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Communicating to Be Over- and Underconfident

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Abstract: In a multi-period trading and learning model, agents initially do not know their abilities which are related to the qualities of private signals. They assess abilities from communicating and comparing quality of their own signals with that of others after each round of trading. Motivated by recent experimental findings that over- and underconfidence in ability are often observed, agents are assumed to credit (blame) themselves strongly for favorable (unfavorable) outcomes. We demonstrate that under reasonable conditions excessive price volatility can be associated with under confidence. The relationship between expected volumes and overconfidence is non-monotone, so are agents' expected profits. We suggest that information communication among agents can alternatively account for a wide range of empirical findings.

Key words: social communication; attribution bias; overconfidence; underconfidence; asset supply uncertainty

JEL codes: D80, G12, G14

We are prone to attribute success to our own dispositions and failure to external forces.

-------- Hastorf, Schneider, and Polefka (1970)

After a big loss in the stock markets, an investor may experience a sense of regret over his decision to invest in stocks; he may interpret his loss as a sign that he is a second-rate investor, thus dealing his ego a painful blow; and he may feel humiliation in front of friends and family when word leaks out.

-------- Barberis, Huang, and Santos (2001)

1. Introduction

Many efforts have been devoted to modeling and testing the psychological findings on decision making. Overconfidence, the tendency of systematic overestimating one's ability and overestimating the accuracy of one's knowledge, is considered to be "perhaps the most robust finding in the psychology of judgement" (De Bondt & Thaler, 1995, p. 389) and is embraced by behavioral economists in understanding many financial market anomalies such as tremendous trading volumes, excess volatility, speculative bubbles, short-term momentum, and long-term reversal. Is overconfidence a really robust human trait? Are there any features in experiment designs that make subjects to behave differently in laboratory settings from real-world settings? In this paper we challenge the robustness of overconfidence, point out the limitation of supporting evidence, and hypothesize that underconfidence is possible when competition, communication and comparison between investors are incorporated into financial models.

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We are motivated by a simple fact that when one learns about her ability or the accuracy of her knowledge, she should naturally take into account the numbers of success and failure of her own and other targets such as peers, neighbors and close friends. By excluding the influence of common factors or pure luck, such considerations help one to reach a more exact estimate. One will only shift upward her beliefs about ability and knowledge accuracy when the number of her success exceeds those of her targets. While in most of traditional experiments, the subjects have to make decisions alone, which often causes them to ignore such comparisons when estimating their abilities. As a result, they are prone to attribute success to their own dispositions and failure to external forces.

We build a multi-period trading and learning model to study the evolution of agents' beliefs about their abilities and the consequences of coexistence of over- and underconfident investors in financial markets. In the beginning of each period, two types of agents with different and unknown abilities observe private signals of different precisions regarding a risky asset's payoff. Based on their beliefs of their abilities and signal precisions, they trade in a rational expectations competition fashion à la Hellwig (1980). At the end of each period, a representative agent from each type collects private signals and analyzes their precision, she then communicates and compares this quality information with the other type. They then update their beliefs, possibly in a biased way, about their own abilities accordingly. The biased belief updating is motivated by recent experimental findings that over- and underconfidence in ability are often observed, agents are assumed to credit (blame) themselves strongly for favorable (unfavorable) outcomes. Asset net supply uncertainty is introduced to prevent equilibrium price from being fully information revealing. We show that agents' ability learning will influence the properties and dynamics of market trading patterns such as price volatility, expected trading volumes, and expected profits.

Four plausible scenarios are considered: all agents are rational in beliefs updating; overconfident and underconfident agents coexist; high-ability agents become overconfident while low-ability ones are rational; and high-ability agents are rational but low-ability ones are underconfident. The main findings are summarized as follows. First, under reasonable conditions, price volatility is the highest in the "only low-ability agent being underconfident" scenario. In contrast, most existing studies show that price volatility is increasing in overconfidence degree. Second, expected trading volumes in the "only high-ability agents being overconfident" scenario are lower than that in the fully rational economy after some initial trading periods. When this happens, a higher overconfidence degree often implies lower expected trading volumes. In contrast, existing studies show that expected trading volumes are always increasing in overconfidence degree. Third, in the "only high-ability agents being overconfident" scenario, high-ability agents' expected profits can be higher with a higher overconfident degree. In contrast, existing studies show that agents' expected profits are always decreasing in overconfident degree. The central message is that overconfidence may not be robust to explain many puzzling phenomena in financial markets.

It turns out that the difference mainly results from asset supply uncertainty which is originally introduced in the literature to circumvent the conceptual difficulties such as the "Grossman-Stiglitz Paradox" (Grossman & Stiglitz, 1980) and the "No-Trade Theorem" (Milgrom & Stokey, 1982). Odean (1998) studies the effect of agent's overconfidence in a dynamic hedging model in which asset supply uncertainty is assumed away for tractability. Consequently, only private signals are incorporated into equilibrium prices. Our model shows that in the presence of asset supply uncertainty, equilibrium price aggregates private signals and asset supply with appropriate intensities. In addition, there is a trade-off between intensities. Even when the variations of private signals and asset supply are identical, if some agents are underconfident and trade less actively in response to private signals so that less information is injected to the economy, under reasonable conditions the intensity associated with asset supply will dominate that of private signals. As a result, the resulting price volatility might be higher than that if some agents are overconfident. Indeed, we show that price volatility is non-monotone in the variance of private signal and asset supply. With this result in mind, it is easy to understand the other findings. Note that expected trading volumes and expected profits are complicated nonlinear functions of price volatility. The simple relationship considered in the existing studies is no longer valid. For instance, when high-ability but overconfident agents trade with rational low-ability ones, the former has a stronger motive to trade more, but this does not necessarily imply that the total trading volumes will be higher than that in a fully rational economy, since the rational low-ability agents may trade much less. The same logic can be applied to the expected profits analysis and the details will be provided in the main text.

The rest of the paper is structured as follows. Section 2 provides critical comments on experimental and behavior economics at large and overconfidence studies in particular. Section 3 develops a multi-period trading and learning model with two types of agents. Agents' beliefs updating is described in four economy scenarios. Section 4 studies the properties and dynamics of price volatility, expected trading volumes and agents' expected profits. Section 5 discusses the role of asset supply uncertainty in a wider context and proposes that direct information communication in financial markets contributes to the observed market trading patterns. Section 6 concludes. All proofs are presented in the appendix.

2. Literature Review

This section first collects our reflections on overconfidence evidence in psychological studies. We then briefly review new experimental findings of over- and underconfidence coexistence, and other analytical studies that challenge the overconfidence explanation of financial market anomalies.

2.1 Why May Overconfidence Evidence Not Be Compelling?

There is indeed a large body of evidence in cognitive and sociological psychology supporting overconfidence. Lichtenstein, Fischhoff, and Phillips (1982) and Odean (1998) provide excellent and extensive reviews. Most papers in economics and finance take this evidence as exogenously given, and explore its consequence and implications. A few endogenize overconfidence but assume individual updates her belief of ability in isolation (e.g., Daniel, Hirshleifer, & Subrahmanyam, 1998; Gervais & Odean, 2001). Perhaps the most cited example is a report that "80% drivers claim that their driving skills are better than the average". At a first glance, this is a strong indicator of overconfidence. However, a second thought reveals that it is not very compelling.

First, actions speak louder than words. Many studies have shown that what people say is often opposite to what people do. For instance, Yezer, Goldfarb and Poppen (1996) succinctly summarize their findings from two experiments in their paper's subtitle "Watch what we do, not what we say or how we play". Analogously, 80% drivers' overconfident claims about their superior driving skills do not imply that the very same subjects drive in any irrational way on real road. Extrapolating from experimental settings to economic settings should be cautioned.

Second, a distinguishing feature of experimental studies is that isolated subjects are not allowed to share information or exchange idea with each other when making decisions, while in reality communication is one of the most prominent sociological traits of all living creatures. In particular, people have strong motives to consult experienced friends, parents and other professionals, or simply search library or Internet resources when they face uncertainty in decision makings. The possible irrational responses in isolated environment will be lessened to a great extent in the real world. In fact, empirical evidence has shown that investors learn actively from their trading experience as well as from others.

Third, experimental studies often offer contradicting evidence on human behavior. While the proponents of overconfidence evidence claim that people attribute failure to external forces and do not blame themselves, “loss aversion” of the prospect theory (Kahneman & Tversky, 1979) and “regret aversion” of the regret theory (Bell, 1982) demonstrate that people feel more pain when facing failure. Barberis, Huang and Santos (2001) vividly describe such feeling as quoted in the beginning of the paper. Barberis, Huang and Santos (2001), Barberis and Huang (2001) show that investor’s loss aversion plays an important role in the well-known “equity premium puzzle” and “value premium puzzle”, both of which are hard to be reconciled with investor’s overconfidence.

2.2 New Experimental Evidence

Recently, the validity of overconfidence evidence has been challenged by a number of psychologists and economists. They either argue that overconfidence is not a robust finding in more carefully designed experiments, or find that underconfident subjects often coexist with overconfident ones when competition and monetary rewards are incorporated into experiments. Erev, Wallsten and Budescu (1994) show that both over- and underconfidence can be obtained from the same set of data, indicating that the results are actually moderated by the research method used. Also the results of Juslin, Winman and Olsson (2000) indicate that the overconfidence bias depends on the selective attention to particular data sets. Kirchler and Maciejovsky (2002) report that under- and overconfidence, as well as well-calibration are often simultaneously observed within the context of an experimental asset market. Klayman et al. (1999) emphasize that overconfidence depends on how the experimenter asks his/her questions, what he/she asks, and whom he/she asks. They conclude that:

> “In the 1980s, the question of bias in confidence judgements seemed settled: People are grossly overconfident on all but the easiest of questions. In the 1990s, the matter was reopened, and a new conclusion was proposed: People are imperfect but generally unbiased judges of confidence; only the choice of questions was biased.”

Even if subjects are asked to compare themselves to an average peer in some experiments, the lack of direct comparisons and the ambiguity of comparison targets may lead to biased estimates. Subjects are mostly overconfident because they are free to choose a comparison in a lower rank or at higher risk. Perloff and Fetzer (1986) and Hoorens and Buunk (1993) show that the bias is reduced when the closest friend is used as specific target. Alicke et al. (1995) argue that the reality constraints that are imposed by more direct comparisons diminish the better-than-average effect. In their experiments, they show that by individuating the target and providing personal contact the magnitude of the effect decreases.

Earlier psychological studies do not neglect the possibility of underconfidence, indeed, they establish the relationship among overconfidence, underconfidence and the difficulty of the judgement task. Lichtenstein, Fischhoff and Phillips (1982) find that overconfidence for difficult questions turns into underconfidence for easy ones. Interestingly, even this seldom questioned evidence is counter to recent studies. Hoelzl and Rustichini (2005) report that choice behavior changes from overconfidence to underconfidence when the task changes from easy and familiar to non-familiar. Moore and Cain (2007) also provide experimental evidence that people believe themselves to be above average on simple tasks, and below average on difficult tasks.

2.3 Other Related Literature

Directly based on the earlier psychological findings, Benos (1998), Odean (1998)[1] develop one-period models with exogenous overconfident investors. All show that price volatility, expected trading volume, market liquidity and price informativeness increase with informed trader's overconfidence. On the other hand, agents' expected welfare decreases. Wang (1998) demonstrates the same holds in a multi-period setup. However, most of these results are questioned by García, Sangiorgi and Urošević (2007) and Xia (2014). The former shows that when rational and overconfident agents coexist and private information acquisition is endogenized, overconfidence does not affect price volatility, information efficiency, and rational agents' welfare. Intuitively, the rational agents respond to the presence of overconfident agents by reducing their information acquisition activities since the aggressive trading of the latter reveals more of their information through prices. The latter demonstrates in an economy that allows informed agents to communicate their information with each other, when investors are more overconfident in their private signals, price volatility and expected trading volume decrease but investors' expected profits increase. These comparative statics are the opposite to the findings in Benos (1998), Odean (1998) and Wang (1998). In addition, Xia (2012) shows that, when facing trade disclosure requirements, overconfident insiders trade less than underconfident ones in order to prevent information from revealing to others. This result again questions the conventional wisdom that overconfident insiders always trade more aggressively.

Most analytical papers in overconfidence literature assume that agent is born with overconfidence which does not change over time. There are a few exceptions. Gervais and Odean (2001), to which the information setup in our model is similar, develop a multi-period market model describing both the process by which a single investor learns about her ability from the number of successful predictions and how a self-attribution bias in this learning can create overconfidence. The patterns in trading volume, price volatility, and expected profits resulting from this endogenous overconfidence are then analyzed. Daniel, Hirshleifer and Subrahmanyam (1998) consider both fixed confidence level and outcome-dependent confidence level. They also assume that a representative agent updates her belief of ability in isolation.

