

# Military Performance Appraising Mechanism Based

## on Combined Incentives

Zou Wenjie (Military Economics Academy of PLA, China)

**Abstract:** A specialized and efficient military performance appraising mechanism is of great significance to the Armed Forces. On the basis of orthodox principal agent theory, this paper introduces discount rate to make another commitment constraint besides of participation constraint and incentive compatible constraint of service personnel. Recommendations on military performance appraising mechanism are to be proposed through a rigorous mathematical analysis here.

Key words: performance appraising; participation constraint; incentive compatible constraint; recessive constraint

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#### **1. Introduction**

Performance appraising is in a hot discussion among scholars all over the world. Thomas Decotiis and André Perit in 1978 analyzed the social and psychological process in which the subjects and objects of performance appraising conduct appraisal by interpersonal interaction and assessment. Latham held in 2000 that an effective appraising mechanism requires not only a set of procedures, regulations and training, but a suite of incentive mechanism in which subjects are inclined to improve performance spontaneously. Mechanism design theory is aimed to create a mechanism that satisfies agent's participation constraint and incentive compatible constraint and maximize principal's expected utility, thus reducing the risk of moral hazard and adverse selection. It works well if we incorporate information economics theories into military performance appraising practice. In this case, the Armed Forces as a whole is the principal and individual service personnel is the agent. Incentives of objective and subjective performance appraising are introduced into orthodox principal-agent model to analyze the behaviors of both the Armed Forces and individual service personnel. Factors that influence optimal expected benefit are also discussed in the paper.

### 2. Model Analysis

Suppose that an individual service personnel has a linear authentic contribution to the Armed Forces Value y, i.e.,  $y = \sum \lambda_i a_i + \varepsilon$  (random variable  $\varepsilon$  represents error in authentic contribution). While the observed value of

Zou Wenjie, Graduate, Military Economics Academy of PLA; research areas/interests: defense economics theory and policy. E-mail: zwjzousha@163.com.

contribution is p, i.e.,  $p = \sum \xi_i a_i + \eta$  (random variable  $\eta$  represents error in observed contribution). Weight factor  $\lambda_i$  represents the degree of an individual service personnel's effect on the authentic benefit of the Armed Forces,  $\xi_i$  represents the degree of an individual service personnel's effect on the observed benefit of the Armed Forces. The dialectical relationships between individual and collective, short-term and long-term, the part and the whole are entailed in p and y. Suppose that the disutility function of an individual service personnel is  $c(a) = \frac{1}{2} \sum a_i^2$ , meeting convex function presumption  $c'(a_i) > 0$ ,  $c''(a_i) > 0$  (Holmsreom & Milgrom, 1991). And suppose  $\varepsilon$  and  $\eta$  follow averaged 0 Normal Distribution.

Presume that the Armed Forces pay service personnel based on observed value of performance:  $w = s + \beta p$ ,  $\beta$  is the incentive coefficient, *s* is the base pay. Then the expected benefit of the Armed Forces is:

$$E[\mathbf{v}(a)] = \sum \lambda_i a_i - \sum \beta_i \xi_i a_i - s$$

Expected benefit of service personnel is:

$$E[u(a)] = s + \sum \beta_i \xi_i a_i - \sum \frac{1}{2} (a_i)^2$$

Expected total benefit is:

$$\pi = E[v(a)] + E[u(a)] = \sum \lambda_i a_i - \sum \frac{1}{2} (a_i)^2$$

To maximize expected benefit of the Armed Forces, participation constraint and incentive compatible constraint are to be satisfied:

$$E[\mathbf{v}(a)] = \sum \lambda_i a_i - \sum \beta_i \xi_i a_i - s$$
  
s.t.  $\operatorname{argmaxs} + \sum \beta_i \xi_i a_i - \sum \frac{1}{2} (a_i)^2$  (1)

$$s + \sum \beta_i \xi_i a_i - \sum \frac{1}{2} (a_i)^2 \ge u(w_0)$$
 (2)

 $u(w_0)$  represents the reservation utility of service personnel. Equation (1) represents incentive compatible constraint; Equation (2) represents participation constraint.

Let  $\partial E[u(a_i)]/\partial a_i = 0$ , then optimal effort is  $a_i = \beta \xi_i$ , and  $a'(\beta_i)_i = \xi_i$ Let  $\partial \pi/\partial a_i = 0$ , then,  $\sum \lambda_i a_i' - \sum a_i a_i' = 0$ 

Optimal incentive coefficient  $\beta_i^*$  is deduced from results above:  $\beta_i^* = \frac{\sum \lambda_i \xi_i}{\sum \xi_i^2}$ 

