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Evaluating Distribution Centers via a Maximizing Set and Minimizing Set Based Fuzzy MCDM Approach

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Abstract: The evaluation and selection of a distribution center has been an important issue for a company to upgrade its distribution efficiency and operation performance in order to fulfill the diverse demands of consumer. This paper suggests a maximizing set and minimizing based fuzzy MCDM model to resolve this problem, where ratings of alternatives versus qualitative criteria and the importance weights of all the criteria are assessed in linguistic values represented by fuzzy numbers. Ranking formulae and membership functions for the final fuzzy evaluation values can be clearly developed for better executing the decision making. A numerical is used to demonstrate the feasibility of the proposed approach.

Key words: fuzzy MCDM; distribution center; location selection; ranking; maximizing set and minimizing set.

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1. Introduction

Manufacturers, customers and suppliers are important members of a supply chain. To some extent, the success of a manufactory depends on its ability to link these members seamlessly. In the real logistics systems, it often requires some distribution centers to connect manufactories and their customers for the improvement of product flow (Yang et al., 2007). Distribution center (DC) is viewed as the competence that links an enterprise with its customers and suppliers. A distribution center is usually supplied by the sources such as manufacturing factories, vendors, etc., and in turn it supplies the consumers or demand locations. In order to reduce transportation cost, enforce operation efficiency and logistics performance, evaluating and selecting a DC location has become one of the most important decision issues for distribution industries (Chen, 2001).

Evaluating a DC location, many conflicting criteria must be considered. These criteria can be classified into two categories: (1) objective — these criteria can be evaluated quantitatively, e.g., investment cost, and (2) subjective — these criteria have qualitative definitions, e.g., expansion possibility, closeness to demand market, etc. In addition, these criteria may have different importance (Bowersox & Closs, 1996; Stevenson, 2014; Sule, 1994; Tompkins et al., 2010). Numerous precision-based methods for location selection problems have been investigated (Aikens, 1985; Alumur & Kara, 2007; Cheng et al., 2005; Colebrook & Sicillia, 2007; Cram et al.,

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2006; Hodder & Dincer, 1986; Malczewski, 2006; Pavić & Babić, 1991; Rietveld & Ouwersloot, 1992; Rodriguez et al., 2006; Şener et al., 2006). A review can be seen in Yuzkaya et al. (2008). All the above methods are developed based on the concept of accurate measure and crisp evaluation, i.e., the measuring values must be numerical and exact. In 2004, Pérez et al. pointed out “Location problems concern a wide set of fields where it is usually assumed that exact data are known. However, in real applications, the location of facility considered can be full of linguistic vagueness, that can be appropriately modeled using networks with fuzzy values.” Moreover, the values for the qualitative criteria are often imprecisely defined for the decision-makers. Obviously, the precision-based methods are not adequate to resolve the DC location selection problem. To resolve the above problems, a fuzzy multiple criteria decision making (MCDM) method is suggested.

Fuzzy set theory, initially proposed by Zadeh (1965), has been extensively applied to objectively reflect the ambiguities in human judgment and effectively resolve the uncertainties in the available information in an ill-defined multiple criteria decision making environment. Numerous approaches have been proposed to solve fuzzy MCDM problems. A review and comparison of many of these methods can be found in Carlsson and Fullér (1996), Chen and Hwang (1992), Kuo (2011) and Triantaphyllou and Lin (1996). Some recent applications on locations evaluation and selection can be found in (Anagnostopoulos et al., 2008; Chou, 2007; Güzel & Erdal, 2015; Wang et al., 2010), and some other recent works in fuzzy MCDM can be found in (Akdag et al., 2014; Chu & Varma, 2012; Chung et al., 2015; Ghorbani et al., 2013). Despite the merits, most of the above papers cannot present membership functions for the final fuzzy evaluation values, nor can they clearly develop defuzzification formulae from the membership functions of the final fuzzy evaluation values, which limit the applicability of the fuzzy MCDM methods available. To resolve the these limitations, this work suggests a maximizing set and minimizing set based fuzzy MCDM approach for the evaluation and selection of distribution centers. Many fuzzy number ranking methods have been studied. A comparison of many of these ranking methods can be found in Wang and Lee (2008). Some recent works can be found in (Abbasbandy & Hajjari, 2009; Asady, 2010; Ezzati et al., 2012; Farhadinia, 2009; Hari Ganeshand Jayakumar, 2014; Rao & Shankar, 2013; Sharma, 2015). However, in spite of the merits, some of these methods are computational complex and difficult to implement and none of them can satisfactorily rank fuzzy numbers in all situations and cases. Herein, the ranking approach of maximizing set and minimizing set (Chen, 1985) is applied for defuzzification due to its simplicity of implementation. Furthermore, defuzzification procedure can be clearly presented and formulae can be developed. Finally, a numerical example demonstrates the computational process of the proposed model.

The rest of this work is organized as follows. Section 2 briefly introduces fuzzy set theory. Section 3 introduces the suggested model. Meanwhile, an example is presented in Section 4 to demonstrate the feasibility of the proposed model and conclusions are made in Section 5.