A few analytical papers allow the coexistence of over- and underconfident investors. De Long et al. (1990) address the potential risks faced by rational arbitrageurs when competing with irrational noise traders who may have overly optimistic or pessimistic beliefs about an asset's payoff. In Kyle and Wang (1997), fund managers are either over- or underconfident in interpreting the precision of private signals when competing against each other.

3. The Model

Two types of agents trades competitively in a multi-period security market à la Hellwig (1980). Trade takes place in each period $t = 1, 2, \cdots$. A risky asset has stochastic payoff $\tilde{V}_t$ to be realized at the end of each period t, unknown to all agents at the beginning of the period. The per-capita net supply of the risky asset is taken to be the realization of a exogenous random variable $\tilde{Z}_t$ for each period t, which can be thought of coming from noise traders. The two types of agents differ only in *ability* that helps them observe one period advantageous private signals correlated to the true value of the risky asset. In a fully rational world, agents realize that abilities are independent. They do not believe so in otherwise situations. For simplicity, a continuum of agents of the same type are assumed to reside in a group, which can be as small as a community or as large as a country. The groups

[1] One model of Odean (1998) has three periods. He also examines the possibility that market maker is overconfident. He finds that overconfident market maker may dampen price volatility.

are labeled as I and J. A measure $m_h \in (0,1)$ of agent population is of the type h (high-ability) while the measure $m_l = 1 - m_h \in (0,1)$ is of the type ℓ (low-ability). We postulate that any agent of one group, say I, does not know the ability of any member of I or that of J at the outset. A casual observer of financial markets would be amazed by the pervasive information communication through social networks. In this economy social communication takes a special form. At the end of each trading period two representative agents $i \in I$ and $j \in J$ exchange some relevant information with each other, from which they update their beliefs regarding abilities and pass new beliefs to members belonging to their own group respectively. How an agent's ability influences her signal's quality and how she assesses her ability will be made clear below.

In short, the snapshot of the market trading and beliefs updating in a single period is illustrated in Figure 1.

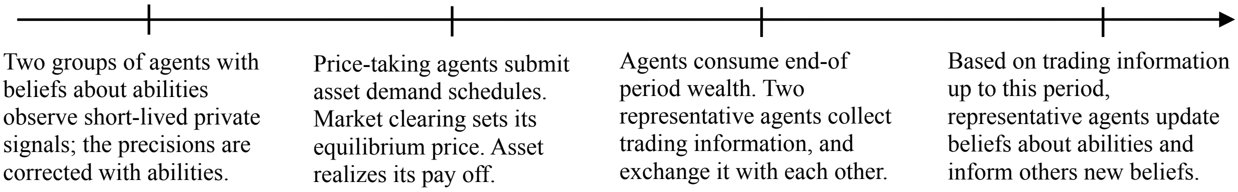

Figure 1 Timeline of Events: This Time Line Demonstrates Signal Acquisition, Market Trading and Beliefs Updating that Occur in A Single Period

It is noteworthy that agents in the model do not need to know the measure of their own groups. The information communication and comparison between two representative agents specified in section 3.2 makes the group measures irrelevant for beliefs updating. For further simplicity, we assume that the measures of two types of agents are equal so that $m_h = m_l = 1/2$ in the numerical simulation.

3.1 Preferences and Information Structure

All agents in the economy have CARA preferences with a common absolute risk aversion coefficient ϱ and they maximize their utility over wealth period by period. Agents' risk aversion is assumed to be invariant to their beliefs regarding ability. It is well known that under CARA preferences agent's demand for risky asset is independent of her initial wealth, so it is set to be 0 for all i for convenience. At the end of each period t agent i is assumed to liquidate asset holdings and consume all of her end-of-period wealth W_{it}. Let x_{it} denote the number of units of the risky asset held by agent i, and let p_t denote its price in period t. In the beginning of each period, agent i chooses the optimal demand to maximize her expected utility conditional on her information set F_{it}.

The information structure and the relationship between ability and signal quality are in the spirits of Gervais and Odean (2001).[2] In the beginning of each period t every agent $i \in [0,1]$ observes a private signal $\tilde{\theta}_{it} = \tilde{V}_{it} + \tilde{\delta}_{it}\tilde{\varepsilon}_{it}^h + (1 - \tilde{\delta}_{it})\tilde{\varepsilon}_{it}^l$ with $\tilde{\delta}_{it} \in \{0,1\}$. The superscript h or ℓ denotes the high or low quality of the signal. All random variables $\tilde{V}_t$, $\tilde{Z}_t$, $\tilde{\varepsilon}_{it}^h$, $\tilde{\varepsilon}_{it}^l$ are jointly and independently Gaussian with zero means and variances Σ, Ω, Φ_h^{-1} and Φ_l^{-1} respectively. We let $\Phi_h^{-1} = \psi\Phi_l^{-1}$ for some $\psi < 1$. Furthermore, for any two

[2] Gervais and Odean (2001) base their single insider trading model on Kyle (1985). Introducing communication between at least two agents into their model is not a trivial extension. As agent's trading intensity choice is affected by her information history, for analytical tractability market-maker has to keep track of each agent's information history. Even so, the multiplicity of agents' information realization imposes great challenge to market-maker's pricing strategy. Specifically, no closed-form solution exists. When there are multiple agents, we circumvent these difficulties by considering trading mechanism à la Hellwig (1980), which excludes the role played by market-makers.

agents the noise terms are independent, and all random variables are independent across periods. For notational simplicity the variance parameters and precision ratio ψ are assumed to be constant across period.[3] Without loss of generality we normalize the payoff of the risky asset so that $\Sigma = 1$.[4]

At the end of each period t, the communication between two representative agents specified in the next subsection enables them to know whether their signals were of high precision ($\tilde{\delta}_t = 1$) or of low precision ($\tilde{\delta}_t = 0$). Apparently agent's signal is more valuable when $\tilde{\delta}_t$ is equal to 1, which is assumed to be the case with probability ã. We interpret ã as the agent's *ability*. A priori, agent's ability is high (ã = H) with probability ϕ_0 and low (ã = L) with probability $1-\phi_0$, where $0 < L < H < 1$ and $0 < \phi_0 < 1$. By assumption, agents in the same group observe diverse signals of the same precision, i.e., the realization of $\tilde{\delta}_{it}$ is the same for all i in one group but the realizations of $\tilde{\varepsilon}_{it}^h$, or $\tilde{\varepsilon}_{it}^l$ are different across i in the same group. We abstract from the possibility that learning and trading may improve agent's ability *per se*.

3.2 Communication and Self-attribution Bias

At the end of period t, agents engage in social communication which is simplified by information exchange and comparison between two representative agents i and j with the aim to learn each other's abilities. This can be achieved by learning the quality of signals that two groups obtained at the beginning of that period, as ability is correlated to signal's precision. To do so, a representative agent can first collect a large number of signals of her group. Standard statistical exercise tells the signals' precision, after that the representative agents communicate with each other and know whether the values of $(\delta_{it}, \delta_{jt})$ are (0, 0), (1, 0), (0, 1) or (1, 1). Very soon we will see how they update their beliefs regarding abilities using such information. The new beliefs then are passed to all agents in the respective groups and affect agents' asset demands in the next period t+1.

At first glance, readers may question the plausibility of the described information collection and communication process. We can alternatively interpret our setup as follows. Consider two full service brokerage firms with unknown abilities that help them to observe signals $\tilde{\theta}_t = \tilde{\delta}_t \tilde{v}_t + (1-\tilde{\delta}_t)\tilde{\varepsilon}_t$ where normal distributions of $\tilde{v}_t$ and $\tilde{\varepsilon}_t$ are identical and independent, as considered by Gervais and Odean (2001). Each firm then provides its own signal with additional independent and idiosyncratic noise terms to its clients, whom are of identical ability.[5] Asset payoff is publicly announced after competitive trading. The communicating firms will know the profile of $(\delta_{it}, \delta_{jt})$ from which they update beliefs regarding abilities.[6] Our purpose is to explore the asset pricing implications of biased belief learning between two parties, no matter whether they are average investors or brokerage firms.

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[3] Relaxing this will not affect any propositions, provided that the varying values of these parameters are common knowledge between agents.

[4] Note that the different precisions of ε_t^h and ε_t^l raise the possibility that the size of θ_t may reveal something about the likelihood of $\tilde{\delta}_t = 1$ before trade and communication take place. This concern is relieved if we vary the precisions of ε_t^h and ε_t^l over time.

[5] Admati and Pfleiderer (1986) show that it is optimal for a monopolistic information owner to sell information with independent and idiosyncratic noise terms to a large number of buyers.

[6] This is the case since $\tilde{v}_t = \tilde{\varepsilon}_t$ happens with zero probability, given the normal distribution is continuous.

Let $\tilde{\Delta}_{it} = \left\{\tilde{\delta}_{i1}, \ldots, \tilde{\delta}_{it}\right\}$ and $\tilde{s}_{it} = \sum_{\tau=1}^{t} \tilde{\delta}_{i\tau}$ be agent *i*'s information history and the number of times that her signal is of high precision in the first *t* periods respectively. After social communication, the representative agents know Δ_{it}, Δ_{jt}, s_{it} and s_{jt}. Note that $\tilde{s}_{it}$ is the sufficient statistics for $\tilde{\Delta}_{it}$ with respect to the probability that agent *i* is of a high-ability type.

In this paper the model with fully rational agents serves as the benchmark. When agent *i* is said to be fully rational, she understands that her ability is independent from others and behaves as a Bayesian. Therefore at the end of period *t*, she ignores Δ_{jt}, s_{jt} and updates her belief that her ability is high given the history according to Bayes' rule:

$$\phi_t(s_i) = \Pr\left\{\tilde{a} = H \middle| \tilde{s}_{it} = \tilde{s}_i\right\} = \frac{H^{s_i}(1-H)^{t-s_i}\phi_0}{H^{s_i}(1-H)^{t-s_i}\phi_0 + L^{s_i}(1-L)^{t-s_i}(1-\phi_0)} \tag{1}$$

Note that the posterior belief at the end of period *t* only depends on s_i and is history-independent. The fully rational agent *i*'s updated expected ability at the end of period *t* is given by

$$\mu_t(s_i) = \mathrm{E}\left[\tilde{a} \middle| \tilde{s}_{it} = \tilde{s}_i\right] = H\phi_t(s_i) + L\left[1 - \phi_t(s_i)\right]$$

By contrast, when agents behave in a boundedly rational way, some plausible behavioral features are assumed as follows. First, agents believe that abilities are interdependent, and each agent cares about the quality comparison of her own information and others. In particular, if one observes that her information is of high precision while others low precision, she will think that the success is due to her high-ability relative to others, and vice versa. Second, agent updates her belief with a biased attribution in the sense that she overweighs success or failure too heavily when applying Bayes' rule to evaluate her own ability. Third, if one observes identical information qualities, she will still update her belief in a fully rational way.[7]

Clearly, this version of beliefs updating departs from the traditional one in that peer's success and failure are taken into account seriously. The earlier psychological studies report that when people succeed, they are inclined to attribute success to their innate abilities rather than to chance or outside factors, and they tend to believe the reverse when they fail. This "self-serving attribution" is exactly where overconfidence comes from. However, discussions in section 2 lead us to consider other possible beliefs updating rule in which one's perceptions of success and failure are attributed in a biased way only if her counterparts have the opposite outcome. This is the case since when the "peer effect" is introduced, the role played by common outside factors is twofold. On the one hand, agents start to respect the outside factors when they have identical outcomes, therefore the self-serving attribution tendency is minimized. On the other hand, when agents had distinct outcomes, their attribution biases are strengthened because no one can exalt or fault outside factors.

More precisely, for any given $\overline{\phi}_{i,t-1}$ and learning bias degrees $\gamma \geq \eta > 1$, we have

$$\overline{\phi}_{it} \equiv \Pr_b\left\{\tilde{a} = H \middle| \overline{\phi}_{i,t-1}, \delta_{it}, \delta_{jt}\right\} = \begin{cases} \dfrac{\gamma H \overline{\phi}_{i,t-1}}{\gamma H \overline{\phi}_{i,t-1} + L(1-\overline{\phi}_{i,t-1})} & \delta_{it} > \delta_{jt} \\ \dfrac{(1-H)\overline{\phi}_{i,t-1}}{(1-H)\overline{\phi}_{i,t-1} + \eta(1-L)(1-\overline{\phi}_{i,t-1})} \quad \text{if} & \delta_{it} < \delta_{jt} \\ \dfrac{H^{\delta}(1-H)^{1-\delta}\overline{\phi}_{i,t-1}}{H^{\delta}(1-H)^{1-\delta}\overline{\phi}_{i,t-1} + | L^{\delta}(1-L)^{1-\delta}(1-\overline{\phi}_{i,t-1})} & \delta_{it} = \delta_{jt} = \delta \end{cases} \tag{2}$$

[7] For simplicity the self-serving attribution bias is fully erased when agents observe information of the same quality. This, of course, can be relaxed without altering the main results.

