A Model of Optimal Dividend Policy to Maximize Shareholder Wealth:

When Taxes are Considered

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Abstract: The article analyzes theoretically how a firm maximizes the value of shareholder’s wealth with its dividend policy. Corporate dividend policy is one of the major puzzles with modern finance. The overall question is whether company should pay out dividend at all. However, the large majority of listed companies pay dividend and they also carry sophisticated dividend policies. In this paper we outline when it is optimal for a company to pay out dividend and when it should reinvest the profit from operations. The model takes taxes into consideration estimating the value of a company, i.e., the present value after deduction for taxes, is used as objective function. Four different taxes are considered. The analysis shows the terms on which it is profitable to receive dividend payout or to reinvest at an arbitrary time. Under the assumption of a unique maximum net present value, the terms at the time for the maximum net present value are also presented.

Key words: dividend; financing; management; reinvestment; taxes

JEL code: C61, E62, G31, G35

1. Introduction

One of the main puzzling questions of modern finance is how a company’s dividend policy affects shareholders’ wealth (for an interesting discussion, see Black, 1976). In this study we construct a model that assumes the main assumptions of Miller-Modigliani’s (1961) classical articles of dividend policy, with the addition of taxes. The aim is to analyze how a firm maximizes the value of shareholder’s wealth with its dividend policy versus reinvestment of the profit from operations, when taxes are considered. This is equivalent to maximizing the market value of the firm. Consequently, the net present value, i.e., the present value after deduction for taxes, is used as objective function in the study.

The basic idea of dividend was, initially, to make equity seems like a loan, where the dividend would give the equity investor a tangible return, thereby enabling value calculation of shares (Frankfurter & Wood, 1997). Dividends would then do share comparable to each other, and stock dividends could consequently be seen as an indicator of company value, in that it shows the relationship cash-flow and price. The dividend issue was early treated by many economic scientists, e.g., Lintner (1956), Gordon (1959, 1962, 1994), Miller and Modigliani (1961), Farrar and Selwyn (1967), Fama (1974), Brennan (1970), Miller and Scholes (1978) and Elston (1996).

Since the seminal theoretical paper by Miller-Modigliani (1961), in which the authors showed the irrelevance...
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of dividend in a frictionless world, the obvious basis when it comes to dividend is why companies should pay it out at all. However, the vast majority of companies pay dividends and they also apply sophisticated dividend policy. Why the real world is not like the academic model is not explained by the Miller-Modigliani (MM) irrelevant arguments. However, there are several explanations complementing MM irrelevant argument given by the academic literature: the signaling theory (Bhattacharya, 1979; Miller & Rock, 1985) claims that dividend is a mean to communicate company information to its shareholders about the company’s future; agency theory (Easterbrook, 1984; Jensen, 1986) claims that dividend is rooted on the conflict interests within a company of different stakeholder groups—stockholders, management, and bondholders; clientele theory (Shleifer & Vishny, 1986; Allen, Bernardo & Welch, 2000) claims that dividend is paid out to satisfy the payout demands from a heterogeneous dividend clienteles; and catering theory (Baker & Wurgler, 2004a; 2004b) claims that stock market mispricing might influence individual firms’ investment decisions. Here managers cater to investors by paying dividends when investors put a relatively high stock price of dividend payers and tend to omit dividends when investors prefer non-payers.

Surveys of dividend policy give a good understanding on the minds of the people responsible for companies’ dividend policy. In 1956 a survey study by Professor John Lintner laid what is considered to be the foundation of modern understanding of dividend policy. The study was conducted by in detail interview managers in charge of dividend policies of 28 companies. The main findings where those managers used long-run target payout ratios to determine dividends, and they change the company’s dividends only when they believe the companies permanent earnings have changed. Since then, several researchers have used the survey method to learn more about the factor that influence the construction of dividend policy (De Jong, Van Dijk & Veld, 2003; Baker, Powell & Veit, 2002). In a study by Brav et al. (2003) the authors compared the treatment of dividend and repurchase of stocks. The study was performed by surveying financial executives with an instrument that focuses on both dividends and repurchases. The study was further developed by conducting one-on-one interviews with a number of CFOs. The findings showed that dividend policy is still very conservative, while repurchase of stocks is more flexible. Also it was showed that companies perceived big penalty for cut, while just small reward for increase.

