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CVA Netting Arbitrage*

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Abstract: After Lehman defaulted (credit crisis which started in 2007), practitioners considered the default risk as a major risk. The Industry began to charge for any default risk associated to derivatives. In this article, we establish the default risky price of a particular space of derivatives based on vanilla CVA and then highlight the arbitrage opportunities resulting of our hedging representation. It is the first time that the CVA premium is not considered as a binary relation.

Key words: CVA; netting arbitrage
JEL codes: C00, C15, C52, C63

1. Introduction

After Lehman Brothers defaulted, the financial industry started treating the counterparty default risk as a major risk. The curve spread ceased to be negligible, and the use of collateral agreements and CVA charges became very standard as protection against a default (see S. Alavian et al., 2010 or Kamtchueng, 2013, for more details).

> "No one is too big to default, everybody owns a default risk."

In this context, how do such fears affect derivatives pricing? How can this be consistent with the usual risk neutral pricing theory?

Focus on the hedging, we would like to establish a relation between a derivative and its vanilla hedging portfolio in a default risky world.

The industry started to consider the CVA for each derivative given by this well-known formula:

$$CVA_{t_0}(\pi) = \int_{t_0}^{T} E^Q[(1-R)N_t(\pi_t)^+ | \tau = t] dQ(\tau < t)$$

The question concerning whether CVA should be considered as a premium or not will be a topic of a subsequent publication. We would like to notify that if the CVA is not a premium, what does mean trading CVA or hedging it?

In this article, we will assume that it is an effective price (e.g., there is a strategy able to replicate the potential loss at default). After describing a set of adequate derivatives, we will elaborate two types of arbitrage opportunities. The first one is naive but need to me mentioned it is a CVA modeling arbitrage, the second is based on netting arbitrage.

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Our conclusions, which can be extended to any other asset classes, are the following; as a market participant who wants to hedge its CVA, we cannot consider it as a one credit spread related product. The CVA as a tradable asset should be computed in consideration of our hedging strategy.

The CVA modeling arbitrage presents an easy way for an underlying desk to insure that the CVA modeling is arbitrage free.

For a derivative P, we define the default risky price $\bar{P}$

$$\bar{P}(t_0, T) = P(t_0, T) - CVA_{t_0}(P)$$

The major result is that for the first time, the Netting Arbitrage is taking into account in the computation of the CVA. This consideration implies that the CVA is no longer an unidirectional risk (in fact it has never been)!

Our hedging representation can represent a big step, in term of time computation but also in term of arbitrage checking.

2. CVA Modeling Arbitrage

Before defining our hedging representation of the derivative P, we want to established straightforward arbitrage relation for the vanilla Calls, $\left(C\left(T_i, K_j\right)\right)_{i,j}$

Considering this set of vanillas $\left(C\left(T_i, K_j\right)\right)_{i,j}$, we would like to prove that the traditional arbitrage conditions stand even for the risky call, $(\bar{C}(T\_i, K\_j\ ))\_(i, j)$

It is well known that the standard no arbitrage vanilla market satisfied the following condition:

- Call Spread : $\frac{\partial C}{\partial K} < 0$
- Butterfly : $\frac{\partial^2 C}{\partial K^2} > 0$
- Calendar Spread : $\frac{\partial C}{\partial T} > 0$

Do the risky Calls, $\left(\bar{C}\left(T_i, K_j\right)\right)_{i,j}$, satisfy these conditions?

We consider a Mirror World (e.g., a world or subset of market participants have the same default identity).

We will proceed using reduction ad absurdum.

We consider a counterparty B which is kind enough to buy from us a spread of two risky calls $\bar{C}_{t_0}(T, K_2) - \bar{C}_{t_0}(T, K_1) > 0$ with $K_1 < K_2$. We will take a position short on this spread.

- In case of default of B, in one hand we will receive from the administrators the recovery of the $R \times C_{\tau_b}(T, K_1)$ and our CVA desk will give $(1 - R) \times C_{\tau_b}(T, K_1)$. On other hand, we will owe $C_{\tau_b}(T, K_2)$. The balance is positive resulting of the classical Call Spread arbitrage.
- If we default, we will receive $C_{\tau_{Me}}(T, K_1)$ from B and give to B via the administration process $R \times C_{\tau_{Me}}(T, K_1)$. The balance is still positive.
- In case of simultaneous defaults, we can consider the fact that our CVA desk manage to give us the loss given default. Therefore B will be entitled to $C_\tau\ \ (T, K_2) - C_\tau\ \ (T, K_1)$ which is negative (given the arbitrage free of $\left(C\left(T_i, K_j\right)\right)_{i,j}$.