Where subscript "*b*" to "Pr" indicates that the probability is calculated by a biased agent. $\bar{\phi}_{jt}$ can be defined similarly.

The case γ > η can be used to capture the possibility that people overweight success more than failure. Meanwhile, the opposite case γ ≤ η is also possible as the loss aversion and regret aversion literature suggest that loss looms larger than gain (Kahneman & Tversky, 1979; Bell, 1982). Whatever is closer to the truth, allowing γ ≠ η will make the posterior beliefs history-dependent and complicate the analysis greatly.[8] For convenience, we set γ = η in the following discussion. This consideration and equation (2) characterize the posterior beliefs in a straightforward way,

$$\bar{\phi}_{it}(s_i, s_j) \equiv \Pr_b\left\{\tilde{a} = H \middle| \tilde{s}_{it} = s_i, \tilde{s}_{jt} = s_j\right\}$$

$$= \begin{cases} \dfrac{\gamma^{s_i - s_j} H^{s_i}(1-H)^{t-s_i}\phi_0}{\gamma^{s_i - s_j} H^{s_i}(1-H)^{t-s_i}\phi_0 + L^{s_i}(1-L)^{t-s_i}(1-\phi_0)} & s_i > s_j \\ \dfrac{H^{s_i}(1-H)^{t-s_i}\bar{\phi}_0}{H^{s_i}(1-H)^{t-s_i}\bar{\phi}_0 + \gamma^{s_j - s_i} L^{s_i}(1-L)^{t-s_i}(1-\phi_0)} \quad \text{if} & s_i < s_j \\ \phi_t(\mathrm{s}_i) & s_i = s_j \end{cases} \tag{3}$$

$$= \frac{\gamma^{s_i - s_j} H^{s_i}(1-H)^{t-s_i}\phi_0}{\gamma^{s_i - s_j} H^{s_i}(1-H)^{t-s_i}\phi_0 + L^{s_i}(1-L)^{t-s_i}(1-\phi_0)}$$

The biased agent *i*'s updated expected ability at the end of period *t* is given by

$$\bar{\mu}_{it}(s_i, s_j) \equiv E_b\left[\tilde{a} \middle| \tilde{s}_{it} = \tilde{s}_i, \tilde{s}_{jt} = \tilde{s}_j\right] = H\tilde{\phi}_{it}(s_i, s_j) + L\left[1 - \tilde{\phi}_{it}(s_i, s_j)\right]$$

$\tilde{\phi}_{jt}(s_j, s_i)$ and $\tilde{\mu}_{jt}(s_j, s_i)$ are defined similarly. For ease of notation, we suppress s_i, s_j and simply use $\tilde{\mu}_{it}, \tilde{\mu}_{jt}$ below. Also, ϕ_0, the a priori belief of ability to be high is set to be 1/2.

Before proceeding, two comments are in order. First, we will see below that even though agents update their beliefs about abilities in a biased manner, they will realize their true abilities in the long run when they become more experienced. Second, even though we hypothesize that over- and underconfidence can coexist, we also explore other plausible scenarios in which one group of agents are fully rational. For instance, after social communication representative agents *i* and *j* observe $s_i < s_j$, the former chooses to ignore s_j and updates her belief

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[8] To see this consider a situation that at the end of two periods, $s_{i2} = s_{j2} = 1$. There are four possible histories: (i) $\Delta_{i2} = \{1,0\}$, $\Delta_{j2} = \{1,0\}$; (ii) $\Delta_{i2} = \{0,1\}$, $\Delta_{j2} = \{0,1\}$; (iii) $\Delta_{i2} = \{1,0\}$, $\Delta_{j2} = \{0,1\}$; (iv) $\Delta_{i2} = \{0,1\}$, $\Delta_{i2} = \{0,1\}$, $\Delta_{j2} = \{1,0\}$. According to the boundedly rational updating rule outlined above, it is easy shown that for agent *i*, the same posterior beliefs are obtained as

$$\phi_{i2} = \frac{H(1-H)\phi_0}{H(1-H)\phi_0 + L(1-L)(1-\phi_0)}$$

for histories (i) and (ii), and as

$$\phi_{i2} = \frac{\gamma H(1-H)\phi_0}{\gamma H(1-H)\phi_0 + \eta L(1-L)(1-\phi_0)}$$

for histories (iii) and (iv). Therefore assuming γ = η in this situation makes $\bar{\phi}_2$ identical no matter what the realized histories are. Even so, allowing history-dependent posterior beliefs will not change the main results in this paper.

in the way as specified in equation (1), while agent *j*'s takes the comparison result into account and the downward bias is embodies in her posterior beliefs through equation (3).

3.3 A Linear Equilibrium

As mentioned above, full rationality model serves as the benchmark. For notational clarity, the equilibrium and its characterizations are often stated in the benchmark scenario. Relevant notations can be adjusted accordingly in other scenarios.

Definition 1 *An equilibrium in the economy consists of a set of trading strategies x_{it} for $i \in I$, x_{jt} for $j \in J$, and a price function P_t such that:*

(1) Each agent's trading strategy maximizes her expected utility of end-of-period wealth given her information set:

$$x_t \in \arg\ \max \mathrm{E}\left[-\exp(-p\tilde{W}_t \middle| F_t\right]$$

Where $F_t = \{\theta_t, s_{i,t-1}, s_{j,t-1}, p_t\}$

(2) The market clears:

$$\int_0^{mh} x_{it}di + \int_0^{ml} x_{jt}dj = Z_t$$

In the equilibrium analysis, it is conjectured that equilibrium prices are linear in the risky asset's payoff and supply, that is, equilibrium prices of the form

$$\tilde{p}_t = P(\tilde{V}_t, \tilde{Z}_t) = \alpha_t \tilde{V}_t + \beta_t \tilde{Z}_t \tag{4}$$

Our objective is to find α_t and β_t that are consistent with this conjecture. The standard technique yields the following result.

Theorem 1 *Given* $\tilde{s}_{i,t-1} = s_{i,t-1}$, $\tilde{s}_{j,t-1} = s_{j,t-1}$ *a unique linear competitive equilibrium price is given by equation (4) with the coefficients α_t and β_t satisfy:*

$$\frac{\alpha_t}{\beta_t} = \lambda_t = \frac{\phi_h}{\rho}\left[\psi + A_t(1-\psi)\right] \tag{5}$$

$$\beta_t = \frac{\lambda_t + \rho\Omega}{\lambda_t(\lambda_t + \rho\Omega) + \Omega} \tag{6}$$

Where

$$A_t = m_h \mu_{h,t-1} + m_l \mu_{l,t-1} \tag{7}$$

A remarkable feature of equilibrium price is that it is ultimately determined by agents' aggregate (or average) expected ability denoted by *A*, as other exogenous parameters are assumed to be constants. In particular, the equilibrium price at period *t* does not depend on the realizations of $\tilde{\delta}_{it}$ for $i \in I$ and $\tilde{\delta}_{jt}$ for $j \in J$, as the noise terms in private signals cancel out when a continuum of agents submit their demands.

In different scenarios, the notations of p_t, α_t, β_t, λ_t and A_t should be adjusted accordingly. For example, when over- and underconfident agents coexist, A_t is replaced by

$$\overline{A}_t = m_h \overline{\mu}_{h,t-1} + m_l \overline{\mu}_{l,t-1}$$

We also explore other scenarios in which one type of agents is fully rational and the other is biased in ability learning. Such considerations help to better understand the relationship between market trading patterns and biased beliefs. Only by doing so can we disentangle the contributing forces of over- and underconfidence. In short run it is possible that high-ability agents may be unlucky and receive fewer high-precision signals than low-ability agents, making the model's description and notations quite awkward. To circumvent these we assume that, without loss of generality, agents in group I are of high-ability, and more importantly, that we always have $s_{it} \geq s_{jt}$ up to any period t, which is true at least in an ex ante perspective. As a consequence, two conceivable scenarios are of particular interests. When all high-ability agents in group I are overconfident while all low-ability agents in group J are rational, their aggregate expected ability is denoted by

$$A_t^0 = m_h \bar{\mu}_{h,t-1} + m_l \mu_{l,t-1}$$

When group I agents are rational while group J agents are underconfident, we have

$$A_t^u = m_h \mu_{h,t-1} + m_l \bar{\mu}_{l,t-1}$$

The economy scenarios are labelled as ε_1, ε_2, ε_3, ε_4 in order. Table 1 summarizes the scenarios, notations and descriptions. The shorthands are used in the legends of figures below.

Table 1 Four Economy Scenarios

Economy	Notation	Shorthand	Description
ε_1	A	All rational	All agents are rational in beliefs updating
ε_2	$\bar{A}$	All Biased	Over- and underconfident agents coexist
ε_3	A^o	Overconfident	Only high-ability agents are overconfident
ε_4	A^u	Underconfident	Only low-ability agents are underconfident

When we abstract the differences in ability and signal precision, then $A_t = \psi = 1$ and the resulting equilibrium conditions in Theorem 1 corresponds to those in Proposition 5.2 in Hellwig (1980). For given variances Σ, Ω, the ratio of the weights of asset payoff $\tilde{V}$ and net supply $\tilde{Z}$, α_t/β_t, being inversely related to agent's degree of risk aversion, and positively related to the precision of noise term in agent's signal, determines the relative contributions of $\tilde{V}$ and $\tilde{Z}$ to variations of the equilibrium price. When agents in the benchmark economy ε_1 are uncertain about their abilities and signal precisions, the ratio α_t/β_t is adjusted by a factor $\psi + A_t(1-\psi) \in [0,1]$. For fixed risk aversion, this implies that agents as a whole behave as if their private signals are less precise; for fixed precision of noise term, agents act as if they are more risk averse. These interpretations become more evident in economies ε_3 and ε_4. For instance, in economy ε_3 when group I agents are overconfident while group J agents are rational, in equilibrium agents altogether behave as if they mistakenly believe the precision of signals to be higher relative to that in the benchmark economy ε_1. Even though the common risk aversion is assumed to be invariant to agent's belief of ability, we can clearly see the close connection between biases in beliefs and risk aversions. In a sense, studying the implications of agents' over- and underconfidence is equivalent to studying the effects of agents' varying risk aversions. Hellwig (1980) examines how agents' different risk aversions change the properties of equilibrium prices, while we extend the analysis to other aspects of market trading such as price volatility, expected trading volumes and agents' expected profits.

4. Properties of the Model

In this section we analyze the effects of agents' learning bias on the properties and dynamics of the economy in equilibrium. We introduce the measure of over- and underconfidence and show that the attribution bias result in dynamically evolving over- and underconfidence. We then look at the effect of varying over- and underconfidence on price volatility, trading volumes, and agents' profits.

4.1 Individual Confidence Bias

Suppose trading in this security market lasts to infinity, we would expect that all rational agents in benchmark economy ε_1 to eventually learn their exact abilities. Does this result still hold when agents learn their abilities with the attribution bias? When the economy is populated by only one insider with an unknown ability, Gervais and Odean (2001) show that the answer might be "no". In particular, they demonstrate that while a high-ability agent will learn her ability precisely in the long run, a low-ability insider may mistakenly do so if her learning bias is sufficiently extreme. Fortunately, when it is common knowledge that there are two different types of agents in the economy, the answer to above question is "yes" since when high-ability agents recognize their exact ability. This is shared with low-ability agents in the presence of social communication. As a consequence, the latter must acknowledge their low-ability eventually.[9]

Proposition 1 *All agents, biased or not, will eventually learn their abilities correctly.*

One central theme of Gervais and Odean (2001) is the evolution of agent's expected level of overconfidence. They show that with self-attribution bias, an trader's expected level of overconfidence is pronounced in the early stages of her career. Then, with more experience, she comes to better recognize her own ability. In other words, their model predicts that more inexperienced agents will be more overconfident than experienced agents. However, Kirchler and Maciejovsky (2002) hypothesize that when predicting asset prices, experiment participants are expected to be more cautious and less certain in the beginning of trading. As participants gain more experience across trading period, they are expected to increasingly place more weight on their predictions, overestimate the accuracy of their judgements and lower the boundaries of the confidence intervals. This hypothesis is confirmed in their studies and is backed by Griffin and Tversky (1992) who report that experts may even be more prone to overconfidence than novices in certain tasks. Nonetheless, this discrepancy is mitigated in our model. To see this more clearly, note that the overconfident agent in Gervais and Odean (2001) will update her belief ϕ_{it}^{GO} in the following way

$$\phi_{it}^{GO}(s_i, s_j) = \phi_{it}(s_i) = \frac{(\gamma H)^{s_i}(1-H)^{t-s_i}\phi_0}{(\gamma H)^{s_i}(1-H)^{t-s_i}\phi_0 + L^{s_i}(1-L)^{t-s_i}(1-\phi_0)}$$

Apparently, we have

$$\overline{\phi}_{it}(s_i, s_j) \le \phi_{it}^{GO}(s_i, s_j)$$

Therefore, it takes a longer time for an agent to become overconfident in our model.