The structure of this article is as following: in Section 2 we present the most important and relevant research on dividend policy; in Section 3 we present the model; in Section 4 we give an analysis of the model; in Section 5 we present an example of a practical case based on the model; and in Section 6 we draw the conclusions of the study.

2. Previous Research

The tax burden on dividends is partly due to the tax on company profits, and the tax treatment of the dividend recipient. To understand the complexity of dividend policy, the taxes on dividend should be set in comparative with the effective capital gain tax. Typically, when the tax on dividend exceeds the tax on capital gain, companies should prefer to retain earnings or to repurchase shares (Auerbach, 1979; Bradford, 1981; Auerbach & Hassett, 2003).

In principle, all research shows that taxes affect dividend policy. However, how much impact taxes have on dividend policy the scholars have different views. Brav et al. (2003); and Julio and Ikenberry (2004) argue that taxes do not have a dominant effect for the dividend decision for most companies. On the other hand, Alzahrani and Lasfer (2012) argue that taxes can have a significant impact on corporate dividend decisions. Alzahrani och
Lasfer showed that companies in countries with double taxation tax system pay significantly lower dividends than companies in partial-imputation system countries. Since taxes are different depending on the type of shareholders, corporate shareholders and institutional ownership prefer dividend before capital gain, while smaller shareholders prefer capital gain. Therefore, the large shareholders may use their power to directly influence dividend payout for tax reasons (Shleifer & Vishny, 1986).

Historically, capital gains have been taxed at a lower rate than dividends, which meant that investors often prefer capital gains before dividends. Furthermore capital gains are not taxed until shares are actually sold; this means that investors can therefore control when capital gains are realized and the tax paid. Dividends, however, investors cannot control because it is the board of the related company which decides on the distribution. Many scholars have evaluated the tax affect on dividend policy due to the tax change that the U.S. Congress passed in 2003. According to the adopted legislation, dividends are taxed at the same rate as long-term capital gains. Studies, such as Chetty and Saez (2005); Brown, Liang, and Weisbenner (2007), and Blouin, Raedy, and Shackelford (2011), have found significant increases in dividend payouts and initiations following the adoption of the legislation. Chetty and Saez (2005) found that the aggregate regular dividends increased by 20% within 1.5 years of the introduction of legislation, prompting the authors to conclude that the tax cuts caused this increase.

In corporate finance, it is generally considered that the finance manager has two operational decision areas to consider: the investment (or capital budgeting) and the financing decisions. The capital budgeting decisions concerns the process of choosing what long-term assets the company should acquire, while the financing decision concerns how these assets should be financed. A third decision area arises, however, when the company begins to generate profits. How much of the earnings, the company should distribute as dividends to shareholders, and how much the company should be reinvested in the business? The ultimate goal of managers should be to maximization of wealth of shareholders, so every action taken by the managers should consequently be to maximize the wealth of the shareholders. The managers do not just have to consider how much of the company’s earnings that is needed for investments, but they also have to consider the effect of their decision on shareholder wealth (Bishop et al., 2000).

Shareholder wealth is defined as the present value of expected future returns to shareholders. These returns are either regular dividend and/or represent revenue from the sale of shares. Shareholder wealth is measured as the market value of the company’s shares. The goal of maximizing shareholder wealth is a long term goal, where shareholder wealth is a function of all future monetary returns to the shareholders. In order to make decisions that maximize shareholder wealth, management must take into account the long-term effects on the company and not just focus on short-term effects (Damodaran, 2006).

3. Model

The starting point in building our tax-including model is the assumptions of Miller and Modligiani (1961), in which the authors assumed an ideal economy characterized by rational behavior and perfect certainty (deterministic approach) and to some extent under risk (stochastic perspective). Their proposition also assumed: no transaction costs exist, and individuals and corporations borrow at the same rates. A limitation in the analysis is that the sum of funds used for dividends and investments is equal to the profits from operations. Thus, neither external financing nor repurchasing of shares is regarded.