Assign the intersection angle between vector quantity  $\lambda$  ( $\lambda_1$ ,  $\lambda_2$ ,...,  $\lambda_n$ ) and  $\xi$  ( $\xi_1$ ,  $\xi_2$ ,...,  $\xi_n$ ) as  $\theta$ , then,

$$\beta^* = \frac{\sum \lambda_i \xi_i}{\sum \xi_i^2} = \frac{[\lambda, \xi]}{[\xi, \xi]} = \frac{\|\lambda\| \|\xi\| \cos \theta}{(\|\xi\|)^2} = \frac{\|\lambda\|}{\|\xi\|} \cos \theta$$

Assign  $\beta^*$  into  $\pi$ , then optimal expected total benefit is:

$$\pi^* = \frac{\|\lambda\|^2}{2} \cos^2\theta$$

There are two kinds of relationships in the structure of the Armed Forces, one is the tangible and dominant wage relationship and the other is the intangible and recessive contract relationship. Levinhson's research revealed

a specific psychological contract, which includes both agreed and definite expectations like wages and bonuses and some vague ones like promotions and career developments. In the military departments, higher authorities are supposed to take not only the objective performances of the service personnel, but their potentials, group dynamics and the particularities of their posts into account, whose recessive benefits rely on subjective judgment of the authorities.

Suppose that there is a specific psychological contract between the Armed Forces and the service personnel, which means that the Armed Forces will provide a recessive incentive commitment of  $\delta$  on the condition that service personnel offers his authentic effort. Then the expected benefit of service personnel will be:

$$E[u(a)] = s + \sum \beta \xi_i a_i + \sum \delta \lambda_i a_i - \sum \frac{1}{2} (a_i)^2$$

In one-stage or finite repeated game, prisoner's dilemma will occur because of the unverifiability of the results (the Armed Forces will deny paying recessive bonus and service personnel will pay no authentic effort). While if it is the case of infinite repeated game, subgame perfect Nash equilibrium will survive. One side is able to trigger the strategy and it will sustain until the other side opts noncooperation (and it will result in a permanent noncooperation).

When an individual service personnel chooses to make a high performance, i.e.,  $y = y^h$ , whether the Armed Forces will obey its incentive commitment is hinged on the discount rate *r*. That is to say, the Armed Forces will compare the short-term benefit of violating the commitment while service personnel did high performance with that of obeying the commitment (Pareto improvement PV under recessive contract). If:

$$y^{h} - (s + \beta p) \le y^{h} - (s + \beta p + \delta y^{h}) + \sum_{n=1}^{\infty} \frac{1}{(1+r)^{n}} \left[ \sum \lambda_{i} a_{i} - \sum \frac{1}{2} (a_{i})^{2} - \pi^{*} \right]$$
  
i.e.,  $\delta y^{h} \le \frac{1}{r} \left[ \sum \lambda_{i} a_{i} - \sum \frac{1}{2} (a_{i})^{2} - \pi^{*} \right]$ 

Then the Armed Forces will choose to keep the recessive contract.

Under the recessive contract, the Armed Forces will maximize its benefit:

$$\max \sum \xi_i a_i(\beta, \delta) - \sum \frac{1}{2} [a_i(\beta, \delta)]^2$$
  
s.t. 
$$\arg \max s + \sum \beta \xi_i a_i + \sum \delta \lambda_i a_i - \sum \frac{1}{2} (a_i)^2$$
(3)

$$s + \sum \beta \xi_i a_i + \sum \delta \lambda_i a_i - \sum \frac{1}{2} (a_i)^2 \ge u(w_0)$$

$$\tag{4}$$

$$\delta y^{h} \leq \frac{1}{r} \left[ \sum \lambda_{i} a_{i} - \sum \frac{1}{2} (a_{i})^{2} - \pi^{*} \right]$$
(5)

Equation (3) represents incentive compatible constraint; Equation (4) represents participation constraint; Equation (5) represents commitment constraint.

Set 
$$\partial \left[ s + \sum \beta \xi_i a_i + \sum \delta \lambda_i a_i - \sum \frac{1}{2} (a_i)^2 \right] / \partial a_i = 0$$
, then,  
 $a_i(\beta, \delta) = \beta \xi_i + \delta \lambda_i$ 
(6)  
And  $\frac{\partial a_i}{\partial a_i} = \lambda$ .

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 $\partial \delta$ 

Set 
$$\partial \left[ \sum \xi_i a_i(\beta, \delta) - \sum \frac{1}{2} [a_i(\beta, \delta)]^2 \right] / \partial \delta = 0$$
, then,  
 $\sum \xi_i a'_i(\beta, \delta) - \sum [a_i(\beta, \delta)] a'_i(\beta, \delta) = 0$   
Then,  $\beta^{**} = (1 - \delta) \frac{\sum \lambda_i \xi_i}{\sum \xi_i^2} = (1 - \delta) \frac{[\lambda, \xi]}{[\xi, \xi]} = (1 - \delta) \frac{\|\lambda\|}{\|\xi\|} \cos \theta = (1 - \delta) \beta^*$   
Assign  $\beta^{**}$  into  $\sum \xi_i a_i(\beta, \delta) - \sum \frac{1}{2} [a_i(\beta, \delta)]^2$ , so  
 $\pi^{**}(\delta) = \pi^* + \frac{\delta}{2} \frac{(2 - \delta)}{2} \|\lambda\|^2 (1 - \cos^2 \theta)$ 