4.2 Price Volatility

Price volatility is measured by variance of the equilibrium price. Shiller (1981) demonstrates that it is hard to understand the excessive price volatility in security markets. Financial economists resort to agent's overconfidence as the underlying contributing forces. Most existing studies reviewed in section 2.3 maintain that

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9 If this is not common knowledge, then the validity of Proposition 1 requires that $\gamma < \left[\left(\frac{L}{H}\right)^{H}\left(\frac{1-L}{1-H}\right)^{1-H}\right]^{\frac{1}{H-L}}$.

price volatility is increasing in agent's overconfidence degree.[10] Nonetheless, this is not always true in our model. Lemma 1 is useful for the comparison of price volatilities in different economy scenarios.

Lemma 1 *In economy ε_1, conditional on agents i and j having received high precision signals s_i and s_j times in the first t periods, the price volatility in period t+1 is given by*

$$var(\tilde{p}_{t+1}|s_i,s_j)=\left[\lambda_{t+1}^2+\Omega\right]\left[\frac{\lambda_{t+1}+\rho\Omega}{\lambda_{t+1}(\lambda_{t+1}+\rho\Omega)+\Omega}\right]^2$$

Where $\lambda_{t+1}=\left[\psi+(1-\psi)A_{t+1}\right]\phi_h/\rho$ *When $5\rho^2\Omega$-4> 0, price volatility is strictly decreasing in A_{t+1} if*

$$\underline{A}\le A_{t+1}\le\overline{A} \tag{8}$$

Where

$$\underline{A}=\frac{\rho^2\Omega-2\psi\phi_h-\rho\sqrt{5\rho^2\Omega^2-4\Omega}}{2(1-\psi)\phi_h}\ \ and\ \ \overline{A}=\frac{\rho^2\Omega-2\psi\phi_h+\rho\sqrt{5\rho^2\Omega^2-4\Omega}}{2(1-\psi)\phi_h}$$

And is strictly increasing in A_{t+1} otherwise. When $5\rho^2\Omega-4\le 0$, price volatility is increasing in A_{t+1}. In other economy scenarios, price volatility and A_{t+1} are adjusted appropriately.

The price volatility depends on agents' aggregate (or average) expected ability *A* because it plays the role of varying agents' risk aversion or varying precision of noise terms in private signals. If we fix the noise terms' precision, a quick inspection of equation (5) reveals that in economy ε_1 rational agents altogether behave as if their risk aversion is $\rho/[\psi+(1-\psi)A]$ rather than ρ when they have identical ability. Given this interpretation, the non-monotonicity of price volatility in risk aversion, or equivalently, agents' aggregate expected ability, is not surprising. When variances of risky asset payoff and net supply and the precision of noise terms are fixed, in general, when agents' risk aversion increases their trading intensity decreases. The weight α of asset payoff $\tilde{V}$ therefore goes down. On the other hand, since there is less information injected into the economy, the weight β of net supply $\tilde{Z}$ goes up. As a consequence, there is a trade-off in terms of price volatility. The calculations show that the decrease in α outweighs the increase in β for sufficiently low risk aversion, hence price volatility is decreasing. When risk aversion becomes higher, then the trade-off changes direction, and price volatility is therefore increasing.[11] The same intuition can be applied to the situation in which agents behave as if their risk aversion is fixed but the precision of noise terms in private signals is practically altered by their aggregate expected ability.

We are in particular interested in the situation when price volatility is strictly decreasing in agents' aggregate expected ability. When this happens, agents in economy scenario ε_4 essentially exhibit the highest risk aversion, the resulting price volatility is therefore the highest. Meanwhile, we observe the lowest price volatility in economy ε_3. In addition, the price volatilities in economies ε_1 and in ε_2 are in-between, with the former being higher than the latter.

To convince readers, we provide a numerical example to show that it is indeed possible that high price volatility is associated with underconfidence rather than overconfidence when reasonable exogenous parameters are chosen. We first set *H* = 0.8 and *L* = 0.4 so that on average agents *i* and *j* receive high precision signals 4 and 2 times in the first 5 periods, 8 and 4 times in the first 10 periods, and so on. Learning bias degree γ and the

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[10] The exceptions include Daniel, Hirshleifer and Subrahmanyam (1998), García, Sangiorgi and Urošević (2004).

[11] The opposite situation happens if we assume that agents only observe private signals correlated with asset net supply prior to trading. Under this assumption, we still have the non-monotonicity result of price volatility.

precision ratio ψ are set to be 2 and 0.1 respectively. This four parameters' influence on the satisfaction of condition (8) are minimal. Next, from preceding discussion we know the net supply variance Ω plays an important role in determining price volatility. Since we have set asset payoff variance Σ = 1, a large Ω will enable the net supply variation to dominate the payoff variation in price volatility, consequently even at a low risk aversion level price volatility will be increasing in ϱ and decreasing in A. Alternatively, we can directly see that a large Ω will make it is easier for the condition (8) to get satisfied. Hence we set Σ = Ω = 1 to create obstacle for the goal. Gervais and Odean (2001) also use these values in their numerical analysis. Third, the magnitude of absolute risk aversion ρ is equally important. Note that for a very low ρ it is more plausible that $5\rho^2\Omega-4 \le 0$, so price volatility is increasing in agents' aggregate expected ability.[12] Therefore the more overconfident agents are, the higher price volatility results in. Although some studies report low value of risk aversion from subjects attending TV game show (Beetsma & Schotman, 2001), other researchers consider ρ to be at least larger than 3 (e.g., Veldkamp, 2006). As a compromise, we set $\rho = 1$, the most common value selected by a large number of papers (e.g., Yuan, 2005). Finally the high precision of noise terms ϕ_h is set to be 1. Given these parameter values, we have $\underline{A} = -0.111$ and $\overline{A} = 1$. Figure 2 shows the expected price volatilities in four economy scenarios as trading and learning unfold.

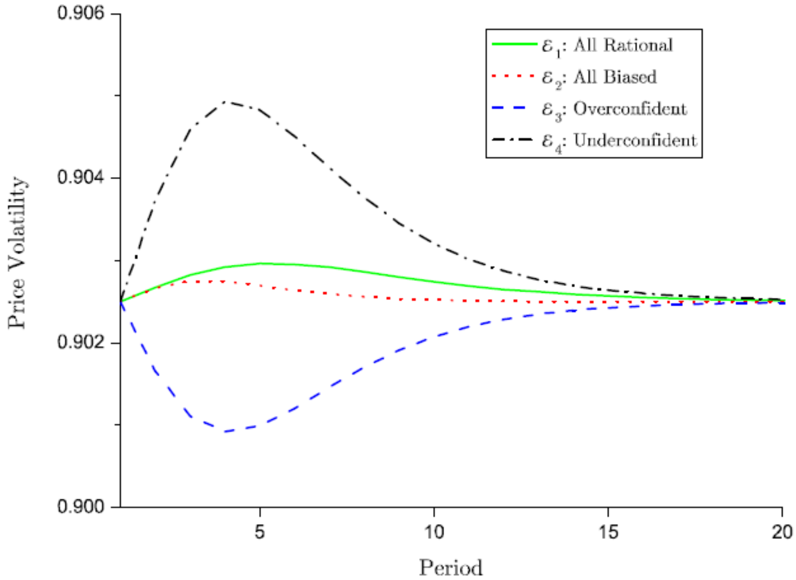

Figure 2 Price Volatilities in Four Economy Scenarios

Parameter values are H = 0.8, L = 0.4, $m_h = m_l = 0.5$, $\phi = 0.5$, $\gamma = 2$, $\Sigma = \Omega = 1$, $\Phi_h = \rho = 1$ and $\psi = 0.1$.

Denote period $t+1$'s price volatility in economy ε_k by $Volatility_{t+1}(\varepsilon_k)$ for $k = 1,\cdots,4$, we have:

Proposition 2 *Given $s_{it} \ge s_{jt}$ in the first t periods and learning bias parameter $\gamma > 1$, when*[13]

$$\underline{A} \le A^u_{t+1} \le A_{t+1} \le \overline{A}_{t+1} \le A^o_{t+1} \le \overline{A} \tag{9}$$

[12] Beetsma and Schotman (2001) use data from a Dutch TV game show and estimate absolute risk aversion to be 0.12-0.20. Nonetheless, the behavior of game-show players may not be representative for behavior outside the studio. Players are often influenced by social pressure from the audience, remarks and directions by the game-show host and the unique event of being on national TV. Consequently, they generally display lower risk aversion. The authors point out "house money effect" can make the estimate to understate player's true risk aversion as players are not yet accustomed to the money they have won so far thus they are more willing to bet their stakes.

[13] Of course $A^u_{t+1} \le A_{t+1} \le \overline{A}_{t+1} \le A^o_{t+1}$ is always satisfied.

we have

$$Volatility_{t+1}(\varepsilon_4) \geq Volatility_{t+1}(\varepsilon_1) \geq Volatility_{t+1}(\varepsilon_2) \geq Volatility_{t+1}(\varepsilon_3)$$

Moreover, when condition (9) is satisfied, the price volatility in economy $\varepsilon_{3(}\varepsilon_{4)}$ are lower (higher) when γ is larger, respectively.

Some comments are in order. First and most importantly, the non-monotonicity of price volatility in some exogenous parameters is crucial for this new finding, which results directly from the randomness of both asset payoff and net supply. Odean (1998) shows that when agents are certain about asset net supply, the resulting price volatility is strictly increasing in the variance of asset's payoff. It is well-known that the introduction of random asset net supply is mainly to overcome the "Grossman-Stiglitz Paradox" (Grossman & Stiglitz, 1980) and the "No-Trade Theorem" (Milgrom & Stokey, 1982), it is new to the literature that such introduction leads to non-monotonicity of price volatility. Second, Third, this comparison result does not depend on the specified updating rule (3). In particular, when condition (9) holds, allowing downward bias degree η to be smaller or larger than the upward bias γ won't change the price volatility patterns. Third, even when the price volatility patterns are reversed for other exogenous parameters so that price volatility in economy ε_3 becomes the highest, we can see that price volatility in economy ε_2 is higher than that in benchmark economy ε_1. Hence the coexistence of over- and underconfident agents is not incompatible with the observed excessive price volatility in security markets. Ruling out the possibility of underconfidence is unnecessary.

4.3 Trading Volumes

The enormousness of trading volumes in security markets is also a big puzzle to economists since the "No-Trade Theorem" of Milgrom and Stokey (1982). The noise or liquidity trading, i.e., the randomness of asset net supply, and agents' heterogeneous prior beliefs have been proposed in the literature as significant motives for trade, among which agents' overconfidence has received special attention because of the supporting evidence from the earlier psychological findings. As reviewed in section 2.3, most existing studies show that expected trading volumes in an economy populated by overconfident agents are higher compared to an otherwise identical but fully rational economy. This paper joins García, Sangiorgi and Urošević (2007) and Xia (2012) and shows that this is not necessarily the case.

In the benchmark equilibrium, agent *i* optimally determines her demand $\tilde{x}_{it}$ for the risky asset. In the appendix, we show that

$$\tilde{x}_{it} = B_{it}\tilde{\theta}_{it} - C_{it}\tilde{p}_t \tag{10}$$

Where

$$B_{it} = \frac{\phi\left[\psi + \mu_{i,t-1}(1-\psi)\right]}{p} \tag{11}$$

$$C_{it} = \frac{\rho\Omega}{\rho^2\Omega + \phi_h[\psi + A_t(1-\psi)} + B_{it} \tag{12}$$

are agent *i*'s trading intensities on private signal and observed price in period *t*, respectively. In other economy scenarios, B_{it} and C_{it} need to be adjusted accordingly.

It is assumed that after asset realizes its payoff at the end of each period, agents liquidate asset holdings and consume all of their end-of-period consumption, the trading volume in period *t*+1 is thus defined by

$$Volume_{t+1} = \frac{1}{2}\left(\int_0^{mh}\left|\tilde{x}_{i,t+1}\right|di + \int_0^{ml}\left|\tilde{x}_{j,t+1}\right|dj\right) \tag{13}$$

Where the coefficient 1/2 corrects the double counting when summing the shares traded over all agents. Lemma 2 provides the value of expected trading volume for the following discussion.