In their 1986 article, Shleifer and Vishny discuss the dividend puzzle example using four taxes. In this
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We consider the same four taxes outlined by Sheifer and Vishny: (1) corporate tax when dividend payout is received; (2) shareholder’s personal income tax on dividends; (3) corporate tax when reinvestment is made and (4) shareholder’s capital gains tax.

3.1 Concepts and Notations

In this study, the net present value, i.e., the present value after deduction for taxes, is used as objective function. In accordance with this, total net present value over time \( t \), \( PV(t) \), is the sum of the net present values of dividends \( PV_d(t) \) and gains of reinvestments \( PV_r(t) \). Thus, \( PV(t) = PV_d(t) + PV_r(t) \).

In the study, the following concepts and denotations are used.

“Gross” means “with no deduction for taxes”; “net” means “after deduction for taxes”.

\( t \) denotes an arbitrary terminal time point,

\( s \) time point ending period \( s \) in the discrete model; time in the continuous model where \( 0 \leq s \leq t \),

\( G(s) \) the shareholder’s gross profit from operations in period \( s \); it is assumed to be non-negative,

\( q(s) \) dividend ratio, i.e., relative dividend fraction of the firm’s gross profit at time point \( s \) where \( 0 \leq q(s) \leq 1 \); \( q \) is decision function,

\( 1 - q(s) \) retention ratio, i.e., relative reinvestment fraction of the firm’s gross profit at time point \( s \) where \( 0 \leq (1 - q(s)) \leq 1 \),

\( r(s) \) market-required rate of return in period \( s \) (interest rate); same notation in both the discrete and continuous model, but for consistency with slightly different values,

\( \delta(s) \) discount factor in period \( s \) in the discrete model, i.e., \( \delta(s) = (1 + r(s))^{-s} \),

\( g(s) \) rate of return on reinvestment in period \( s \) (growth rate); same notation in the discrete and continuous model, but for consistency with slightly different values,

\( \gamma(s) \) growth factor in period \( s \), the discrete model, \( \gamma(s) = 1 + g(s) \).

Different tax rates, \( \tau_i(s) \), all \( 0 \leq \tau_i(s) < 1 \):

\( \tau_1(s) \) denotes corporate tax rate on gross profit in period \( s \), when dividend is received,

\( \tau_2(s) \) personal income tax rate on gross dividend payout in period \( s \),

\( \tau_3(s) \) corporate tax rate on gross profit in period \( s \), when reinvestment is made,

\( \tau_4(s) \) capital gains tax rate, i.e., personal income tax rate in period \( s \) on the capitalized reinvestments after a deduction for corporate tax; \( \tau_4(s) = \tau_4(t) \) for all \( s \leq t \) where \( t \) is sale date,

\( PV(t) \) total net present value over \( t \) periods,

\( PV_d(t) \) net present value of dividends over \( t \) periods,

\( PV_r(t) \) net present value of capital gains of reinvestments over \( t \) periods.

3.2 Net Present Value of Dividends

The net dividend in period \( s \) equals \( G(s)q(s)(1 - \tau_1(s))(1 - \tau_2(s)) \) for \( s = 0, 1, \ldots \).

By discounting and summing over \( s \) we get

\[
PV_d(t) = G(0)q(0)(1 - \tau_1(0))(1 - \tau_2(0)) + G(1)q(1)(1 - \tau_1(1))(1 - \tau_2(1))\delta(1) + \\
+ G(2)q(2)(1 - \tau_1(2))(1 - \tau_2(2))\delta(1)\delta(2) + \ldots + G(t)q(t)(1 - \tau_1(t))(1 - \tau_2(t))\delta(1) \cdots \delta(t) = \\
\sum_{s=0}^{t} G(s)q(s)(1 - \tau_1(s))(1 - \tau_2(s)) \prod_{u=0}^{s} \delta(u)
\]

where \( \delta(0) = 1 \).
For convenience, we now make the discrete expression (1) continuous in time; thus, a change occurs from discontinuous to continuous time. Then $\delta(u)$ is replaced by the continuous version $\exp(-r(u))$. We get

$$PV_D(t) = \int_0^t G(s)q(s)(1 - \tau_1(s))(1 - \tau_2(s)) \cdot \exp\{-\int_0^s r(u)du\} ds$$  \hspace{1cm} (2)