2. Fuzzy Set Theory

2.1 Fuzzy Sets

$A = \{(x, f_A(x)) \mid x \in U\}$, where U is the universe of discourse, x is an element in U, A is a fuzzy set in U, $f_A(x)$ is the membership function of A at x (Kaufmann & Gupta, 1991). The large $f_A(x)$, the stronger the grade of membership for x in A.

2.2 Fuzzy Numbers

A real fuzzy number A is described as any fuzzy subset of the real line R with membership function f_A which possesses the following properties (Dubois & Prade, 1978):

(a) f_A is a continuous mapping from R to [0,1];

(b) $f_A(x) = 0, \forall x \in (-\infty, a]$;

(c) f_A is strictly increasing on $[a, b]$;

(d) $f_A(x) = 1, x \in [b, c]$;

(e) f_A is strictly decreasing on $[c, d]$;

(f) $f_A(x) = 0, \forall x \in [d, \infty)$;

where $a \le b \le c \le d$, A can be denoted as [a, b, c, d]. The membership function f_A of the fuzzy number A can also be expressed as:

$$f_A(x) = \begin{cases} f_A^L(x), a \le x \le b \\ 1, \qquad b \le x \le c \\ f_A^R(x), c \le x \le d \\ 0, \qquad \text{otherwise} \end{cases} \tag{1}$$

where $f_A^L(x)$ and $f_A^R(x)$ are left and right membership functions of A, respectively (Kaufmann & Gupta, 1991). A fuzzy triangular number can be denoted as (a, b, c).

2.3 α-cuts

The α-cuts of fuzzy number A can be defined as $A^\alpha = \{x \mid f_A(x) \ge \alpha\}, \alpha \in [0,1]$, where A^α is a non-empty bounded closed interval contained in R and can be denoted by $A^\alpha = [A_l^\alpha, A_u^\alpha]$, where A_l^α and A_u^α are its lower and upper bounds, respectively (Kaufmann & Gupta, 1991).

2.4 Arithmetic Operations on Fuzzy Numbers

Given fuzzy numbers A and B, $A, B \in R^+$, the α-cuts of A and B are $A^\alpha = [A_l^\alpha, A_u^\alpha]$ and $B^\alpha = [B_l^\alpha, B_u^\alpha]$, respectively. By the interval arithmetic, some main operations of A and B can be expressed as follows (Kaufmann & Gupta, 1991):

$$(A \oplus B)^\alpha = [A_l^\alpha + B_l^\alpha, A_u^\alpha + B_u^\alpha] \tag{2}$$

$$(A \ominus B)^\alpha = [A_l^\alpha - B_u^\alpha, A_u^\alpha - B_l^\alpha] \tag{3}$$

$$(A \otimes B)^\alpha = [A_l^\alpha \cdot B_l^\alpha, A_u^\alpha \cdot B_u^\alpha] \tag{4}$$

$$(A \oslash B)^\alpha = \left[A_l^\alpha / B_u^\alpha , A_u^\alpha / B_l^\alpha \right] \tag{5}$$

$$(A \otimes r)^\alpha = [A_l^\alpha \cdot r, A_u^\alpha \cdot r], r \in R^+ \tag{6}$$

2.5 Linguistic Values

A linguistic variable is a variable whose values are expressed in linguistic terms. Linguistic variable is a very helpful concept for dealing with situations which are too complex or not well-defined to be reasonably described by traditional quantitative expressions (Zadeh, 1975). For example, “importance” is a linguistic variable whose values include UI (unimportant), LI (less important), I (important), MI (more important) and VI (very important). These linguistic values can be further represented by triangular fuzzy numbers such as UI = (0.0, 0.0, 0.25), LI = (0.0, 0.25, 0.5), I = (0.25, 0.5, 0.75), MI = (0.50, 0.75, 1.00) and VI = (0.75, 1.00, 1.00).

3. Model Development

Suppose decision makers D_t, t = 1,2...,l, are responsible for evaluating alternatives A_i, i = 1,2,…,m, under selected criteria, C_j, j = 1,2,…,n. Criteria are categorized into three groups such as benefit qualitative criteria C_j, j = 1,…,g, benefit quantitative criteria C_j, j = g+1,…,h, and cost quantitative criteria C_j, j = h+1,…,n. The proposed model is developed as the following steps.

3.1 Aggregate Ratings of Alternatives versus Qualitative Criteria

Assume $x_{ijt} = (a_{ijt}, b_{ijt}, c_{ijt})$, $i = 1,...,m,\ j = 1,...,g,\ t = 1,...,l,$

$$x_{ij} = \frac{1}{l} \otimes (x_{ij1} \oplus x_{ij2} \oplus ... \oplus x_{ijl}) \tag{7}$$

Where $a_{ij} = \frac{1}{l}\sum_{t=1}^{l} a_{ijt}$, $b_{ij} = \frac{1}{l}\sum_{t=1}^{l} b_{ijt}$, $c_{ij} = \frac{1}{l}\sum_{t=1}^{l} c_{ijt}$. x_{ijt} denotes ratings assigned by each decision maker for each alternative versus each qualitative criterion. x_{ij} denotes averaged rating of each alternative versus each qualitative criterion.