Lemma 2 *In economy* ε_1, *conditional on the agents i and j having received high precision signals* s_i *and* s_j *times in the first t periods, expected trading volume in period t+1 is given by*[14]

$$E(Volume_{t+1} \mid s_i, s_j) = \sqrt{\frac{1}{2\pi}}\left[m_h\sqrt{var(\tilde{x}_{h,t+1} \mid s_i, s_j)} + m_l\sqrt{var(\tilde{x}_{l,t+1} \mid s_i, s_j)}\right] \tag{14}$$

Where $var(\tilde{x}_{h,t+1} \mid s_i, s_j)$ *is given as (A7) in the appendix.* $var(\tilde{x}_{l,t+1} \mid s_i, s_j)$ *can be calculated similarly. In other economy scenarios, expected trading volume is adjusted accordingly.*

From the appendix we see that the exact forms of $var(\tilde{x}_{h,t+1} \mid s_i, s_j)$ and $var(\tilde{x}_{l,t+1} \mid s_i, s_j)$ are quite involved. Unlike the price volatility in which the expected abilities μ_{ht} and μ_{lt} of two types of agents play their roles collectively through aggregate expected ability A_{t+1}, expected trading volume in period t+1 is determined not only by A_{t+1} but also by μ_{ht}, μ_{lt} individually. The way that μ_{ht}, μ_{lt} and A_{t+1} enter into the expression of $E(Volume_{t+1} \mid s_i, s_j)$ makes it a formidable task to analyze the resulting comparative statics. The ultimate dependence of $E(Volume_{t+1} \mid s_i, s_j)$ on learning bias degree γ is equally ambiguous. To see this more clearly, in period t+1 consider a relatively simple comparison of economies ε_1 and ε_3. As low-ability agents in group J are always rational under these two scenarios, the difference in expected trading volume is ultimately caused by the distinctions of μ_{ht} and $\overline{\mu}_{ht}$. Even so, it is still extremely hard to answer whether expected trading volume in economy ε_3 is always higher than that in economy ε_1. First of all, let's take a close look of the high-ability agents' demands for risky asset in economies ε_1 and ε_3. Suppose in some period the realizations of signal $\tilde{\theta}$ and price $\tilde{p}$ in both economies are very close, then the magnitude of trading intensities B_h and C_h in equation (10) of high-ability agents in group I determines whose demand is larger. On the one hand, in economy ε_3 overconfident agents will definitely increase their trading intensity B_h on private signals because either they overestimate the precision of signals or they become less risk averse. On the other hand, in determining their demands, overconfident agents also respond to observable price, on which the change of their trading intensity C_h is indefinite. This can be easily seen through a scrutiny of equation (12). It is likely that overconfident agents' larger trading intensity B_h on signals implies a larger variation in price, leading the very agents to increase their trading intensity C_h on price too. As a result we are uncertain whether high-ability agents' demand in economy ε_3 is higher or lower because the form of equation (10). When we take into account group J agents' asset demands and the resulting total expected trading volume (which involves sum of the absolute value of demands) in both economies, the matter becomes more complicated. In addition, a component of expected trading volume-the variance of agents' demand-is affected by agents' trading intensities as well as price volatility. A quick examination of equation (A6) in the appendix, that is,

$$var(\tilde{x}_{i,t+1} \mid s_i, s_j) = B_{i,t+1}^2 var\left(\tilde{\theta}_{i,t+1}\middle| s_i, s_j\right) + C_{i,t+1}^2 var(\tilde{p}_{t+1} \mid s_i, s_j) - 2B_{i,t+1}C_{i,t+1}\alpha_{t+1}$$

[14] Let $\tilde{y}$ be a normally distributed random variable with mean zero, then $E\left|\tilde{y}\right| = \sqrt{2var(\tilde{y})/\pi}$.

reveals that the interactions among them are far from simple. Preceding analysis has shown that the price volatility, affected by variations in asset's payoff and net supply, is non-monotone in several exogenous parameters, which complicates the property of expected trading volume. Even though we understand the comparative statics of price volatility, its influence, when compounded by the influence of *B* and *C*, on the expected trading volume is still ambiguous.

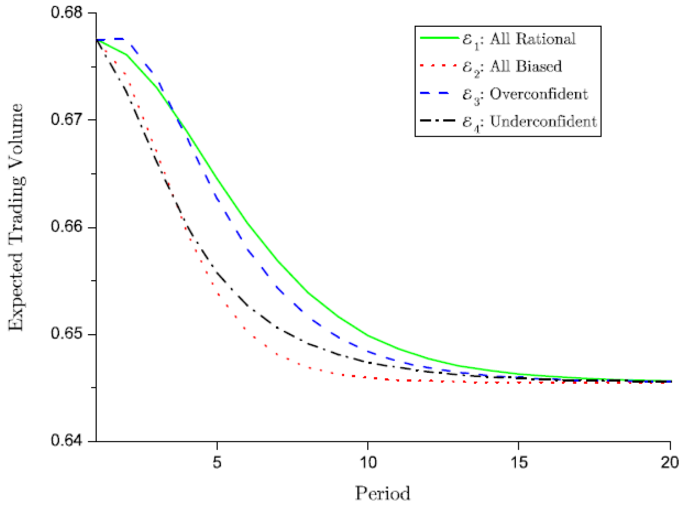

Figure 3 Expected Trading Volumes in Four Economy Scenarios

Parameter values are H = 0.8, L = 0.4, $m_h = m_l = 0.5$, $\phi_0 = 0.5$, $\gamma = 2$, $\Sigma = \Omega = 1$, $\Phi_h = \rho = 1$ and $\psi = 0.1$.

We choose to rely on numerical simulation in Figure 3 to examine the properties and patterns of the expected trading volumes in different scenarios, using the previously chosen exogenous parameters and the assumption that agents observe high precision signals in proportion to their true abilities. A number of remarkable features stand out. First, except for the first period in which agents have common prior, expected trading volumes in scenarios ε_1 and ε_3 are higher than those in scenarios ε_2 and ε_4 until agents realize their true abilities. Even so, a direct analytical comparison of expected trading volumes in some scenarios, for instance, economies ε_1 and ε_4, is still pretty difficult.

Second, we see that expected trading volumes in different scenarios exhibit diminishing patterns for all trading periods until agents' beliefs converge with the only possible exception being that in economy ε_3, expected trading volume could be increasing for some initial periods. This is understandable. When trading begins, agents learn their abilities through information communication over time. In general, high-ability agents will increase their trading volumes as they indeed observe signals of high precision or they are prone to believe so, while low-ability agents will do the opposite. Numerical simulation shows that on average the magnitude of decreasing volumes outweighs that of increasing ones, despite the same measure of agents in groups *I* and *J*. In economy ε_3 analysis in section 3.1 reveals that the degree of high-ability agents in group *I* overestimating their ability is most severe in some initial periods, they trade far more aggressively than what they do in later periods, while the changes of trading volume coming from low-ability but rational agents in group *J* is not that much. This interplay leads to the increasing portion of expected trading volumes in economy ε_3. All these discussions can be seen evidently in a decomposition exercise, as shown in Figure 4, where we roughly measure the expected trading volumes of two types of agents by $m_h\sqrt{var(\tilde{x}_{g,t+1}\,|\,s_i s_j\,/\,2\pi}$ and $m_l\sqrt{var(\tilde{x}_{l,t+1}\,|\,s_i s_j\,/\,2\pi}$ respectively.[15]

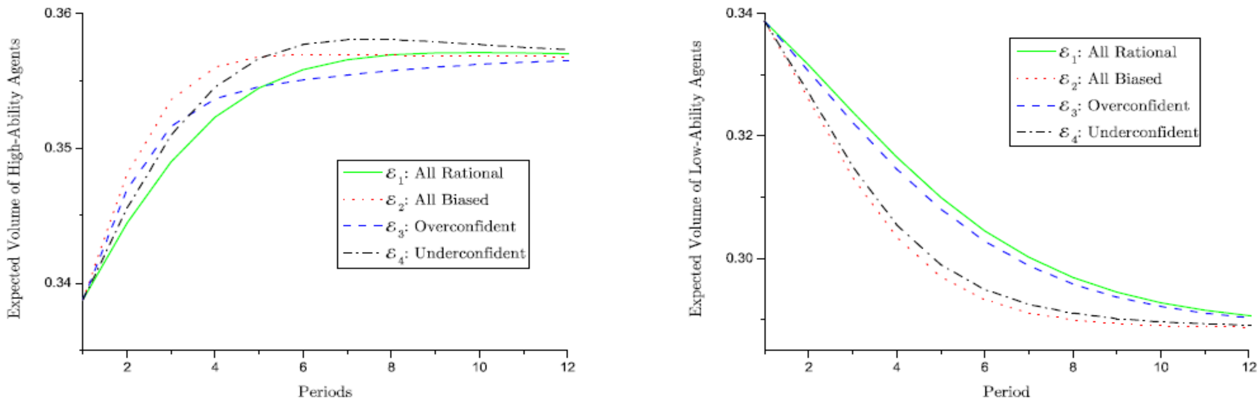

Figure 4 Decomposition of Expected Trading Volumes in Four Economy Scenarios

Finally and most relevantly for our purpose, Figure 3 reveals that in several initial periods the expected trading volume in economy ε_3 is higher relative to that in economy ε_1, but this pattern is completely reversed later on. Similar pattern applies to economies ε_2 and ε_4 where the distinction also comes from whether high-ability agents are overconfident or rational in learning. The decomposition exercise in Figure 4 shows that expected trading volume of the rational low-ability agents in economy ε_3 is always lower than those of the very agents in economy ε_1, while the overconfident high-ability agents' expected trading volume in economy ε_3 is initially higher but then dominated by those of rational high-ability agents in economy ε_1. Putting together, we observe a new pattern of expected trading volume. It is partially consistent with the existing studies but we also see a sharp contrast. Admittedly, an analytical proof is hard to obtain. As explained before, the difference in expected trading volumes is solely determined by the distinction of group *I* agents' expected abilities $\overline{\mu}_{ht}$ and μ_{ht}. However, the way that group *I* agents' expected ability enters into the form of expected trading volume makes the comparative statics analysis intractable. For instance, numerical simulation demonstrates that slight changes of $\overline{\mu}_{ht}$-μ_{ht}, which are always of the same sign for $t > 1$, will dramatically change the sign of the difference between expected trading volumes in two scenarios at some point.

It is noteworthy to highlight that the pattern of expected trading volumes in four economy scenarios is retained for many sets of parameters specifying the economy and information structures (perhaps for all sets, no numerical counterexample is found).[16] This is even the case when the patterns of price volatility are reversed for some sets of parameters so that the price volatility in "overconfident" scenario ε_3 is the highest while that in "underconfident" ε_4 the lowest.

Denote period t+1's expected trading volume in economy ε_k by $Volume_{t+1}(\varepsilon_k)$ for $k = 1, .. ,4$, the findings in numerical simulation is summarized as follows.

Proposition 3 *For many sets of parameters specifying the economies, given $s_{it} \geq s_{jt}$ for all periods t, there*

[15] It is very interesting to note that, in the left panel of Figure 4, the rough measure of high-ability agents' trading volume in economy ε_4 is always higher than that in economy ε_1 although agents are always rational in learning their ability (The opposite is true for low-ability agents in economies ε_4 and ε_1, see the right panel of Figure 4). The distinction solely results from agent's different trading intensities on price. In economy ε_4 where low-ability agents are underconfident, their demand affects the equilibrium price in such a way that it is optimal for high-ability agents to react more to price than they do in economy ε_1. The same is true for high-ability agents' trading volumes in economies ε_2 and ε_3 for similar reason.

[16] I fix $\Sigma = 1$, $m_h = m_l = 1/2$ and $\phi_0 = 1/2$ for numerical simulations.

exists some periods t > 1 such that

$$Volume_{t+1}(\varepsilon_1) > Volume_{t+1}(\varepsilon_3) > Volume_{t+1}(\varepsilon_4) > Volume_{t+1}(\varepsilon_2)$$

Another specific question we want to reexamine is the relationship between expected trading volume and overconfidence degree. Most existing studies maintain that when agents mistakenly believe that their private signals are more precise, expected trading volume is higher primarily because overconfident agents trade more aggressively. For our purpose the attention is restricted to “only high-ability agents being overconfident” scenario ε_3 Our model disagrees with the simple monotone relationship. Before we go to the detail, we need to emphasize that the economy ε_3 is distinct from the economies considered in other studies in two aspects. First, rational agents are usually assumed away in those studies. Either a single overconfident agent plays a strategic game with market-makers, or many overconfident agents trade competitively. Second, even when rational agents trade against overconfident agents, both are certain about the types of each other and the overconfidence degree is constant.