At default, the payoff implies a positive balance for us. Therefore, there is clear arbitrage since B paid for an option which is against him whatever the event. A similar demonstration could be done for the Calendar and Butterfly.

We also want to note that these arbitrage relationships are still valid for $R_{Me} > R_b$ and$\lambda_{Me} > \lambda_B$ (the last inequality means that the default probability term structure of B is higher than ours).

Remark: the conservation of the vanilla arbitrage free conditions are not guaranteed, indeed our CVA/Exposure modeling can lead to arbitrage opportunities.

3. Hedging Representation

Each derivative is associated to a hedging strategy. In order to make sense, this strategy is composed by position on liquid products called "vanilla".

Therefore for a given $P \in D(S)$, there is one portfolio (at least one) π^P composed by vanilla position able to replicate it.

$$\begin{aligned} P_t &= \pi_t^P \\ &= \sum_{i=1}^{N} w_t^i C_t^i\big(T_{\varsigma(i)}, K_{\varepsilon(i)}\big) \\ &+ w_t^0 S_t \end{aligned}$$

Where $(w_i)_i$ are stochastic allocation processes describing a hedging strategy.

The temptation is huge to pass directly to this equality to the one linking the risky price of a derivative and the associated risky hedging portfolio.

$$\begin{aligned} \bar{P}_t &= \bar{\pi}_t^P \\ &= \sum_{i=1}^{N} w_t^i \bar{C}_t^i\big(T_{\varsigma(i)}, K_{\varepsilon(i)}\big) \\ &+ w_t^0 \bar{S}_t \end{aligned}$$

The first equality is the result of the classical pricing theory. Given our assumption regarding the CVA, the premium of the default option associated to$P$should be equal to the one associated to the hedging portfolio π_t^P.

This is the basic of the Classical Pricing Theory that will lead us to derive other arbitrage type in the next section.

We have to determine the value of the CVA related to the hedging representation of P, π_t^P:

$$\begin{aligned} CVA_{t_0}(P) &= CVA_{t_0}(\pi^P) \\ &= \int_{t_0}^{T} E^Q[(1-R)N_t(\pi_t^P)^+ | \tau = t] dQ(\tau < t) \\ &= \int_{t_0}^{T} E^Q\left[(1-R)N_t\left(\sum_{i=0}^{N} w_t^i C_t^i\big(T_{\varsigma(i)}, K_{\varepsilon(i)}\big)\right)^+ | \tau = t\right] dQ(\tau < t) \\ CVA_{t_0}(P) &\neq \sum_{i=0}^{N} w_{t_0}^i CVA_{t_0}\left(C^i\left(T_{\varsigma(i)}, K_{\varepsilon(i)}\right)\right) \\ &= \sum_{i=0}^{N} w_{t_0}^i \int_{t_0}^{T} E^Q\left[(1-R)N_t\left(C_t^i\big(T_{\varsigma(i)}, K_{\varepsilon(i)}\big)\right)^+ | \tau = t\right] dQ(\tau < t) \end{aligned}$$

The first equality is a pure deduction from the equality of fair premium called as well risk free premium. Unfortunately we do not have any linearity of the CVA operator (resulting of the no linearity of the positive value operator).

In order to retrieve the linearity property of the CVA, we consider a subset of derivatives $D^{S,+}$ defined as the set of positive derivatives statically replicable. We restrict ourselves to the positive derivatives with constant allocation (e.g., $sign(w_t^i)$ is constant for all *t*).

We have:

$$CVA_{t_0}(P) \quad = \quad \sum_{i=0}^{N} w_{t_0}^i CVA_{t_0}\left(C^i\left(T_{\varsigma(i)}, K_{\varepsilon(i)}\right)\right)$$

The representation of the risky price combine to risky price of the vanilla can help us to identify CVA modeling inconsistencies. By considering this sub set of derivative, an underlying desk can easily check the arbitrage opportunities related to the risky prices of simple vanillas.

Remark: We have implicitly considered the CVA related to a Mirror entity.

4. Netting Arbitrage

In this section, we consider a third party Das hedge seller. We construct the same type of portfolio relaxing our Mirror context hypothesis.