3.2 Normalize Values of Alternatives versus Quantitative Criteria

Herein, Chen's (2001) method is applied to normalize values of alternatives versus quantitative criteria, including benefit and cost, in order to make data dimensionless for calculation rationale. Benefit quantitative data has the characteristics: the larger the better; whereas cost quantitative data has the characteristics: the smaller the better. Suppose y_{ij} = (o_{ij}, ρ_{ij}, q_{ij}) denotes evaluation value of alternative i versus benefit quantitative criteria j, j = g+1,...,h, as well as cost quantitative criteria j, j = h+1,...,n. And x_{ij} denotes the normalized value of y_{ij}

$$x_{ij} = \left(\frac{o_{ij}}{q_{ij}^*}, \frac{p_{ij}}{q_{ij}^*}, \frac{q_{ij}}{q_{ij}^*}\right),\ q_{ij}^* = \max q_{ij},\ j \in B, \tag{8}$$

$$x_{ij} = \left(\frac{o_{ij}^*}{q_{ij}}, \frac{o_{ij}^*}{p_{ij}}, \frac{o_{ij}^*}{o_{ij}}\right), \qquad o_{ij}^* = \min o_{ij},\ \ j \in C. \tag{9}$$

For calculation convenience, assume $x_{ij} = (a_{ij}, b_{ij}, c_{ij})$, $j = g+1,...,n$.

3.3 Average Importance Weights

Assume $w_{jt} = (d_{jt}, e_{jt}, f_{jt}),\ w_{jt} \in R^+,\ j = 1,...,n,\ t = 1,...,l,$

$$w_j = \frac{1}{l} \otimes (w_{j1} \oplus w_{j2} \oplus ... \oplus w_{jl}) \tag{10}$$

Where $d_j = \frac{1}{l}\sum_{t=1}^{l} d_{jt}$, $e_j = \frac{1}{l}\sum_{t=1}^{l} e_{jt}$, $f_j = \frac{1}{l}\sum_{t=1}^{l} f_{jt}$. w_{jt} represents the weight assigned by each decision maker for each criterion and w_j represents the average importance weight of each criterion.

3.4 Develop Membership Functions

The membership function of the final fuzzy evaluation value, G_i, i = 1,…,n, of each alternative can be developed as equation (11). In equation (11), the first two parts are additive weighted ratings under benefit criteria. The third part is under cost criteria but given a negative sign. Therefore, the larger the G_i value, the better performance A_i will have.

$$G_i = \sum_{j=1}^{g} w_j \otimes x_{ij} + \sum_{g+1}^{h} w_i \otimes x_{ij} - \sum_{j=h+1}^{n} w_j \otimes x_{ij} \ , \tag{11}$$

The membership functions are developed as:

$$G_i^{\alpha} = \sum_{j=1}^{g} w_j^{\alpha} \otimes x_{ij}^{\alpha} + \sum_{j=g+1}^{h} w_j^{\alpha} \otimes x_{ij}^{\alpha} - \sum_{j=h+1}^{n} w_j^{\alpha} \otimes x_{ij}^{\alpha}, \tag{12}$$

$$w_j^{\alpha} = [(e_j - d_j)\alpha + d_j, (e_j - f_j)\alpha + f_j], \tag{13}$$

$$x_{ij}^{\alpha} = [(b_{ij} - a_{ij})\alpha + a_{ij}, (b_{ij} - c_{ij})\alpha + c_{ij}]. \tag{14}$$

From equations (13) and (14), we can develop equation (15) as follows:

$$\sum w_j^{\alpha} \otimes x_{ij}^{\alpha} = \Big[\sum (e_j - d_j)(b_{ij} - a_{ij})\alpha^2 + \sum (a_{ij}(e_j - d_j) + d_j(b_{ij} - a_{ij}))\alpha + \sum a_{ij} d_j, \\ \sum (b_{ij} - c_{ij})(e_j - f_j)\alpha^2 + \sum (c_{ij}(e_j - f_j) + f_j(b_{ij} - c_{ij}))\alpha + \sum c_{ij} f_j \Big] \tag{15}$$