Figure 5 plots expected trading volume in economy ε_3 with a varying learning bias degree γ. Interestingly, after several initial periods, expected trading volumes are strictly decreasing when high-ability agents’ learning bias degree is larger. At this moment we should be well prepared for these findings. The decomposition exercise, plotted in Figure 6, shows that it is indeed the case that high-ability agents will trade more as they become more experienced (the opposite, to a larger extent, is true for low-ability agents), but there is no guarantee that they will trade more when they become more overconfident (interestingly, low-ability agents trade even less when this happens). Under the complicated interactions among two types of agents’ asset demands, information quality and market-clearing prices, more overconfident agents do not necessarily trade more aggressively. In fact, these simulation exercises further confirm preceding discussions from a new perspective. It is deserve mentioning that similar patterns of expected trading volumes with varying learning bias degrees are retained in economy ε_3 for many sets of exogenous parameters (no numerical counterexample is found).

Proposition 4 *In economy ε_3 where high-ability agents in group I and low-ability agents in group J are overconfident and rational in learning their abilities respectively, higher expected trading volumes are not necessarily associated with a larger learning bias degree γ.*

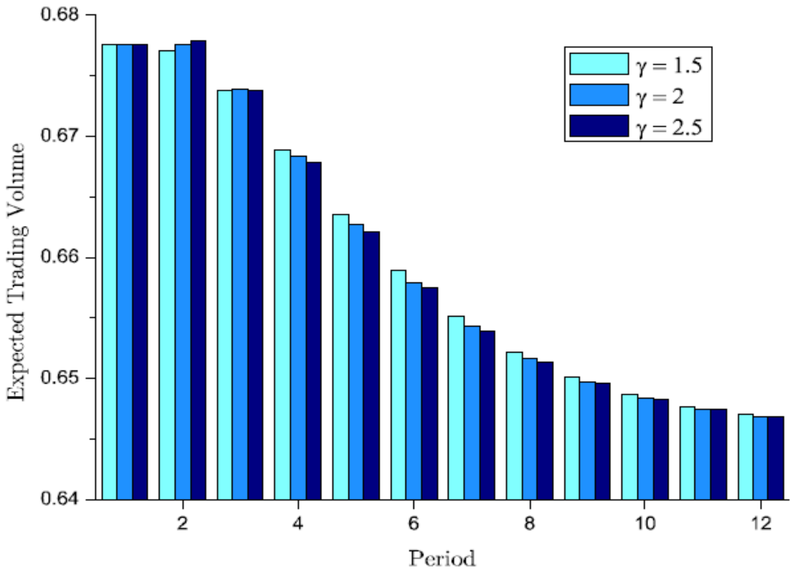

Figure 5 Expected Trading Volumes with A Varying Learning Bias Degree In Economy ε_3

Parameter values are H = 0.8, L = 0.4, $m_h = m_l = 0.5$, $\phi_0 = 0.5$, $\gamma = 2$, $\Sigma = \Omega = 1$, $\Phi_h = \rho = 1$ and $\psi = 0.1$

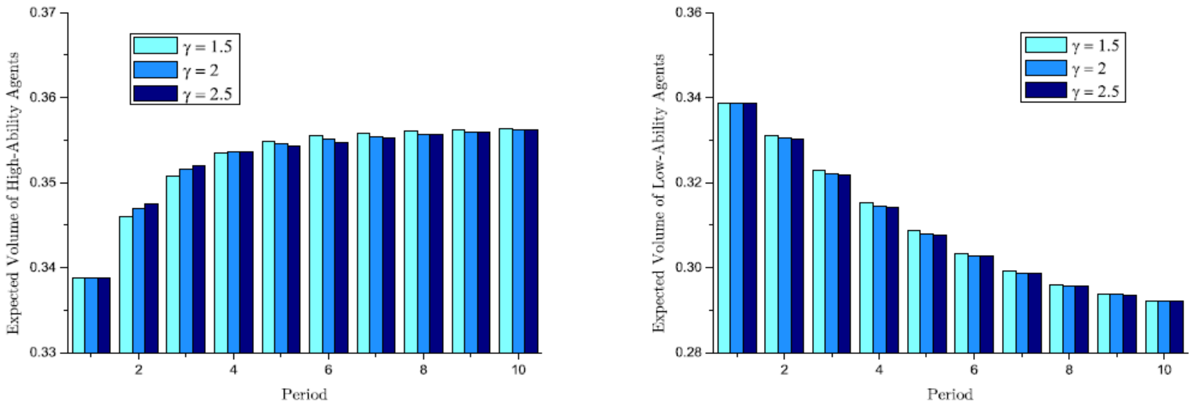

Figure 6 Decomposition of Expected Trading Volumes with a Varying Learning Bias Degree in Economy ε_3

4.4 Expected Profits

We turn to address the effects of agents' learning bias on the properties and dynamics of their expected profits in equilibrium. They are important because of two concerns. Models of irrational behavior at large and of overconfidence in particular are criticized by the argument that rational agents will outperform irrational agents and eventually drive the latter to the margins of markets. This view has been challenged by De Long et al. (1990), and Hirshleifer and Luo (2001), among others. These authors argue that irrational agents may earn higher expected profits than rational ones by bearing a larger amount of risk created by higher demands. Yet, Odean (1998) and Gervais and Odean (2001) show that a single insider's expected profit is decreasing in her overconfidence degree.

Lemma 3 *Conditional on the agents i and j having received high precision signals s_i and s_j times in the first t periods, expected profit of agent i in period t+1 is given by*

$$E(\pi_{i,t+1} \mid s_i, s_j) = \frac{\rho\Omega\Phi_h^3\Lambda(\mu_{it})\Lambda^2(A_{t+1}) + \rho^3\Omega^2\Phi_h\Lambda(\mu_{it})\left[1+2\Phi_h\Lambda(A_{t+1})\right] + \rho^5\Omega^3\left[1+\Phi_h\Lambda(\mu_{t+1})\right]}{\left[\Phi_h^2\Lambda^2(A_{t+1}) + \rho^2\Omega(1+\Phi_h\Lambda(A_{t+1}))\right]^2} \tag{15}$$

where

$$\Lambda(y) = \psi + y(1-\psi) \tag{16}$$

Given $s_{it} \geq s_{jt}$ for all t, high-ability agent' expected profit is higher than that of low-ability agent.

Not unexpectedly, we once again face the situation that agent *i*'s expected profit in period *t*+1 is determined by her expected ability μ_{it} as well as the aggregate expected ability A_{t+1}, which affect agent's trading behavior and equilibrium price respectively. The ultimate influences of learning bias degree on expected profits in different economy scenarios are obscure, to say the best. We have to rely on numerical simulations to gain insights about patterns of expected profits. Fortunately, before doing so an affirmative conclusion can be drawn: From the ex ante perspective in the sense that agents observe high precision signal in proportion to their real abilities, then high-ability agents, whatever rational or overconfident, on average earn more than low-ability agents. This is evident in Figure 7 under the same exogenous parameters used before. The intuition behind this result is straightforward. No matter high-ability agents are rational or overconfident, their private signals are indeed more precise on average. Other things equal, this implies more expected profits. Moreover, as trading and learning unfold their evaluation of ability become more accurate. Consequently, their expected profits rise over time and converge to the highest level. The opposite characterizes the dynamics of low-ability agents' expected profits.

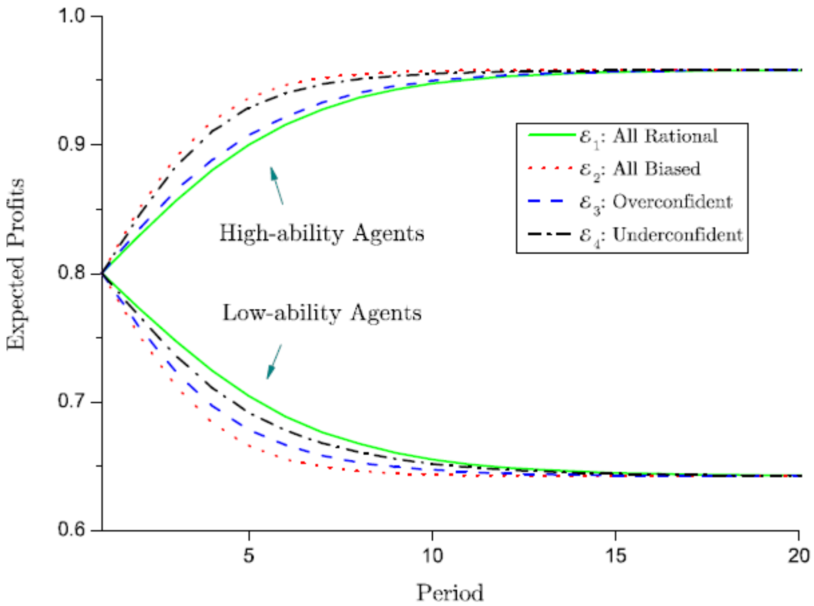

Figure 7　Expected Trading Profits in Four Economy Scenarios

Parameter values are H = 0.8, L = 0.4, $m_h = m_l = 0.5$, $\phi_0 = 0.5$, $\gamma = 2$, $\Sigma = \Omega = 1$, $\Phi_h = \rho = 1$ and $\psi = 0.1$

Some other interesting patterns of expected profits arise. First, comparing Figure 7 with left panel of Figure 5, we see that expected profits are more or less related to agents' trading behavior. For instance, on average high-ability agents trade more and earn more in economy ε_4 relative to economy ε_1, and same is true in economy ε_2 relative to ε_3 However, we should also note that trading more does not necessarily imply that earning more, and vice versa. Although high-ability agents' expected profits can be ranked orderly in Figure 7, we clearly can't rank their expected trading volume in the same way. At the same time, comparing Figure 7 with right panel of Figure 5, we can rank both expected profits and expected trading volume of low-ability agents, but the one-to-one relationship does not exist. For instance, low-ability agents trade more but earn less in economy ε_3 relative to economy ε_4. Second, when two types of agents compete together, rational agents' expected profits may be higher (resp. lower) than overconfident (resp. underconfident) agents' when their competitors behave differently. For instance, for high-ability agents rational ones in ε_4 on average earn more than overconfident ones in economy ε_3, and for low-ability agents rational ones in economy ε_3 earn less than underconfident ones in economy ε_4.

We can understand these findings from another perspective. Recall that in left panel of Figure 6 when high-ability agents become more overconfident while low-ability agents are always rational in economy ε_3, the formers may trade less after several initial periods. The following Figure 8, using the same exogenous parameters chosen before, reveals that high-ability agents may still earn more.

Proposition 5 *In economy ε_3 where group I agents and group J agents are overconfident and rational in learning their abilities respectively, lower expected profits is not necessarily associated with larger learning bias degree.*

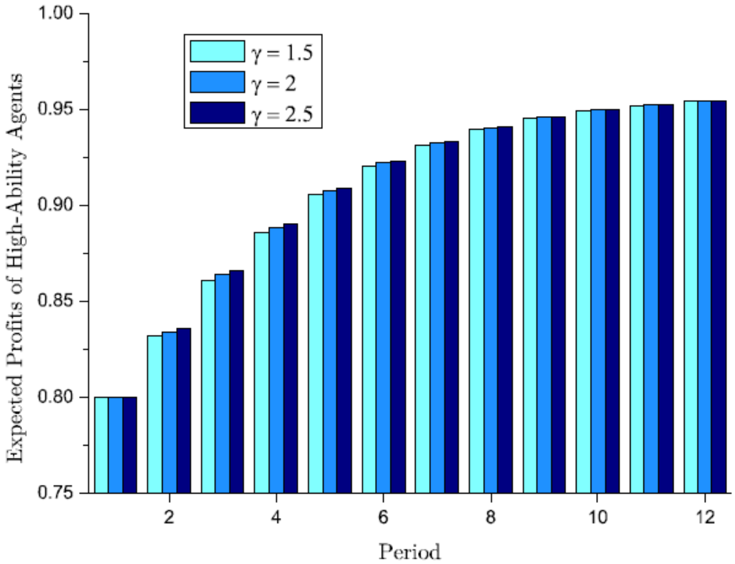

Figure 8 Expected Profits of High-Ability But Overconfident Agents in Economy ε_3

Parameter values are H = 0.8, L = 0.4, $m_h = m_l = 0.5$, $\phi_0 = 0.5$, $\gamma = 2$, $\Sigma = \Omega = 1$, $\Phi_h = \rho = 1$ and $\psi = 0.1$

The logic behind this result is not surprising, as we are already familiar with the nonlinear relationship between expected profits and price volatility. To see this more clearly, remember informed agents' profits come from noise traders' loss. When high-ability but overconfident agents trade more aggressively, they take a larger share of profits relative to rational low-ability agents. Their higher risk is compensated by higher return. Note that this is not always true, it is possible that overconfident agent's expected profit is decreasing in her overconfident degree. For instance, when $\rho = 0.2$, we have such an example. For low-ability but rational agents, for most sets of parameter their expected profits are decreasing in the learning bias parameter.