### 3.3 Net Present Value of Reinvestments

The net reinvestment gain at time point $s$ equals $G(s)(1-q(s))(1-\tau_3(s))$. This is increased by future growth up to time $t$, where it is taxed by the ratio $\tau_4(s)$ and discounted back to time $s = 0$. With the convention $\tau_4(s) = \tau_4(t)$ for $0 \leq s < t$ we get

$$PV_R(t) = G(0)(1 - q(0))(1 - \tau_3(0))(1 - \tau_4(0)) \cdot \prod_{u=1}^t \gamma(u) \cdot \prod_{u=1}^t \delta(u) +$$

$$+ G(1)(1 - q(1))(1 - \tau_3(1))(1 - \tau_4(1)) \cdot \prod_{u=2}^t \gamma(u) \cdot \prod_{u=1}^t \delta(u) +$$

$$+ G(t)(1 - q(t))(1 - \tau_3(t))(1 - \tau_4(t)) \cdot \prod_{u=1}^t \delta(u) =$$

$$= \prod_{u=1}^t \delta(u) \cdot \sum_{s=0}^t G(s)(1 - q(s))(1 - \tau_3(s))(1 - \tau_4(s)) \cdot \prod_{u=s+1}^t \gamma(u)$$  \hspace{1cm} (3)

where $\prod_{u=s+1}^t \gamma(u)$ is defined as 1.

As above, a change is made from discrete to continuous time, and simultaneously $\delta(u)$ and $\gamma(u)$ are replaced by their continuous analogues $r$ and $g$. We obtain

$$PV_R(t) = \exp\{-\int_0^t r(u)du\} \cdot \int_0^t G(s)(1 - q(s))(1 - \tau_3(s))(1 - \tau_4(s)) \cdot \exp\{\int_s^t g(u)du\} ds$$  \hspace{1cm} (4)

### 3.4 Net Present Value of Both Dividends and Reinvestments

Adding Formulas (2) and (4) results in

$$PV(t) = \int_0^t G(s)q(s)(1 - \tau_1(s))(1 - \tau_2(s)) \cdot \exp\{-\int_0^s r(u)du\} ds +$$

$$+ \exp\{-\int_0^t r(u)du\} \cdot \int_0^t G(s)(1 - q(s))(1 - \tau_3(s))(1 - \tau_4(s)) \cdot \exp\{\int_s^t g(u)du\} ds$$  \hspace{1cm} (5)

### 4. Analysis

Rearranging Equation (5) by assembling all terms that contain the decision function $q$ we get

$$PV(t) = \exp\{-\int_0^t r(u)du\} \cdot \int_0^t G(s)(1 - \tau_3(s))(1 - \tau_4(s)) \cdot \exp\{\int_s^t g(u)du\} ds +$$
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\[ + \int_0^t G(s)q(s)(1 - \tau_1(s))(1 - \tau_2(s)) \cdot \exp\{ -\int_0^s r(u)du \} ds - \]

\[ - \exp\{ -\int_0^t r(u)du \} \cdot \int_0^t G(s)(1 - \tau_3(s))(1 - \tau_4(s)) \cdot \exp\{ \int_s^t g(u)du \} ds = \]

\[ = \exp\{ -\int_0^t r(u)du \} \cdot \int_0^t G(s)(1 - \tau_3(s))(1 - \tau_4(s)) \cdot \exp\{ \int_s^t g(u)du \} ds + \]

\[ + \int_0^t G(s)q(s)h(s) \cdot \exp\{ -\int_0^s r(u)du \} ds \]

(6a)

(6b)

where \( h(s) = (1 - \tau_1(s))(1 - \tau_2(s)) - (1 - \tau_3(s))(1 - \tau_4(s)) \cdot \exp\{ \int_s^t g(u)du \} \).