By applying Eq. (15) to Eq. (12), three equations are developed:

$$\sum_{j=1}^{g} w_j^{\alpha} \otimes x_{ij}^{\alpha} = [\sum_{j=1}^{g} (e_j - d_j)(b_{ij} - a_{ij})\alpha^2 + \sum_{j=1}^{g} (a_{ij}(e_j - d_j) + d_j(b_{ij} - a_{ij}))\alpha + \sum_{j=1}^{g} a_{ij} d_{ij} \ , \\ \sum_{j=1}^{g} (b_{ij} - c_{ij})(e_j - f_j)\alpha^2 + \sum_{j=1}^{g} (c_{ij}(e_j - f_j) + f_j(b_{ij} - c_{ij}))\alpha + \sum_{j=1}^{g} c_{ij} f_j]. \tag{16}$$

$$\sum_{j=g+1}^{h} w_j^{\alpha} \otimes x_{ij}^{\alpha} = [\sum_{j=g+1}^{h} (e_j - d_j)(b_{ij} - a_{ij})\alpha^2 + \sum_{j=g+1}^{h} (a_{ij}(e_j - d_j) + d_j(b_{ij} - a_{ij}))\alpha + \sum_{j=g+1}^{h} a_{ij} d_{ij} \ , \\ \sum_{j=g+1}^{h} (b_{ij} - c_{ij})(e_j - f_j)\alpha^2 + \sum_{j=g+1}^{h} (c_{ij}(e_j - f_j) + f_j(b_{ij} - c_{ij}))\alpha + \sum_{j=g+1}^{h} c_{ij} f_j]. \tag{17}$$

$$\sum_{j=h+1}^{n} w_j^{\alpha} \otimes x_{ij}^{\alpha} = [\sum_{j=h+1}^{n} (e_j - d_j)(b_{ij} - a_{ij})\alpha^2 + \sum_{j=h+1}^{n} (a_{ij}(e_j - d_j) + d_j(b_{ij} - a_{ij}))\alpha + \sum_{j=h+1}^{n} a_{ij} d_{ij} \ , \\ \sum_{j=h+1}^{n} (b_{ij} - c_{ij})(e_j - f_j)\alpha^2 + \sum_{j=h+1}^{n} (c_{ij}(e_j - f_j) + f_j(b_{ij} - c_{ij}))\alpha + \sum_{j=h+1}^{n} c_{ij} f_j]. \tag{18}$$

Assume:

$$A_{i1} = \sum_{j=1}^{g} (e_j - d_j)(b_{ij} - a_{ij}), \ A_{i2} = \sum_{j=g+1}^{h} (e_j - d_j)(b_{ij} - a_{ij}), \ A_{i3} = \sum_{j=h+1}^{n} (e_j - d_j)(b_{ij} - a_{ij}),$$

$$B_{i1} = \sum_{j=1}^{g} [a_{ij}(e_j - d_j) + d_j(b_{ij} - a_{ij})], \ B_{i2} = \sum_{j=g+1}^{h} [a_{ij}(e_j - d_j) + d_j(b_{ij} - a_{ij})],$$

$$B_{i3} = \sum_{j=h+1}^{n} [a_{ij}(e_j - d_j) + d_j(b_{ij} - a_{ij})], \ C_{i1} = \sum_{j=1}^{g} (b_{ij} - c_{ij})(e_j - f_j),$$

$$C_{i2} = \sum_{j=g+1}^{h} (b_{ij} - c_{ij})(e_j - f_j), \ C_{i3} = \sum_{j=h+1}^{n} (b_{ij} - c_{ij})(e_j - f_j),$$

$$D_{i1} = \sum_{j=1}^{g} [c_{ij}(e_j - f_j) + f_j(b_{ij} - c_{ij})], \ D_{i2} = \sum_{j=g+1}^{h} [c_{ij}(e_j - f_j) + f_j(b_{ij} - c_{ij})],$$

$D_{i3} = \sum_{j=h+1}^{n}[c_{ij}(e_j - f_j) + f_j(b_{ij} - c_{ij})]$, $O_{i1} = \sum_{j=1}^{g} a_{ij}d_j$, $O_{i2} = \sum_{j=g+1}^{h} a_{ij}d_j$, $O_{i3} = \sum_{j=h+1}^{n} a_{ij}d_j$,

$P_{i1} = \sum_{j=1}^{g} b_{ij}e_j$, $P_{i2} = \sum_{j=g+1}^{h} b_{ij}e_j$, $P_{i3} = \sum_{j=h+1}^{n} b_{ij}e_j$, $Q_{i1} = \sum_{j=1}^{g} c_{ij}f_j$, $Q_{i2} = \sum_{j=g+1}^{h} c_{ij}f_j$, $Q_{i3} = \sum_{j=h+1}^{n} c_{ij}f_j$.

By applying the above assumption, equations (16)-(18) can be arranged as:

$$\sum_{j=1}^{g} w_j^{\alpha} \otimes x_{ij}^{\alpha} = [A_{i1}\alpha^2 + B_{i1}\alpha + O_{i1}, C_{i1}\alpha^2 + D_{i1}\alpha + Q_{i1}], \quad (19)$$

$$\sum_{j=g+1}^{h} w_j^{\alpha} \otimes x_{ij}^{\alpha} = [A_{i2}\alpha^2 + B_{i2}\alpha + O_{i2}, C_{i2}\alpha^2 + D_{i2}\alpha + Q_{i2}], \quad (20)$$

$$\sum_{j=h+1}^{n} w_j^{\alpha} \otimes x_{ij}^{\alpha} = [A_{i3}\alpha^2 + B_{i3}\alpha + O_{i3}, C_{i3}\alpha^2 + D_{i3}\alpha + Q_{i3}]. \quad (21)$$