It is receivable that the military performance has got Pareto improvement after recessive contract being introduced in. Recessive contract rent SS is used to define the Pareto improvement here:

$$SS = \pi^{**} - \pi^{*} = \frac{\delta (2 - \delta)}{2} \|\lambda\|^{2} (1 - \cos^{2}\theta)$$

Further, suppose that y follows a two-point distribution that is  $P(y = y^h | a) = a$ 

Assign (6) into (5): 
$$\delta \left[ \delta - 2 + \frac{2r}{\|\lambda\|^2 (1 - \cos^2 \theta)} \right] \le 0$$

Influenced by such factors as flexible disposals, group dynamics and loyalty degrees, some indexes are hard to be objectively measured, while they could be subjectively estimated otherwise. So it is more often the case that a combination of dominant and recessive incentives functions in performance appraising, i.e.:

$$0 \le 2 - 2r / \left\|\lambda\right\|^2 (1 - \cos^2 \theta) \le 1$$

As a result:

Interval of military human resources condition:  $(r/1 - \cos^2\theta, 2r/1 - \cos^2\theta)$ Optimal dominant incentive:  $\beta^* = \frac{|2r - ||\lambda||^2 (1 - \cos^2\theta) |\cos \theta|}{||\lambda||| |\xi|| (1 - \cos^2\theta)}$ Optimal recessive incentive:  $\delta^* = 2 - \frac{2r}{||\lambda||^2 (1 - \cos^2\theta)}$ Recessive contract rent:  $SS = 2r - \frac{2r^2}{||\lambda||^2 (1 - \cos^2\theta)}$ Optimal expected benefit:  $\pi^{**} = \frac{||\lambda||^2 \cos^2\theta}{2} - \frac{2r^2}{||\lambda||^2 (1 - \cos^2\theta)} + 2r$ 

#### 3. Conclusion

(1) Optimal expected benefit  $\pi^{**}$  is in a positive correlation with  $\|\lambda\|$  when  $\cos^2\theta$  is kept constant. Recessive incentive  $\delta^*$  should be intensified since service personnel's effort impacts much on the army's benefit. For instance, key staff and intellectual capitals in the Armed Forces are exposed to a greater number of contingencies and in need of more favorable rights of flexible handling, which is some kind of recessive incentive psychological contract. Besides, because the value of service personnel has a positive effect on the intensity of incentive and

organizational performance, the Armed Forces is expected to raise the value of  $\|\lambda\|$  by training service personnel for an overall objective. For example, a more efficient selection and recommendation mechanism is to be established to create a platform for elites of all levels to receive training from the headquarters so as to be well equipped for larger missions.

(2)  $\delta^*$  and discount rate *r* are in a negative correlation. So the Armed Forces is supposed to increase the intensity of recessive incentive to encourage service personnel to choose long-term interests and collaboration rather than immediate interests. And the value of *r* will decrease as the duration of cooperation between the Armed Forces and service personnel lasts longer, then the service personnel will focus more on the long-term development of the army. At the present stage, the PLA is brewing the professionalism of military officers, which is a scheme that will give service personnel long-term incentives. Directorate generals should mount communications and collaborations with key officers of great growth potentials to draw up performance goals and indexes in accord with the requirements of their posts. Self-management is favorable by prudent decentralization of authorities on the condition that their basic objectives have been attained. Besides, the PLA may also adopt long-term incentive strategies such as postponing payment to improve the military performance according to their practical situations.

(3) Optimal expected benefit  $\pi^{**}$  is in a negative correlation with  $\cos^2\theta$ . Organizational performance will be dwarfed as the regularization of performance appraising mechanism becomes more stiff. This is not unusual in the Armament Research and Development Establishment. Rigid and structured target architecture and performance-control means will suppress the creativity of research staff. So it is hard to measure the authentic contributions of this group of personnel by sole objective appraising and will have impact on incentive effects. So military staff should be installed and cultivated selectively. For example, for higher-level officers, performance appraising mechanism should be well regulated to meet their hazy demands; for ordinary officers, performance appraising mechanism should be well regulated so as to raise military expected benefits.

(4) Dominant incentive is in lack of elasticity while recessive incentive is in redundance of elasticity. The former will induce service personnel to care more about individual short-term interests than long-term benefits of the whole army; the latter will undermine the validity of performance appraising. So dominant and recessive incentives are supposed to work in combination. The Armed Forces should establish a well-regulated base pay system in accord with the military scale, work performance and technical complexity, and set bonuses in response to the contributions, potentials and official ranks of service personnel. Besides, remunerations and discursive powers are expected to be linked up with the long-term performances of the army, soservice personnel would have the stimulation to care more about the long-term development of the Armed Forces where they serve.

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