5. Discussion

In this section we discuss why our model generates asset pricing and welfare implications in sharp contrast with existing studies. It turns out that the uncertainty of risky asset net supply is a crucial element which separates our findings from others. Furthermore, if we cannot rely on agent's overconfidence to explain the observed market trading patterns such as excessive price volatility, trading volume, what are other possible contributing factors? We suggest that information communication among agents is a plausible candidate.

5.1 The Role of Asset Random Net Supply

As mentioned before, randomness of asset net supply, or noise/liquidity trading, is introduced into competitive or strategic rational expectation frameworks in order to circumvent the troubles of "Grossman-Stiglitz Paradox" and "No-Trade Theorem". Our paper shows that this uncertainty has somewhat unexpected effects on the non-montonicity of price volatility, trading volume, and expected profits, at least in the competitive rational expectation framework. Odean (1998) examines the role of overconfidence à la a multi-period version of Hellwig (1980). He shows that price volatility and expected volume are increasing in an agent's overconfidence degree. Notably, the net supply of risky asset is assumed to be constant in every trading period. Although it is understandable to make this simplification in a complicated dynamic hedging model for the tractability purpose, the important roles of supply uncertainty on price volatility and expected volume are compromised.

It is indeed true that the role of asset random net supply is minimal in affecting price volatility and expected volume in some strategic rational expectation models such as the risk-neutral version of Kyle (1985). For instance, the variance of noise trading does not affect price volatility because agents scale up their trading intensities on private signals in response to an increase in the amount of noise trading. Nonetheless, the supply uncertainty has much greater influence if risk-neutral agents are replaced by risk-averse ones (Subramanyam, 1991).

5.2 The Role of Information Communication

Trading activity is economic as well as sociological. Recently a new and growing empirical literature has documented that information communication affects individual trading behavior and market trading patterns in financial markets (e.g., Wysocki, 1998; Antweiler & Frank, 2004; Hong, Kubik & Stein, 2005; Ivković & Weisbenner, 2007). Prompted by these findings, Ozsoylev (2005) builds analytical model and establishes that when investors directly and truthfully share information in established social network prior to competitive trading, the resulting equilibrium may account for the observed high price volatility and trading volume. Xia (2014) shows in a context of strategic trading that agent's information communication in social network generates higher price volatility and trading volume but lower expected profits compared to those in a no-communication economy. In particular, Xia (2014) allows agents to be overconfident in private signals in the sense that they incorporate a disproportionally large weight of private signals relative to information received from communication. He demonstrates that price volatility, expected trading volume are strictly decreasing in agents' overconfidence degree, but agents' expected profits are strictly increasing in the overconfidence degree. These results are directly opposite to findings in the overconfidence literature reviewed in section 2.3. Very interestingly, Xia (2014) further argues that information communication in financial markets can alternatively explain some intriguing empirical findings which are usually attributed to overconfidence. In short, the information communication explanation reproduces the empirical observations and is complementary to the overconfidence explanation.

6. Conclusion

The Poincaré Conjecture was one of the most important and difficult open problem in mathematics. After nearly a century of effort by mathematicians, a series of papers made available in 2002 and 2003 by Grigori Perelman sketched a solution. Three groups of mathematicians have produced works filling in the details of Perelman's proof since then. In 2006, Perelman was awarded the Fields Medal which is widely considered to be the top honor a mathematician can receive. However, the reclusive Perelman declined to accept the award for some unknown reason. According to a close friend, when a leading Russian mathematical institute failed to reelect him as a member, Perelman was left feeling an absolutely ungifted and untalented person and he had a crisis of confidence and cut himself off. Some even said that he had abandoned mathematics entirely (Lobastova & Hirst, 2006).

This somewhat dramatic story, combined with other new findings in psychological experiments, lead us to cast doubt on the widely accepted notion that people tend to exhibit overconfidence in ability and knowledge. This paper summarizes several key elements ignored in previous studies and stresses the potentially important underconfidence tendency in decision making. We establish that the observed high price volatility and trading volume, and agents' low expected profits can be contributed by agents' underconfidence rather than overconfidence. To gain more support, we plan to carefully design asset market experiments that allow subjects to share information and exchange idea before decision making and directly test the asset pricing implications of the possible coexistence of over- and underconfidence agents in competitive trading. Because both the overconfidence and the information communication explanations can account for puzzling individual trading behavior and market trading patterns in financial markets. More analytical and empirical works should be directed to examine them in the future research.

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References:

Admati Anat R. and Paul Peiderer (1986). "A monopolistic market for information", *Journal of Economic Theory*, Vol. 39, pp. 400-438.

Alicke Mark D., M. L. Klotz, David L. Breitenbecher, Tricia J. Yurak and Debbie S. Vredenburg (1988). "Personal contact, individuation, and the better-than-average effect", *Journal of Personality and Social Psychology*, Vol. 68, pp. 804-825.

Antweiler Werner and Murray Z. Frank (2004). "Is all that talk just noise? The information content of internet stock message boards", *Journal of Finance*, Vol. 59, pp. 1259-1294.

Barber Brad M. and Terrance Odean (2002). "Online investors: Do the slow die first?", *Review of Financial Studies*, Vol. 15, pp. 455-487.

Barberis Nicholas and Ming Huang (2001). "Mental accounting, loss aversion, and individual stock returns", *Journal of Finance*, Vol. 56, pp. 1247-1291.

Barberis Nicholas, Ming Huang and Tano Santos (2001). "Prospect theory and asset prices", *Quarterly Journal of Economics*, Vol. 116, pp. 1-53.

Beetsma Roel M. W. J. and Peter C. Schotman (2001). "Measuring risk attitudes in a natural experiment: Data from the television game show lingo", *Economic Journal*, Vol. 111, pp. 821-848.

Bell David E. (1982). "Regret in decision making under uncertainty", *Operations Research*, Vol. 30, pp. 961-981.

Benos Alexandros (1998). "Aggressiveness and survival of overconfident traders", *Journal of Financial Markets*, Vol. 1, pp. 353-383.

Daniel Kent D., David Hirshleifer and Avanidhar Subrahmanyam (1998). "Investor psychology and security market under- and overreactions", *Journal of Finance*, Vol. 53, pp. 1839-1885.

De Bondt, Werner F. M. and Richard H. Thaler (1995). "Financial decision-making in markets and firms: A behavioral perspective", in: Robert A. Jarrow, Vojislav Maksimovic, & William T. Ziemba (Eds.), *Handbook in Operations Research and Management Science*, Vol. 9, pp. 385-410, North-Holland, Amsterdam.

Erev Ido, Thomas S. Wallsten and David V. Budescu (1994). "Simultaneous over- and underconfidence: The role of error in judgement processes", *Psychological Review*, Vol. 101, pp. 519-527.

García Diego, Francesco Sangiorgi and Branko Urosevic (2007). "Overconfidence and market efficiency with heterogeneous agents", *Economic Theory*, Vol. 30, pp. 313-336.

Gervais Simon and Terrance Odean (2001). "Learning to be overconfident", *Review of Financial Studies*, Vol. 14, pp. 1-27.

Gri¢n Dale and Amos Tversky (1992). "The weighing of evidence and the determinants of confidence", *Cognitive Psychology*, Vol. 24, pp. 411-435.

Grossman Sanford J. and Joseph E. Stiglitz (1980). "On the impossibility of informationally efficient markets", *American Economic Review*, Vol. 70, pp. 393-408.

Hastorf Albert H., David J. Schneider and Judith Polefka (1970). *Person Perception*, Addison-Wesley: Reading, MA.

Hellwig Martin F. (1980). "On the aggregation of information in competitive markets", *Journal of Economic Theory*, Vol. 22, pp. 477-498.

Hoelzl Erik and Aldo Rustichini (2005). "Overconfident: Do you put your money on it?", *Economic Journal*, Vol. 115, pp. 305-318.

Hong Harrison, Jeffrey D. Kubik and Jeremy C. Stein (2005). "Thy neighbor's portfolio: Word-of-mouth effects in the holdings and trades of money managers", *Journal of Finance*, Vol. 60, pp. 2801-2824.

Hoorens V. and B. Buunk (1993). "Social comparison of health risks: Locus of control, the person-positivity bias, and unrealistic optimism", *Journal of Applied Social Psychology*, Vol. 23, pp. 291-302.

Ivkovic Zoran and Scott Weisbenner (2007). "Information diffusion effects in individual investors' common stock purchases: Covet thy neighbors' investment choices", *Review of Financial Studies*, Vol. 20, pp. 1327-1357.

Juslin Peter, Anders Winman and Henrik Olsson (2000). "Naive empiricism and dogmatism in confidence research: A critical examination of the hard-easy effect", *Psychological Review*, Vol. 107, pp. 384-396.

Kahneman Daniel and Amos Tversky (1979). "Prospect theory: An analysis of decision under risk", *Econometrica*, Vol. 47, pp. 263-291.

Kirchler Erich and Boris Maciejovsky (2002). "Simultaneous over- and underconfidence: Evidence from experimental asset markets", *Journal of Risk and Uncertainty*, Vol. 25, pp. 65-85.

Klayman Joshua, Jack B. Soll, Claudia González-Vallejo and Sema Barlas (1999). "Overconfidence: It depends on how, what, and whom you ask", *Organizational Behavior and Human Decision Processes*, Vol. 79, pp. 216-247.

Kyle Albert S. (1985). "Continuous auctions and insider trading", *Econometrica*, Vol. 53, pp. 1315-1335.

Kyle Albert S. (1989). "Informed speculation with imperfect competition", *Review of Economic Studies*, Vol. 56, pp. 317-356.

Kyle Albert S. and F. Albert Wang (1997). "Speculation duopoly with agreement to disagree: Can overconfidence survive the market test?", *Journal of Finance*, Vol. 52, pp. 2073-2090.

LeRoy Stephen F. and Richard D. Porter (1981). "The present-value relation: Tests based on implied variance bounds", *Econometrica*, Vol. 49, pp. 555-574.

Lichtenstein Sarah, Baruch Fischoff and Lawrence Phillips (1982). "Calibration of probabilities: The state of the art to 1980", in: Daniel Kahneman, Paul Slovic, & Amos Tversky (Eds.), *Judgment under Uncertainty: Heuristics and Biases*, chap. 22, pp. 306-334, Cambridge University Press, Cambridge.

Lobastova Nadejda and Michael Hirst (2006). "World's top maths genius jobless and living with mother", *The Daily Telegraph*.

Milgrom Paul and Nancy Stokey (1982). "Information, trade and common knowledge", *Econometrica*, Vol. 26, pp. 17-27.

Moore Don A. and Daylian M. Cain (2007). "Overconfidence and underconfidence: When and why people underestimate (and overestimate) the competition", *Organizational Behavior and Human Decision Processes*, Vol. 103, pp. 197-213.

Odean Terrance (1998). "Volume, volatility, price and prot when all traders are above average", *Journal of Finance,* Vol. 53, pp. 1887-1934.

Perloff Linda S. and Barbara K. Fetzer (1986). "Self-other judgments and perceived vulnerability to victimization", *Journal of Personality and Social Psychology*, Vol. 50, pp. 502-510.

Shiller Robert J. (1981). "Do stock prices move too much to be justified by subsequent changes in dividends?", *American Economic Review*, Vol. 71, pp. 421-498.

Subrahmanyam Avanidhar (1991). "Risk aversion, market liquidity, and price efficiency", *Review of Financial Studies*, Vol. 4, pp. 417-441.

Veldkamp Laura L. (2006). "Media frenzies in markets for financial information", *American Economic Review*, Vol. 96, pp. 577-601.

Wang F. Albert (1998). "Strategic trading, asymmetric information and heterogeneous prior beliefs", *Journal of Financial Markets*, Vol. 1, pp. 321-352.

Wysocki Peter D. (1998). "Cheap talk on the web: The determinants of postings on stock message boards", University of Michigan Business School Working Paper No. 98025.

Xia Chun (2012). "Do insiders always overreact?", Mimeo The University of Hong Kong.

Xia Chun (2015). "Communication and confidence in financial networks", *Journal of Business and Economics*.

Yezer Anthony M., Robert S. Goldfarb and Paul J. Poppen (1996). "Does studying economics discourage cooperation: Watch what we do, no what we say or how we play", *Journal of Economic Perspectives*, Vol. 10, pp. 177-186.

Yuan Kathy (2005). "Asymmetric price movements and borrowing constraints: A rational expectations equilibrium model of crises, contagion, and confusion", *Journal of Finance*, Vol. 60, pp. 379-411.