Applying equations (19)-(21) to equation (12) to produce equation (22):

$$\begin{aligned} G_i^{\alpha} = [&(A_{i1} + A_{i2} - C_{i3})\alpha^2 + (B_{i1} + B_{i2} - D_{i3})\alpha + (O_{i1} + O_{i2} - Q_{i3}), \\ &(C_{i1} + C_{i2} - A_{i3})\alpha^2 + (D_{i1} + D_{i2} - B_{i3})\alpha + (Q_{i1} + Q_{i2} - O_{i3})]. \end{aligned} \quad (22)$$

The right and left membership functions of G_i can be obtained as shown in equation (23) and equation (24) as follows:

$$\alpha = f_{G_i}^{L}(x) = \frac{-(B_{i1} + B_{i2} - D_{i3}) + [(B_{i1} + B_{i2} - D_{i3})^2 + 4(A_{i1} + A_{i2} - C_{i3})(x - (O_{i1} + O_{i2} - Q_{i3}))]^{1/2}}{2(A_{i1} + A_{i2} - C_{i3})} \quad (23)$$

If $O_{i1} + O_{i2} - Q_{i3} \le x \le P_{i1} + P_{i2} - P_{i3}$;

$$\alpha = f_{G_i}^{R}(x) = \frac{-(D_{i1} + D_{i2} - B_{i3}) + [(D_{i1} + D_{i2} - B_{i3})^2 + 4(C_{i1} + C_{i2} - A_{i3})(x - (Q_{i1} + Q_{i2} - O_{i3}))]^{1/2}}{2(C_{i1} + C_{i2} - A_{i3})} \quad (24)$$

If $P_{i1} + P_{i2} - P_{i3} \le x \le Q_{i1} + Q_{i2} - O_{i3}$.

3.5 Rank Fuzzy Numbers

In this research, Chen's maximizing set and minimizing set (Chen, 1985) is applied to rank all the final fuzzy evaluation values. This method is one of the most commonly used approaches of ranking fuzzy numbers in fuzzy decision making.

The maximizing set M is defined as:

$$f_M(x) = \begin{cases} \left(\dfrac{x_{R_i} - x_{\min}}{x_{\max} - x_{\min}}\right)^k, x_{\min} \le x_{R_i} \le x_{\max}, \\ 0, \text{otherwise}. \end{cases} \quad (25)$$

The minimizing set N is defined as:

$$f_N(x) = \begin{cases} \left(\dfrac{x_{L_i} - x_{\max}}{x_{\min} - x_{\max}}\right)^k, x_{\min} \le x_{L_i} \le x_{\max}, \\ 0, \text{otherwise}, \end{cases} \quad (26)$$

where $x_{\min} = \inf_x S$, $x_{\max} = \sup_x S$, $S = \cup_{i=1}^{n} S_i$, $S_i = \{x \big| f_{A_i}(x) > 0\}$, usually k is set to 1.

The right utility of A_i is defined as:

$$U_M(A_i) = \sup_x (f_M(x) \wedge f_{A_i}(x)), i = 1 \sim n. \tag{27}$$

The left utility of A_i is defined as:

$$U_N(A_i) = \sup_x (f_M(x) \wedge f_{A_i}(x)), i = 1 \sim n. \tag{28}$$

The total utility of A_i is defined as:

$$U_T(A_i) = \frac{1}{2}(U_M(A_i) + 1 - U_N(A_i)), i = 1 \sim n. \tag{29}$$

The total utility $U_T(A_i)$ is applied to rank fuzzy numbers. The larger the $U_T(A_i)$, the larger the fuzzy number A_i. Applying equations (25)-(29) to equations (23)-(24), the total utility of fuzzy number G_i can be obtained as:

$$\begin{aligned} U_T(G_i) &= \frac{1}{2}(U_M(G_i) + 1 - U_N(G_i)),\ i = 1 \sim n\ , \\ &= \frac{1}{2}\Big[\frac{-(D_{i1}+D_{i2}-B_{i3}) - [(D_{i1}+D_{i2}-B_{i3})^2 + 4(C_{i1}+C_{i2}-A_{i3})(x_{R_i} - (Q_{i1}+Q_{i2}-O_{i3}))]^{\frac{1}{2}}}{2(C_{i1}+C_{i2}-A_{i3})} \\ &+1 - \frac{-(B_{i1}+B_{i2}-D_{i3}) + [(B_{i1}+B_{i2}-D_{i3})^2 + 4(A_{i1}+A_{i2}-C_{i3})(x_{L_i} - (O_{i1}+O_{i2}-Q_{i3}))]^{\frac{1}{2}}}{2(A_{i1}+A_{i2}-C_{i3})}\Big]. \end{aligned} \tag{30}$$