Consider \( PV(t) \) as a functional of \( q() \), all other functions given. The term (6a) is independent of \( q \). Since \( G(s) \) and \( \exp\{ -\int_0^s r(u)du \} \) are non-negative, it is found that \( PV(t) \) is maximized by choosing \( q(s) = 1 \) whenever \( h(s) > 0 \), and \( q(s) = 0 \) whenever \( h(s) < 0 \). Since all \( \tau_i:s \) fulfill \( 0 \leq \tau_i < 1 \) this entails that optimum for an arbitrary time \( s \) is obtained by choosing

\[ q(s) = \begin{cases} 
1 & \text{if } \exp\{ \int_s^t (g(u) - r(u))du \} < \frac{(1 - \tau_1(s))(1 - \tau_2(s))}{(1 - \tau_3(s))(1 - \tau_4(s))} \\
0 & \text{if } \exp\{ \int_s^t (g(u) - r(u))du \} > \frac{(1 - \tau_1(s))(1 - \tau_2(s))}{(1 - \tau_3(s))(1 - \tau_4(s))} 
\end{cases} \]

(7)

We can write \( \exp\{ \int_s^t (g(u) - r(u))du \} = \frac{1 + g_{T}(s,t)}{1 + r_{T}(s,t)} \), where \( g_{T}(s,t) \) is the total rate of return on reinvestment during the period \( s \) to \( t \) and \( r_{T}(s,t) \) is the corresponding total market-required rate of return; taxes not included. The time length \( t \) can be chosen arbitrarily but fixed.

Taking the derivative of \( PV(t) \) with respect to \( t \) (in points of differentiability) we get

\[ PV'(t) = \exp\{ -\int_0^t r(u)du \} \cdot G(t) \cdot [g(t)(1 - \tau_1(t))(1 - \tau_2(t)) + (1 - q(t))(1 - \tau_3(t))(1 - \tau_4(t))] + \]

\[ + (g(t) - r(t)) \cdot PV_{R}(t) \]

(8)

The first term is the present value of the sum of net dividend and net reinvestment at \( t \). \( PV_{R}(t) \) in the second term is the present value of the accumulated, capitalized net reinvestments up to \( t \) (see Equation (4)), \( g(t) \cdot PV_{R}(t) \) is the present value of the revenue of this reinvested capital and \( r(t) \cdot PV_{R}(t) \) is the present value of the cost to keep this capital.

At optimum \( q(t) \) is either 0 or 1 and the first term is then the present value of either the net dividend (\( q(t) = 1 \)) or the net investment (\( q(t) = 0 \)), in both cases after taxes.

We now extend our model to comprise the assumption that the rate of return on reinvestment in period \( s \) depends on the size of the reinvestment. To do this \( G(s) \) is divided into monetary units ordered from 0 and upwards. The rate of return at time \( t \) of the \( vth \) unit reinvested at time \( s \) is denoted \( g(s,t,v) \) and the dividend ratio of the \( vth \) unit at time \( s \) is denoted \( q(s,v) \).
Then Equation (6) is extended as follows

\[
PV(t) = \exp\{-\int_0^t r(u)du\} \cdot \left(1 - \frac{1}{(1 - \tau_1(s))(1 - \tau_2(s))(1 - \tau_3(s))(1 - \tau_4(s))}\right) \int_0^t \exp\{\int_s^t g(s, u, v)du\}dvds + \\
+ \int_0^t \int_0^G(s) q(s, v)h(s, v) \cdot \exp\{-\int_0^s r(u)du\}dvds
\]

where \( h(s, v) = (1 - \tau_1(s))(1 - \tau_2(s))(1 - \tau_3(s))(1 - \tau_4(s)) \cdot \exp\{\int_s^t (g(s, u, v) - r(u))du\} \).