It is clear from the derivative hedging representation that the long vanilla positions will induce sensitivity to the credit spread of *D* (see Figure 2 in Appendix). As a derivatives seller, we cannot allow the buyer of a derivative to have any netting benefit. Even if he does not have the ability to see it, we will be entitled to a certain loss (or gain). The CVA cannot be a function of only our potential default, indeed as seller the diversification of our hedging portfolio exposed us to others default entities.

We have to transfer this risk back to the derivative buyer.

We will consider a simple and concrete example, the sell of a digital. A digital can be replicated statistically via a call spread position. As explained Kamtchueng (2014) in Chapter 4, it is not a vanilla option.

This sell will push us to enter in a long CVA position. As demonstrate, in Section 1 and 2, the hedge portfolio will cover the Digital (or Call Spread); what are the consequences regarding our CVA position?

By considering a static hedging, $Digital(T, K) \in D^{S,+}$we can determine the CVA of our portfolio as function of the CVA of its constituents.

$$\begin{aligned} CVA_{t_0}(Digital(T,K)) &= CVA_{t_0}\left(\pi^{Digital(T,K)}\right) \\ &= CVA_{t_0}\left(C(T \;\;, K_1)\right) \; - CVA_{t_0}\left(C(T \;\;, K_2)\right) \end{aligned}$$

$$K_1 < K_2$$

These equalities are resulting of the previous section. We can derive furthermore the computation by taking into account of the hedge counterparty default identity.

$$\begin{aligned} CVA_{t_0}(Digital(T,K))[A \to B] \quad &\neq \quad CVA_{t_0}\left(C(T \;\;, K_1)\right)[B \to D] \\ &- \quad CVA_{t_0}\left(C(T \;\;, K_2)\right)[C \to B] \end{aligned}$$

In term of credit sensitivity, we have

$$\begin{aligned} CVA_{t_0}(Digital(T,K))[\tau_B] \quad &\neq \quad CVA_{t_0}\left(C(T \;\;, K_1)\right)[\tau_D] \\ &- \quad CVA_{t_0}\left(C(T \;\;, K_2)\right)[\tau_B] \end{aligned}$$

Table 1 CVA (Integral Definition) and CVA Netting Arbitrage (CVA of the Hedging Portfolio) as Function of Default Distribution

Default Probabilities				
Pd^B	Pd^A	Pd^D	CVA	CVA_{NN}
$\theta^{\pm}$	θ	θ	0.00214	0.00234
$\theta^{\mp}$	θ	$\theta^{\pm}$	0.00223	0.00203
$\theta^{\mp}$	θ	θ^{-}	0.00223	0.00182
θ^{+}	θ	θ^{+}	0.00225	0.00186
θ^{-}	θ	θ^{-}	0.00212	0.00251
θ	θ	$\theta^{-,-}$	0.00219	0.00199
θ^{+}	θ	θ	0.00225	0.00167

In Table 1, we illustrate the arbitrage opportunities regarding our static hedge of the Call Spread

In order to deliver the Digital by taking a static position on a Call Spread, we cannot accept to have a CVA charge superior to the one we will pay for our hedging portfolio position. Without communication between desks, we can have a non arbitrage free structure even for a simple payoff.

5. Conclusion

We have established for first time the hedging cost of specific Risky derivatives taking into account the netting arbitrage. Indeed as a tradable asset, our hedging strategy involves our credit spread but also the ones of our hedge sellers.

This concept is as important as the modeling of the CVA itself (which involves forward market dynamics and an intensive consistent multi-factor simulator factory). The communication between traders and quants is essential; some trading choices will impact the pricing. The impact is not only related to the discount part but also to our sensitivities as we have shown with the Digital option example.

Our view of the CVA as a premium will be discussed in a subsequent publication.

Our work can be extended to more exotic products via the semi-static replication completeness.

Future works will expose the use of a representation of the risky vanilla.

One of the major issues in regards to counterparty risk is the wrong-way risk. The consideration of the hedge sellers credit rating, could be seen as a second order wrong-way risk. Indeed it could be defined by the correlation between the credit quality of the counterparty and the one of the hedging seller.

