{G(4+5G)}}{1+G}}$, $G = \frac{\omega_0 B}{1-\omega_0 F}$, $\eta = \frac{G-R_\infty}{G-R_0}$, $R_\infty = \left(\frac{1-\bar{\mu}_\infty}{1+\bar{\mu}_\infty}\right)^2$, $R_0 = \left(\frac{2+\bar{\mu}_\infty}{2-\bar{\mu}_\infty}\right)R_\infty$, $\xi = -R_\infty \exp\left(-\frac{\nu}{c}\tau_{\mathbf{ef}}\right)$, $\frac{\nu}{c} = \frac{\bar{K}_{\mathrm{d}}(\infty)}{c}\,\frac{7+2\bar{\mu}_\infty^2-\bar{\mu}_\infty^4}{3-\bar{\mu}_\infty^2}$	
14	Lee et al. (2005) [30]	$\frac{\bar{K}_{\mathbf{d}}(0\to\tau)}{c} = \frac{\bar{K}_{\mathbf{d}}(0\to\tau_{\mathbf{m}})}{c}\,\frac{\bar{\mu}_{\mathbf{d}}(\tau_{\mathbf{m}})}{\bar{\mu}_{\mathbf{d}}(\tau)}$, $\frac{\bar{K}_{\mathbf{d}}(0\to\tau_{\mathbf{m}})}{c} = (1+0.005\,\theta_{\mathbf{0}})(1-\omega_0)+3.47\,\omega_0 B$, $\tau_{\mathbf{m}} = \frac{\ln 10}{\bar{K}_{\mathbf{d}}(0\to\tau_{\mathbf{m}})/c}$, $\bar{\mu}_{\mathbf{d}}(\tau) = \left[d_{\mathrm{E}}/\bar{\mu}_{\mathbf{d,\,dif}}(\tau)+(1-d_{\mathrm{E}})/\bar{\mu}_{\mathbf{d,\,dir}}(\tau)\right]^{-1}$, where θ_0 is the solar zenith angle in degrees (43.0° in the current study)	T6-T8, K3-K5
15	Pottier et al. (2005) [31]	$T(0\to\tau) = \frac{4\alpha}{(1+\alpha)^2\exp(\delta)-(1-\alpha)^2\exp(-\delta)}$, $\alpha = \sqrt{\frac{1-\omega_0}{1-\omega_0+2\omega_0 B}}$, $\delta = \tau_{\mathbf{ef}}\sqrt{(1-\omega_0)(1-\omega_0+2\omega_0 B)}$	No
16	Kubelka (1948) [23] and Rogatkin (2007) [32]	$T(0\to\tau) = \frac{\beta}{\alpha\sinh(k\tau_{\mathrm{ef}})+\beta\cosh(k\tau_{\mathrm{ef}})}$, $\alpha = 1+\frac{1-\omega_0}{\omega_0 B}$, $\beta = \sqrt{\alpha^2-1}$, $k = \sqrt{(1-\omega_0)(1-\omega_0+2\omega_0 B)}$	No
17	Rogatkin (2007) [32]	$T(0\to\tau) = \frac{(1-P^2)\exp(-k\tau_{\mathrm{ef}})}{1-P^2\exp(-2k\tau_{\mathrm{ef}})}$, $k = \omega_0 FL$, $P = \frac{\beta_1-L}{\beta_2}$, $L = \sqrt{\beta_1^2-\beta_2^2}$, $\beta_1 = \omega\times\frac{A-\ln F+\ln\left[1-\omega+\sqrt{\omega^2-B^2\exp(-2A)}\right]}{\sqrt{\omega^2-B^2\exp(-2A)}}$, $\beta_2 = \frac{B\exp(-A)}{\omega}\beta_1$, $A = \frac{1-\omega_0}{\omega_0 F}$, $\omega = \frac{1-(1-2B)\exp(-A)}{2}$	No
18	Nechad and Ruddick (2010) [15]	$\frac{\bar{K}_{\mathbf{d}}}{c} = m_0(1-\omega_0)+\omega_0 Bm_1\left(1-\frac{\omega_0 B}{1-\omega_0}\right)$, $m_0 = 1.09+0.49\cosh(\theta_0)\cosh(0.7C)-0.56\cosh(\theta_0 C)$, $m_1 = [4.266-4.56\cosh(\theta_0)\cosh(0.73C)+5.51\cosh(\theta_0 C)]m_0^2$, θ_0 in radians (0.750); C is the cloud coverage (0.345)	T1-T3, T5-T9, K1-K6
19	Pan and Zimmerman (2010) [3]	**P & Z**: $T(0\to\tau) = d_{\mathrm{E}}\exp\left(-\frac{K_{\mathbf{d,dif}}}{c}\tau\right)+(1-d_{\mathrm{E}})$ $\times\exp\left\{-\frac{1}{\bar{\mu}_{\mathbf{d,dir}}(\tau)}\left[\frac{K_{\mathbf{d},\infty}}{c}\tau+\frac{(1-\omega_0 F-K_{\mathbf{d},\infty}/c)(1-e^{-P\tau})}{P}\right]\right\}$, $\frac{K_{\mathrm{d,\ dif}}}{c} = 1.317-(1.399-1.012\sqrt{B}-0.939\,B)\omega_0$ $+(0.047-0.244\sqrt{B}-1.120\,B)\omega_0^2$, $\frac{K_{\mathrm{d}}(\infty)}{c} = 1-(0.959-2.346\sqrt{B}+0.747\,B)\omega_0$ $-(0.046+1.807\sqrt{B}-0.888\,B)\omega_0^2$, $P = 0.817-0.877\sqrt{\omega_0}+(0.193+0.421\,\omega_0+0.741\,\omega_0^2)\sqrt{B}$	T4, T6-T8, K2-K5
20	Gege (2012) [16]	$\frac{\bar{K}_{\mathrm{d}}}{c} = \frac{1-\omega_0 F}{\bar{\mu}_{\mathrm{d}}}$, $\bar{\mu}_{\mathrm{d,\ dir}} = \mu_{\mathrm{i}}$, $\bar{\mu}_{\mathrm{d,\ dif}} = [1.1156+0.5504(1-\mu_{\mathrm{i}})]^{-1}$	T6, T7, K3, K4
21	Sokoletsky and Shen (2014) [2]	MQSSA: Eqs. (18), (20), (23)-(32) by Sokoletsky and Shen (2014) [2]	T7, T8, K4,