where

$$\begin{aligned} x_{R_i} &= -(2(C_{i1}+C_{i2}-A_{i3})x_{\min} + (x_{\min}-x_{\max})(D_{i1}+D_{i2}-B_{i3}+x_{\min}-x_{\max})) \\ &\quad -(x_{\max}-x_{\min})[(D_{i1}+D_{i2}-B_{i3}+x_{\min}-x_{\max})^2 \\ &\quad +4(C_{i1}+C_{i2}-A_{i3})(x_{\min}-Q_{i1}-Q_{i2}+O_{i3})]^{\frac{1}{2}} / 2(C_{i1}+C_{i2}-A_{i3}). \end{aligned} \tag{31}$$

$$\begin{aligned} x_{L_i} &= -(2(A_{i1}+A_{i2}-C_{i3})x_{\max} + (x_{\max}-x_{\min})(B_{i1}+B_{i2}-D_{i3}+x_{\max}-x_{\min})) \\ &\quad +(x_{\min}-x_{\max})[(B_{i1}+B_{i2}-D_{i3}+x_{\max}-x_{\min})^2 \\ &\quad +4(A_{i1}+A_{i2}-C_{i3})(x_{\max}-O_{i1}-O_{i2}+Q_{i3})]^{\frac{1}{2}} / 2(A_{i1}+A_{i2}-C_{i3}). \end{aligned} \tag{32}$$

In equation (31), formula for x_{R_i} is developed as follows:

$$\Rightarrow \frac{x_{R_i} - x_{\min}}{x_{\max} - x_{\min}} = \frac{-(D_{i1}+D_{i2}-B_{i3})}{2(C_{i1}+C_{i2}-A_{i3})} - \frac{[(D_{i1}+D_{i2}-B_{i3})^2 + 4(C_{i1}+C_{i2}-A_{i3})(x_{R_i} - (Q_{i1}+Q_{i2}-O_{i3}))]^{\frac{1}{2}}}{2(C_{i1}+C_{i2}-A_{i3})}.$$

$$\begin{aligned} &\Rightarrow -(D_{i1}+D_{i2}-B_{i3}) - [(D_{i1}+D_{i2}-B_{i3})^2 + 4(C_{i1}+C_{i2}-A_{i3})(x_{R_i} - (Q_{i1}+Q_{i2}-O_{i3}))]^{\frac{1}{2}} \\ &= \frac{x_{R_I} - x_{\min}}{x_{\max} - x_{\min}} 2(C_{i1}+C_{i2}-A_{i3}). \end{aligned}$$

$$\Rightarrow x_{R_i} = \frac{-2(C_{i1}+C_{i2}-A_{i3})x_{\min}+(x_{\min}-x_{\max})(D_{i1}+D_{i2}-B_{i3}+x_{\min}-x_{\max})}{2(C_{i1}+C_{i2}-A_{i3})}$$

$$-[\frac{(x_{\max}-x_{\min})(D_{i1}+D_{i2}-B_{i3}+x_{\min}-x_{\max})^2}{2(C_{i1}+C_{i2}-A_{i3})}$$

$$+\frac{(x_{\max}-x_{\min})4(C_{i1}+C_{i2}-A_{i3})(x_{\min}-Q_{i1}-Q_{i2}+O_{i3})}{2(C_{i1}+C_{i2}-A_{i3})}]^{\frac{1}{2}}.$$

In equation (32), formula for x_{L_i} is developed as follows:

$$\frac{x_{L_i}-x_{\max}}{x_{\min}-x_{\max}} = \frac{-(B_{i1}+B_{i2}-D_{i3})}{2(A_{i1}+A_{i2}-C_{i3})}$$

$$+\frac{[(B_{i1}+B_{i2}-D_{i3})^2+4(A_{i1}+A_{i2}-C_{i3})(x_{L_i}-(O_{i1}+O_{i2}-Q_{i3}))]^{\frac{1}{2}}}{2(A_{i1}+A_{i2}-C_{i3})}.$$

$$\Rightarrow -(B_{i1}+B_{i2}-D_{i3})+[(B_{i1}+B_{i2}-D_{i3})^2+4(A_{i1}+A_{i2}-C_{i3})(x_{L_i}-(O_{i1}+O_{i2}-Q_{i3}))]^{\frac{1}{2}}$$

$$=\frac{x_{L_i}-x_{\max}}{x_{\min}-x_{\max}}2(A_{i1}+A_{i2}-C_{i3}).$$

$$\Rightarrow x_{L_i} = \frac{-2(A_{i1}+A_{i2}-C_{i3})x_{\max}+(x_{\max}-x_{\min})(B_{i1}+B_{i2}-D_{i3}+x_{\max}-x_{\min})}{2(A_{i1}+A_{i2}-C_{i3})}$$

$$+[\frac{(x_{\min}-x_{\max})(B_{i1}+B_{i2}-D_{i3}+x_{\max}-x_{\min})^2}{2A_{i1}+A_{i2}-C_{i3}}$$

$$+\frac{(x_{\min}-x_{\max})4(A_{i1}+A_{i2}-C_{i3})(x_{\max}-O_{i1}-O_{i2}+Q_{i3})}{2A_{i1}+A_{i2}-C_{i3}}]^{\frac{1}{2}}.$$

4. Numerical Example

Assume that a logistics company is looking for a suitable city to set up a new distribution center. Three decision makers, D_1, D_2 and D_3 of this company are responsible for the evaluation of three distribution center candidates, A_1, A_2 and A_3. Four benefit qualitative criteria such as expandability (C_1), convenience to acquiring materials (C_2), closeness to market (C_3), human resources (C_4); one benefit quantitative criterion such as area size (C_5); and one cost quantitative criterion such as investment cost (C_6) are chosen for evaluating the distribution centers.