In the same way as above it is seen that \( PV(t) \) is maximized by choosing

\[
q(s, v) = \begin{cases} 
1 & \text{if } \exp\{\int_s^t (g(s, u, v) - r(u))du\} < \frac{(1 - \tau_1(s))(1 - \tau_2(s))}{(1 - \tau_3(s))(1 - \tau_4(s))} \\
0 & \text{if } \exp\{\int_s^t (g(s, u, v) - r(u))du\} > \frac{(1 - \tau_1(s))(1 - \tau_2(s))}{(1 - \tau_3(s))(1 - \tau_4(s))}
\end{cases}
\]  

(9)

If \( g \) and \( r \) are continuous functions and \( g \) non-increasing the solution (9) gives an upper limit \( v^* = v^*(s, t) \) for optimal reinvestment.

So far all functions are considered as known. Let this assumption be relaxed by assuming the rates of growth and interest to be random variables and that we want to maximize the expected value of the present value \( PV(t) \). The same arguments as above lead to the same decision rules (7) and (9), but with the left hand side replaced by its expected value. Since by Jensen’s inequality (Feller, 1966) the expected value \( E(\exp\{U\}) \geq \exp\{E(U)\} \) for any random variable \( U \) it is seen that \( q(s) = 0 \) more often than expressed by inserting the expected rates in the rules (7) and (9). The expectation is then taken conditionally on the \( r \) and \( g \) processes up to time \( s \). Thus random rates of growth and interest should increase the retention rate compared to known rates if the expected rates are the same.

5. An Example of A Practical Case

Above we have shown theoretically the “equilibrium” terms on which it is profitable to change from receiving dividend to reinvestment or vice versa, when four different taxes are considered.

In the simple example below, we will calculate the average, annual rate of return on reinvestment—under the above-mentioned terms—during a ten-year period under the following hypothetical terms

- corporate tax rate on gross profit (\( \tau_1 \)) is 28%, when dividend is received,
- personal income tax rate on gross dividend payout (\( \tau_2 \)) is 30%,
- corporate tax rate on gross profit (\( \tau_3 \)) is 20%, when reinvestment is made,
- capital gains tax rate (\( \tau_4 \)) is 20%, and
- annual, market-required rate of return is 5% during the period,
- annual rate of return on reinvestment is \( g\% \) during the period.

According to the discrete version of the solution (7), we get the breakpoint between \( q = 0 \) and \( q = 1 \) when

\[
\frac{1.0g^{10}}{1.05^{10}} = \frac{(1 - 0.28) - (1 - 0.30)}{(1 - 0.20) - (1 - 0.20)}, \text{ which gives } g = 2.521\ldots
\]
In this case, the average, annual rate of return on reinvestment is 2.5 percent during the period, thus equivalent to a market-required annual rate of 5 percent.

6. Conclusions

Under the assumptions that the sum of funds used for dividends and reinvestments is equal to the profits from operations, we find from Equation (7), that the value of the shareholder’s wealth (i.e., equivalent to the value of the firm) is affected by dividend policy in a world both with and without taxes. Dividend/reinvestment policy is irrelevant in this respect only if the ratio \( \frac{1 + g_t(s, t)}{1 + r_t(s, t)} \) is equal to the ratio \( \frac{(1 - \tau_1(s))(1 - \tau_2(s))}{(1 - \tau_3(s))(1 - \tau_4(s))} \) of tax rates; \( g_t(s, t) \) is the total rate of return on reinvestment during the period \( s \) to \( t \) and \( r_t(s, t) \) is the corresponding total market-required rate of return.

We find from Equation (7) that it is advantageous for shareholders to receive dividend payout at time \( s \), when the ratio \( \frac{1 + g_t(s, t)}{1 + r_t(s, t)} \) is less than the ratio \( \frac{(1 - \tau_1(s))(1 - \tau_2(s))}{(1 - \tau_3(s))(1 - \tau_4(s))} \) of tax rates. Otherwise, if this relationship is larger, it is better to reinvest.

However, in the case of random rates of growth and interest, the retention rate should be increased compared to known rates of return, if the expected rates are the same.

The derivative of \( PV(t) \) (see Equation (8)) set equal to zero determines the relationship between the included components at the time of maximum net present value. Here we assume that the present value, as a function of time, has a unique maximum. Then, the time for maximum net present value occurs when the sum of the net dividend and the net reinvestment is equal to the difference between the cost to keep the accumulated net reinvested capital and the revenue of the same capital.

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