			K5
22	Sobolev (1956, 1975) [33] and Barun and Ivanov (2015) [34]	$T(0 \to \tau) = \dfrac{(1 - R_\infty^2)\exp(-k\tau_{\mathrm{ef}})}{1 - R_\infty^2 \exp(-2k\tau_{\mathrm{ef}})}$, $R_\infty = \dfrac{1 - \omega_0 F - \sqrt{(1 - \omega_0 F)^2 - (\omega_0 B)^2}}{\omega_0 B}$, $k = \dfrac{\omega_0 B(1 - R_\infty^2)}{2R_\infty^2}$	No

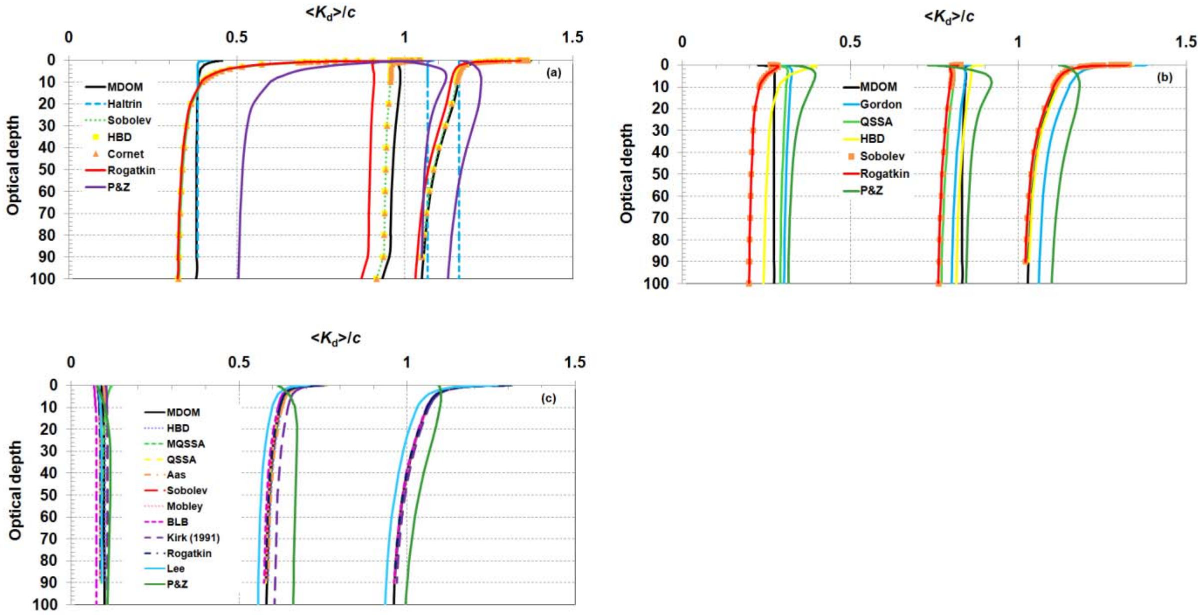

Fig. 1 The vertical profiles of $\overline{K}_{\mathrm{d}}(\tau)/c$ computed by the numerical (MDOM, black curves) and analytical (color curves) methods for three different $p(\theta)$ with the following sets of {g, B}: {0.00193, 0.4986} (a); {0.5033, 0.1559} (b); and {0.9583, 0.00869} (c) at $d_{\mathrm{E}} = 0.345$, $\theta_{\mathrm{i}} = 30.5°$, and three values of ω_0: 0.1, 0.5, and 0.95 (from right to left). The average cosines have been derived here from the MDOM by Eqs. (3) and (4).

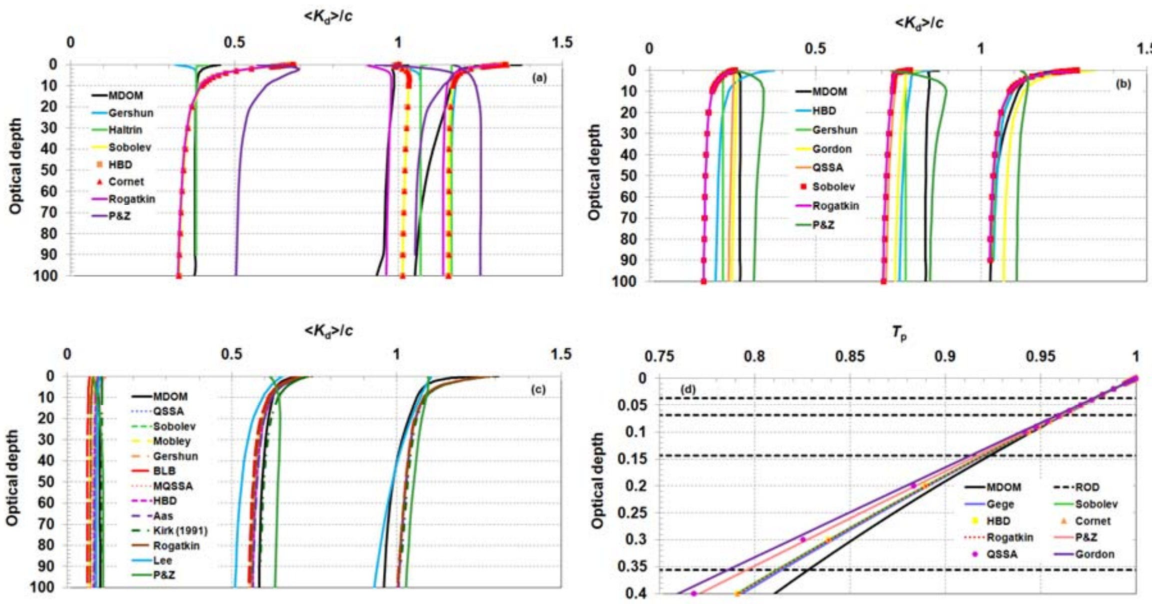

Fig. 2 The plots (a)-(c) are similar to those plotted on the Fig. 1a-c, but the average cosines have been computed here using Eqs. (5)-(8). The plot (d) indicates the plane transmittance computed for the Rayleigh $p(\theta)$ at $\theta_{\mathrm{i}} = 30.5^{\circ}$ and $\omega_0 = 0.9999$. The Rayleigh optical depths (ROD) at wavelengths of 400, 500, 600, and 700 nm (from the bottom to the top) are also shown on Fig. 2d by the black dash curve.

5. Conclusions

The most important conclusion following on from this study is that even though different methods may reveal different accuracies under different atmospheric and underwater situations, the Sobolev, HBD, Rogatkin, Gershun, and QSSA models seem to be relevant for any realistic optical conditions with a high accuracy. Thus, these models are greatly recommended for use in both the ocean and atmospheric optics as simple yet highly accurate analytical approximations.

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