Further assume that linguistic values and their corresponding triangular fuzzy numbers shown in Table 1 are used to evaluate each distribution center candidate versus each qualitative criterion. Ratings of distribution center candidates versus qualitative criteria are given by decision makers as shown in Table 2. Through equation (7), averaged ratings of distribution center candidates versus qualitative criteria can be obtained as also displayed in Table 2. In addition, suppose values of distribution center candidates versus quantitative criteria are present as in Table 3. According to equations (8) and (9), values of alternatives under benefit and cost quantitative criteria can be normalized as shown in Table 4. The linguistic values and its corresponding fuzzy numbers, shown in section 2.5, are used by decision makers to evaluate the importance of each criterion as displayed in Table 5. The average weight of each criterion can be obtained using equation (10) and can also be shown in Table 5.

Apply equations (11)-(22) and $g = 4$, $h = 5$, $n = 6$ to the numerical example to produce A_{i1},A_{i2},A_{i3}, B_{i1},B_{i2},B_{i3}, C_{i1},C_{i2},C_{i3}, D_{i1},D_{i2},D_{i3}, O_{i1},O_{i2},O_{i3}, P_{i1},P_{i2},P_{i3}, Q_{i1},Q_{i2},Q_{i3} for each candidate as displayed in Table 6. The calculation values for $A_{i1}+A_{i2},-C_{i3}$, $B_{i1}+B_{i2}-D_{i3}$, $O_{i1}+O_{i2}-Q_{i3}$, $C_{i1}+C_{i2}-A_{i3}$, $D_{i1}+D_{i2}-B_{i3}$, $P_{i1}+P_{i2}-P_{i3}$, $Q_{i1}+Q_{i2}-Q_{i3}$ are shown in Table 7.

Table 1 Linguistic Values and Fuzzy Numbers for Ratings

Very low(VL) /Very difficult(VD) /Very far(VF)	(0.00,015,0.30)
Low(L)/Difficult(D)/Far(F)	(0.15,0.30,0.50)
Medium(M)	(0.30,0.50,0.70)
High(H)/Easy(E)/Close(C)	(0.50,0.70,0.85)
Very high(VH)/Very easy(VE)/Very close(VC)	(0.70,0.85,1.00)

Table 2 Ratings of Distribution Center Candidates versus Qualitative Criteria

Candidates	Criteria	D_1	D_2	D_3	Averaged Ratings
A_1	C_1	VH	H	VH	(0.63,0.80,0.95)
	C_2	VE	E	M	(0.50,0.68,0.85)
	C_3	C	VC	VC	(0.63,0.80,0.95)
	C_4	M	H	H	(0.43,0.63,0.80)
A_2	C_1	VH	VH	H	(0.63,0.80,0.95)
	C_2	M	M	E	(0.37,0.57,0.75)
	C_3	C	C	VC	(0.57,0.75,0.90)
	C_4	VH	VH	VH	(0.70,0.85,1.00)
A_3	C_1	L	L	H	(0.27,0.43,0.62)
	C_2	VE	E	VE	(0.63,0.80,0.95)
	C_3	M	M	C	(0.37,0.57,0.75)
	C_4	L	M	H	(0.32,0.50,0.68)

Table 3 Values of Distribution Center Candidates versus Quantitative Criteria

Criteria	Distribution Center Candidates			Units
	A_1	A_2	A_3	
C_5	100	80	90	hectare
C_6	2	5	10	million

Table 4 Normalization of Quantitative Criteria

Criteria	Distribution Center Candidates		
	A_1	A_2	A_3
C_5	1	0.8	0.9
C_6	1	0.4	0.2

Table 5 Averaged Weight of Each Criterion

	D_1	D_2	D_3	Averaged weights
C_1	MI	VI	IM	(0.50,0.75,0.92)
C_2	IM	MI	LI	(0.25,0.50,0.75)
C_3	LI	LI	VI	(0.25,0.53,0.67)
C_4	UI	IM	VI	(0.33,0.50,0.67)
C_5	MI	VI	IM	(0.50,0.75,0.92)
C_6	VI	VI	VI	(0.75,1.00,1.00)

Through equations (23) and (24), the left, $f_{G_i}^{L}(x)$, and right, $f_{G_i}^{R}(x)$, membership functions of the final fuzzy evaluation value, G_i, $i=1,...,n$, of each distribution center candidate can be obtained and displayed in Table 8.