6. Numerical Test

6.1 Notations

- CVA^{NN}: credit valuation adjustment without netting agreement
- Pd^X: the credit spread of the entity X
- Pd^{Me}: credit spread term structure of the derivative seller
- Pd^{Ccy}: credit spread of the derivative buyer
- $\theta \coloneqq Q(\tau < t_i) \quad i = 1 \dots N$ a default probability from a credit term structure
- $\theta^* \coloneqq Q(\tau < t_i) * \epsilon \quad i = 1 \dots N$ with $* \in \{-, +, \pm, \mp\}$, the term structure is shifted by the absolute value $\epsilon = 0.00001$
  - If $* = \pm$ then $Q(\tau < t_i) + \epsilon, \forall i < \frac{N}{2}$ and $Q(\tau < t_i) - \epsilon, \forall i > \frac{N}{2}$

- If $* = \mp$ then $Q(\tau < t_i) - \epsilon, \forall i < \frac{N}{2}$ and $Q(\tau < t_i) + \epsilon, \forall i > \frac{N}{2}$

- $\theta^{*,*} := Q(\tau < t_i) * \epsilon * \epsilon \quad i = 1 \dots N$ with $* \in \{-, +, \pm, \mp\}$, the term structure is shifted by the absolute value $\epsilon = 0.00001$
- $dQ(\tau < t) = Q(\tau \in t \pm dt) \quad dt$

6.2 Parameter Values

- $T = 2Y$
- $K_1 = 135$
- $K_2 = 150$
- $S_{t_0} = 150$
- $\theta^S = \{v_0^s = 0.0048, v_\infty = 0.2278, k = 3.1266, \sigma^v = 0.4145, \rho = 0.3\}$
- $\theta := \{0.0015\%, 0.0036\%, 0.0064\%, 0.01\%, 0.14\%, 0.02\%, 0.289\%, 0.0412\%\}$
- $(t_i)_{i=1 \dots N} := \{0.25, 0.5, 0.75, 1., 1.25, 1.5, 1.75, 2\}$

6.3 Figures

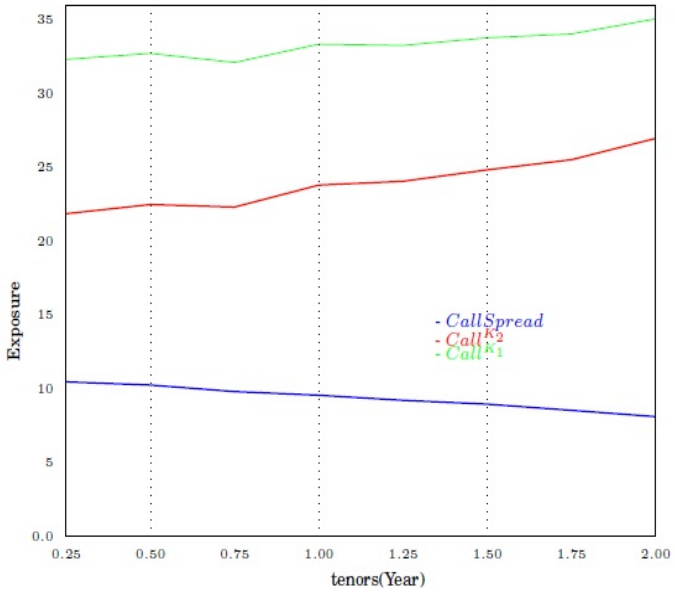

Figure 1 Expected Exposure Profile

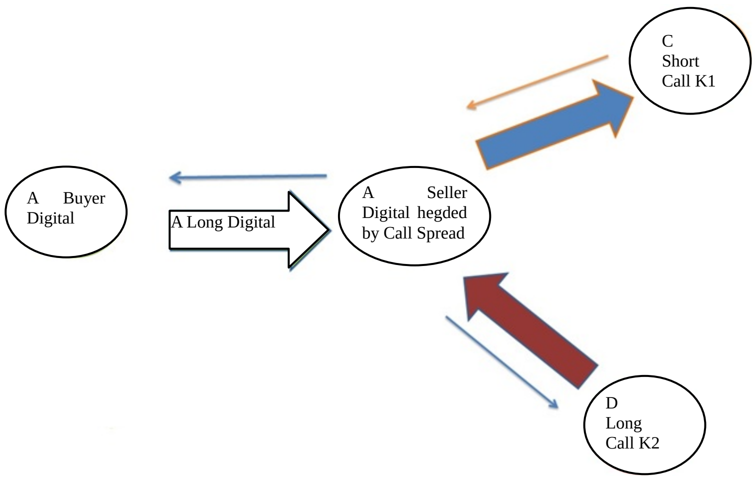

Figure 2 Netting Arbitrage for a Digital Option

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