Appendix Proofs of Main Text Results

Proof of Theorem 1. We conjecture that $\tilde{p}_t = \alpha_t \tilde{V}_t - \beta_t \tilde{Z}_t$ Consider agent i of type h, we can show that, when $\delta_{ht} = 1$, which happens with probability $\mu_{h,t-1}$,

$$x_{ht}(\delta_{ht}=1) = \frac{E\left(\tilde{V}_t \mid \theta_{ht}, p_t, s_{h,t-1}\right) - p_t}{\rho var\left(\tilde{V}_t \mid \theta_{ht}, p_t, s_{h,t-1}, s_{l,t-1}\right)}$$
$$= \frac{\beta_t^2 \Sigma\Omega\theta_{ht} + \left[\left(\alpha_t - \alpha_t^2\right)\Sigma\Phi_h^{-1} - \left(\Sigma + \Phi_h^{-1}\right)\beta_t^2\Omega\right]p_t}{\rho\beta_t^2\Sigma\Omega\Phi_h^{-1}}$$

by the Projection Theorem. When $\delta_{ht} = 0$, which happens with probability $1-\mu_{h,t-1}$,

$$x_{ht}(\delta_{ht}=0) = \frac{\beta_t^2 \Sigma\Omega\theta_{ht} + \left[\left(\alpha_t - \alpha_t^2\right)\Sigma\Phi_l^{-1} - \left(\Sigma + \Phi_l^{-1}\right)\beta_t^2\Omega\right]p_t}{\rho\beta_t^2\Sigma\Omega\Phi_l^{-1}}$$

Applying the law of large number to $x_{ht} = \mu_{h,t-1} x_{ht}(\delta_{ht}=1) + (1-\mu_{h,t-1}) x_{ht}(\delta_{ht}=0)$ yields

$$\begin{aligned} X_{ht} = {} & \mu_{h,t-1}\frac{\beta_t^2 \Sigma\Omega V_t + \left[\left(\alpha_t - \alpha_t^2\right)\Sigma\Phi_h^{-1} - \left(\Sigma + \Phi_h^{-1}\right)\beta_t^2\Omega\right]p_t}{\rho\beta_t^2\Sigma\Omega\Phi_h^{-1}} \\ & + (1-\mu_{h,t-1})\frac{\beta_t^2 \Sigma\Omega V_t + \left[\left(\alpha_t - \alpha_t^2\right)\Sigma\Phi_l^{-1} - \left(\Sigma + \Phi_l^{-1}\right)\beta_t^2\Omega\right]p_t}{\rho\beta_t^2\Sigma\Omega\Phi_l^{-1}} \end{aligned} \qquad \text{(A1)}$$

X_{lt} is obtained by replacing $\mu_{h,t-1}$ in (A1) with $\mu_{l,t-1}$.

From the market clearing condition $m_h X_{ht} + m_l X_{lt} = Z_t$ and $\Phi_h^{-1} = \psi\Phi_l^{-1}$ we obtain

$$p_t = \frac{\beta_t^2\left[\psi + (1-\psi)A_t\right]\Sigma\Omega V_t - \rho\beta_t^2\Sigma\Omega\Phi_h^{-1}Z_t}{A_t\left[(\alpha_t^2-\alpha_t)\Sigma\Phi_h^{-1} + (\Sigma+\Phi_h^{-1})\beta_t^2\Omega\right] + \psi(1-A_t)\left[(\alpha_t^2-\alpha_t)\Sigma\Phi_l^{-1} + (\Sigma+\Phi_l^{-1})\beta_t^2\Omega\right]}$$

Where A_t is defined in (7).

Equilibrium condition dictates that

$$\frac{\alpha_t}{\beta_t} = \lambda_t = \frac{\Phi_h}{\rho}\left[\psi + (1-\psi)A_t\right]$$

and

$$\beta_t = \frac{1}{\lambda_t + (\rho\Sigma)^{-1} + (\beta_t\lambda_t^2 - \lambda_t)(\rho\beta_t\Omega)^{-1}} \qquad \text{(A2)}$$

Solving (A2) yields

$$\beta_t = \frac{\Sigma(\lambda_t + \rho\Omega)}{\lambda_t\Sigma(\lambda_t+\rho\Omega) + \Omega}$$

When $\Sigma = 1$, (5) and (6) follow in the main text.

Proof of Propsition 1. Suppose $(\tilde{a}_i, \tilde{a}_j) = (H, L)$ for $i \in I$ and $j \in J$, we show agents i's updated posterior beliefs $\overline{\varphi}_{it}$ will converge to 1 almost surely as $t \to \infty$. An agent is expected to observe signal of high precision a fraction a of the time. When $t \to \infty$, for an unbiased agent i, we have

$$\phi_{it} \to \frac{H^{Ht}(1-H)^{t-Ht}\phi_0}{H^{Ht}(1-H)^{t-Ht}\phi_0 + L^{Ht}(1-L)^{t-Ht}(1-\phi_0)} = \left\{1+\left(\frac{1-\phi_0}{\phi_0}\right)\left[\left(\frac{L}{H}\right)^H\left(\frac{1-L}{1-H}\right)^{1-H}\right]^t\right\}^{-1}$$

To show $\phi_{it}\to 1$ as $t\to\infty$. We only have to show $\left(\frac{L}{H}\right)^H\left(\frac{1-L}{1-H}\right)^{1-H} < 1$. Define $f(x) = H\ln x + (1-H)\ln(1-x)$, it is easily shown that $f(x)$ is strictly increasing if $0 < x \le H$. Note that

$$\begin{aligned}\ln\left(\frac{L}{H}\right)^H\left(\frac{1-L}{1-H}\right)^{1-H} &= H(\ln L - \ln H) + (1-H)\left[\ln(1-L) - \ln(1-H)\right] \\ &= f(L) - f(H) < 0\end{aligned}$$

This yields the desired result for a rational agent i.

When $t\to\infty$, for a biased agent i, we have

$$\begin{aligned}\overline{\phi}_{it} &\to \frac{\gamma^{Ht-Lt}H^{Ht}(1-H)^{t-Ht}\phi_0}{\gamma^{Ht-Lt}H^{Ht}(1-H)^{t-Ht}\phi_0 + L^{Ht}(1-L)^{t-Ht}(1-\phi_0)} \\ &= \left\{1+\left(\frac{1-\phi_0}{\phi_0}\right)\left[\left(\frac{1}{\gamma}\right)^{H-L}\left(\frac{L}{H}\right)^H\left(\frac{1-L}{1-H}\right)^{1-H}\right]^t\right\}^{-1}\end{aligned}$$

Obviously $\overline{\phi}_{it}$ will converge to 1 as $t\to\infty$.

Since it is common knowledge that agents of two different types compete against each other and the convergence results of ϕ_{it} $\overline{\phi}_{it}$ are shared with agent j, biased or not. Agent $j\in J$ will eventually know her low-ability.

Proof of Lemma 1. Given $(\tilde{s}_{i,t} = s_i, \tilde{s}_j = s_j)$ and the equilibrium conditions specified in Theorem 1, we can calculate that

$$var(\tilde{p}_{t+1} \mid s_i, s_j) = \alpha_{t+1}^2 + \beta_{t+1}^2\Omega = (\lambda_{t+1}^2 + \Omega)\left[\frac{\lambda_{t+1} + \rho\Omega}{\lambda_{t+1}(\lambda_{t+1} + \rho\Omega) + \Omega}\right]^2 \tag{A3}$$

Where $\lambda_{t+1} = \left[\psi + (1-\psi)A_{t+1}\right]\Phi_h / \rho$. Straightforward differentiation of the expression for price variance with respect to A_{t+1} yields

$$\frac{\left[A_{t+1} + \frac{\psi\Phi_h + \rho^2\Omega}{(1-\psi)\Phi_h}\right]\left[A_{t+1} - \frac{\rho^2\Omega - 2\psi\Phi_h - \rho\sqrt{5\rho^2\Omega^2 - 4\Omega}}{2(1-\psi)\Phi_h}\right]\left[A_{t+1} - \frac{\rho^2\Omega - 2\psi\Phi_h + \rho\sqrt{5\rho^2\Omega^2 - 4\Omega}}{2(1-\psi)\Phi_h}\right]}{\left[\left[\psi\Phi_h + (1-\psi)A_{t+1}\Phi_h\right]^2 + \rho^2\left[1 + \psi\Phi_h + (1-\psi)A_{t+1}\Phi_h\right]\right]^3}$$

It is easily seen that the monotonicity of price variance depends on the last two terms in the numerator.

Proof of Lemma 2. Assume $\Sigma = 1$, in the proof of Theorem 1, for an agent $i\in I$ we have

$$\begin{aligned}\tilde{x}_{i,t+1} &= \mu_{h,t}\frac{\beta_{t+1}^2\Omega\tilde{\theta}_{i,t+1} + \left[(\alpha_{t+1} - \alpha_{t+1}^2)\Phi_h^{-1} - (1+\Phi_h^{-1})\beta_{t+1}^2\Omega\right]\tilde{p}_{t+1}}{\rho\beta_{t+1}^2\Omega\Phi_h^{-1}} \\ &\quad + (1-\mu_{i,t})\frac{\beta_{t+1}^2\Omega\tilde{\theta}_{i,t+1} + \left[(\alpha_{t+1} - \alpha_{t+1}^2)\Phi_l^{-1} - (1+\Phi_l^{-1})\beta_{t+1}^2\Omega\right]\tilde{p}_{t+1}}{\rho\beta_{t+1}^2\Omega\Phi_l^{-1}} \\ &= B_{h,t+1}\tilde{\theta}_{i,t+1} - C_{h,t+1}\tilde{p}_{t+1}\end{aligned} \tag{A4}$$

we therefore obtain (10), (11) and (12) in the main text. Given $\tilde{s}_{it} = s_i, \tilde{s}_{jt} = s_j$, we calculate that

$$\begin{aligned} \mathrm{var}\left(\tilde{\theta}_{i,t+1} \middle| s_i, s_j\right) &= E\left[\tilde{\theta}^2_{i,t+1} \middle| s_i, s_j\right] = E\left[E\left(\tilde{\theta}^2_{i,t+1} \middle| \delta_{i,t+1}, s_i, s_j\right) \middle| s_i, s_j\right] \\ &= 1 + \Phi_h^{-1}\left(\mu_{ht} + \frac{1-\mu_{ht}}{\psi}\right) \end{aligned} \tag{A5}$$

Since all random variables are mutually independent across period, given $\tilde{s}_{it} = s_i, \tilde{s}_{jt} = s_j$ and (A4) we have

$$var\left(\tilde{x}_{i,t+1} \middle| s_i, s_j\right) = B^2_{h,t+1} var\left(\tilde{\theta}_{i,t+1} \middle| s_i, s_j\right) + C^2_{h.t+1} var\left(\tilde{p}_{t+1} \middle| s_i, s_j\right) - 2B_{h,t+1}C_{h,t+1}\alpha_{t+1} \tag{A6}$$

Substituting in (A3) and (11), (12) and (A5), we calculate

$$\begin{aligned} var(\tilde{x}_{i,t+1} \mid s_i, s_j) = &\frac{\Phi_h \Lambda^2(\mu_{ht})\left[1-\mu_{ht} + \psi(\mu_{ht} + \Phi_h)\right]}{\rho^2 \psi} \\ &+ \frac{\left[\rho^2\Omega + \Phi_h^2 \Lambda^2(A_{t+1})\right]\left[\rho^2\Omega + \Phi_h \Lambda(\mu_{ht})(\rho^2\Omega + \Phi_h\Lambda(A_{t+1}))\right]^2}{\rho^2\left[\Phi_h^2\Lambda^2(A_{t+1}) + \rho^2\Omega(1+\Phi_h\Lambda(A_{t+1}))\right]^2} \\ &- \frac{2\Phi_h^2 \Lambda(\mu_{ht})\Lambda(A_{t+1})\left[\rho^2\Omega + \Phi_h\Lambda(\mu_{ht})(\rho^2\Omega + \Phi_h\Lambda(A_{t+1}))\right]}{\rho^2\left[\Phi_h^2\Lambda^2(A_{t+1}) + \rho^2\Omega(1+\Phi_h\Lambda(A_{t+1}))\right]} \end{aligned} \tag{A7}$$

Where $\Lambda(.)$ is defined in (16). (A7) can be used in (14).

Proof of Lemma 3. Given $(\tilde{s}_{it} = s_i, \tilde{s}_{jt} = s_j)$, because of (10), the expected profit for an agent i in period t+1 can be calculated as

$$E(\tilde{\pi}_{i,t+1} \mid s_i, s_j) = E\left[\tilde{x}_{i,t+1}\left(\tilde{V}_{t+1} - \tilde{p}_{t+1}\right) \middle| s_i, s_j\right] = B_{i,t+1} - \alpha_{t+1}(B_{i,t+1} + C_{i,t+1} var(\tilde{p}_{t+1} \mid s_i, s_j)$$

which equals (15) in the main text by substituting (11) and (12).