Table 6 Values for A_{i1}, A_{i2}, A_{i3}, B_{i1}, B_{i2}, B_{i3}, C_{i1}, C_{i2}, C_{i3}, D_{i1}, D_{i2}, D_{i3}, O_{i1}, O_{i2}, O_{i3}, P_{i1}, P_{i2}, P_{i3}, Q_{i1}, Q_{i2}, Q_{i3}

	A_1	A_2	A_3
A_{i1}	0.17	0.17	0.17
A_{i2}	0.00	0.00	0.00
A_{i3}	0.00	0.00	0.00
B_{i1}	0.77	0.76	0.62
B_{i2}	0.25	0.16	0.23
B_{i3}	0.25	0.10	0.05
C_{i1}	0.12	0.12	0.12
C_{i2}	0.00	0.00	0.00
C_{i3}	0.00	0.00	0.00
D_{i1}	-1.11	-1.11	-1.08
D_{i2}	-0.17	-0.13	-0.15
D_{i3}	0.00	0.00	0.00
O_{i1}	0.74	0.78	0.49
O_{i2}	0.50	0.40	0.45
O_{i3}	0.75	0.30	0.15
P_{i1}	1.68	1.71	1.28
P_{i2}	0.75	0.60	0.68
P_{i3}	1.00	0.40	0.20
Q_{i1}	2.68	2.70	2.23
Q_{i2}	0.92	0.73	0.83
Q_{i3}	1.00	0.40	0.20

Table 7 Values for $A_{i1}+A_{i2}-C_{i3}$, $B_{i1}+B_{i2}-D_{i3}$, $O_{i1}+O_{i2}-Q_{i3}$, $C_{i1}+C_{i2}-A_{i3}$, $D_{i1}+D_{i2}-B_{i3}$, $P_{i1}+P_{i2}-P_{i3}$, $Q_{i1}+Q_{i2}-O_{i3}$

	A_1	A_2	A_3
$A_{i1}+A_{i2}-C_{i3}$	0.17	0.17	0.17
$B_{i1}+B_{i2}-D_{i3}$	1.02	0.92	0.84
$O_{i1}+O_{i2}-Q_{i3}$	0.24	0.78	0.74
$C_{i1}+C_{i2}-A_{i3}$	0.12	0.12	0.12
$D_{i1}+D_{i2}-B_{i3}$	-1.53	-1.34	-1.28
$P_{i1}+P_{i2}-P_{i3}$	1.43	1.91	1.75
$Q_{i1}+Q_{i2}-O_{i3}$	2.84	3.13	2.91

Table 8 Left and Right Membership Functions of G_i

$f_{G_1}^L(x)$	$\dfrac{-1.02+\left[(1.02)^2+4(0.17)(x-0.24)\right]^{\frac{1}{2}}}{2\times 0.17}$, $0.24\le x\le 1.43$
$f_{G_1}^R(x)$	$\dfrac{1.53+\left[(-1.53)^2+4(0.12)(x-2.84)\right]^{\frac{1}{2}}}{2\times 0.12}$, $1.43\le x\le 2.84$
$f_{G_2}^L(x)$	$\dfrac{-0.92+\left[(0.92)^2+4(0.17)(x-0.78)\right]^{\frac{1}{2}}}{2\times 0.17}$, $0.78\le x\le 1.91$
$f_{G_2}^R(x)$	$\dfrac{1.34+\left[(-1.34)^2+4(0.12)(x-3.13)\right]^{\frac{1}{2}}}{2\times 0.12}$, $1.91\le x\le 3.13$
$f_{G_3}^L(x)$	$\dfrac{-0.84+\left[(0.84)^2+4(0.17)(x-0.74)\right]^{\frac{1}{2}}}{2\times 0.17}$, $0.74\le x\le 1.75$
$f_{G_3}^R(x)$	$\dfrac{1.28+\left[(-1.28)^2+4(0.12)(x-2.91)\right]^{\frac{1}{2}}}{2\times 0.12}$, $1.75\le x\le 2.91$

By equations (25)-(32), the total utilities, $U_T(G_i)$ x_{R_i} and x_{L_i} can be obtained and shown in Table 9.

Table 9 Total Utilities $U_T(G_i)$, x_{R_i} and x_{L_i}

Alternatives	G_1	G_2	G_3
x_{R_i}	1.97	2.26	2.12
x_{L_i}	1.39	1.40	1.33
$U_T(G_i)$	0.315	0.551	0.517

Then according to values in Table 9, candidate A_2 has the largest total utility, $U_T(G_2)$ = 0.551. Therefore A_2 becomes the most suitable distribution center candidate for this company.

5. Conclusions

A fuzzy MCDM model is proposed for the evaluation and selection of the locations of distribution centers, where ratings of alternatives versus qualitative criteria and the importance weights of all the criteria are assessed in linguistic values represented by fuzzy numbers. Membership functions of the final fuzzy evaluation values can be developed through interval arithmetic and α-cuts of fuzzy numbers. Chen's maximizing set and minimizing set is applied to defuzzify the final fuzzy evaluation values in order to rank all the alternatives. Ranking formulae are clearly developed for better executing the decision making. Finally a numerical has demonstrated the computational procedure of the proposed approach.

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