Journal of Business and Economics, ISSN 2155-7950, USA
April 2013, Volume 4, No. 4, pp. 302-319

http://www.academicstar.us

Health Status and Portfolio Choices for Elder People

Shishu Zhang
(H-E-B School of Business, University of the Incarnate Word, San Antonio, Texas 78209, USA)

Abstract: As baby boomers enter the age of retirement, the portfolio choice of the elder people began to draw more and more attention. Although the empirical evidence suggests that elder people tend to hold less risky assets, there is a lack of theoretical research to explain through which channel this happens. The paper sets up a theoretical model that includes the health condition into the utility function and the health expenditure in the constraint to analyze the elder people's portfolio choice in a rigorous way. The paper analytically solved the model and obtained a numerical solution to analyze the change of parameters of the model on the effect of risk aversion and expected health. In addition, the paper proves the robustness of the analytical solution by solving other two models with different form of utility functions. The numerical solution proves that the elders' maximum consumption/investment and their health status are positively related.
Key words: health status; portfolio choices; elder people; dynamic programming
JEL codes: G11, I10, G00

1. Introduction

The portfolio choices of elder people attract more and more attention because of the following reasons: (1) The elderly holds a disproportionately large amount of total wealth; (2) Projected future Social Security privatization involves an expansion of individually controlled and managed pension accounts; (3) More and more of the baby boom generation will be entering retirement over the coming decades. Therefore, it is now a particularly important and pressing time to understand how individuals allocate their financial portfolios as they age during retirement.

A large number of empirical literature (Kimball, 1990; Mayers and Smith, 1983) has examined the portfolio choices of the elder people and conclude that there are three factors contribute to create continued declining stock share during retirement: (1) Increasing probability of getting sick with age causes individuals to reduce their exposure to the stock market; (2) With a dissimulated financial wealth during retirement, the same amount of health expenditure becomes more risky, which will induce investors to invest more cautiously; (3) Individuals tend to disserve more slowly with health uncertainty than without, which reduces the importance of retirement income as an implicit risk-free asset in total financial wealth. Hence, the optimal portfolio allocation is to invest more in explicit risk-free assets, like bonds, and less on risky assets as age increases.

Although empirical evidences suggest that elder people hold fewer risky assets, we lack the knowledge of through what channel this effect happens. It is needed to formally set up a theoretical model to capture the factors which may affect the elder's portfolio choices. In my paper, I set up a dynamic life-time model which incorporates

Shishu Zhang, Assistant Professor of Economics, University of the Incarnate Word; research areas: health economics, educational economics, development economics. E-mail: zhangshishu123456@hotmail.com.

the health factor into the utility function and includes the health expenditure to the constraints so as to analyze the portfolio choices of the elder people in a more vigorous way.

2. Background

2.1 Portfolio Choices

Portfolio decisions play an important role in wealth accumulation, accounting for perhaps 90 percent of total returns (Ibbotson and Kaplan, 2000). All things equal, advancing age leaves less time remaining before death, or a shortening investment horizon. Empirically speaking, both traditional investment advice and observed portfolio shares suggest that risk taking declines with age (Ameriks and Zeldes, 2004; Guiso, Jappelli and Pistaferri, 2002). A rule of thumb is that the percentage of a portfolio that is held in the stock market should equal 100 minus the age of the investor, so that a 30-year-old would hold 70 percent of his/her financial wealth in stocks, while a 70-year-old would hold 30 percent in stocks (Malkiel, 1996).

Bodie and Crane (1997) find evidence of a strong negative relationship between the age of participants and the percentage of financial wealth in retirement accounts held in equities. Vanderhei, Galer, Quick, and Rea (1999) and Holden, Vanderhei (2001) find that the average share held in stocks declines from 76.8 percent in their twenties to 53.2 percent for participants in their sixties. Heaton and Lucas (2000) find that age profiles are generally decreasing: the ratio of stocks to financial assets is lower for households older than 65 than for younger households. Heaton and Lucas (2000) finds that the share of financial wealth in equities increases over the working life, and then declines after retirement, generating a "hump-shaped" pattern.

But there are few theoretic models to explain these empirical results. The first dynamic theory of portfolio choice by Samuelson (1969) states that the optimal portfolio share should be constant over the life cycle, independent of both age and wealth. The concept of "businessman's risk" (i.e., holding risky stocks are only advisable for young businessmen, not for widows) is explored and rejected as invalid. This conclusion is reached under the assumptions of independently and identically distributed returns and requires frictionless markets and the absence of labor income.

There have been numerous efforts to reconcile theory with empirical patterns since Samuelson published their results. Much of the research examining levels of consumption, saving, and wealth, as well as their responsiveness to policy, has been done using a life-cycle model with the simplifying assumption of perfect certainty. In recent paper (Hubbard et al., 1994), the author examine a life-cycle model of consumption, saving, and wealth accumulation by combining uncertainty about earnings, medical expenses, and length of life.

Recent researches by Jagannathan and Kocherlakota (1996), Elmendorf and Kimball (2000), Campbell and Viceira (2002) have focused on the role of labor income and the risk associated with it. The intuition is as follows: when labor income risk is not highly correlated with stock market, it can be regarded as a substitute for risk-free assets. The implicit risk-free asset holdings in the form of labor income lose importance as the investor ages, leading him/her to hold more risk-free assets explicitly in his/her financial portfolio. But this rationale only explains changes in portfolio choice leading up to retirement, not continuous declines with age. In the same way, many researches provide compelling explanations for the vast differences in portfolios that we see between young and old investors, but few can explain continued declines in risk taking with age after retirement.

Recent theoretical results suggest that strong bequest motives can explain non-increasing equity shares with age after retirement (Gomes and Michaelides, 2005). However, a key problem with this explanation is that bequests may be primarily unintended. It seems premature to assign too much explanatory weight to bequests when they are not fully understood.

As stated above, the former models which include medical expenses, labor income and bequest motives can only explain partial of the puzzle of elder people's saving behavior and portfolio choices. New theoretical models need to be set up to explain the empirical results of the decline of holding risky assets and the slow dissaving behavior. This paper reconciles the portfolio choice theory with empirical patterns by including health status to utility maximization problems and adding uncertain future medical expenses as another important source of background constraint for the retirees' spending behavior.

2.2 Saving Puzzle and Health Expenditure

According to early studies, the elderly engage in no dissaving, but instead continue to amass wealth as they grow older (Mirer, 1979). Many elderly keep large amounts of assets until very late in life. Furthermore, the more income they earned during their working years, the slower they run down their assets. More recent articles report less dramatic conclusions: on average, wealth increases during the first few years of retirement and then decreases with age, although too slowly to be consistent with the simple life cycle model (King and Dicks-Mireaux, 1982; Diamond and Hausman, 1984; Hurd, 1987). This underspending puzzle can be explained, at least in part, by modeling sources of uncertainty confronting the elderly, which to a large extent refers to the uncertain health expenditure.

Tanner (1998) shows that health spending is not large on average among U.S. retirees, but French and Jones (2004) reveals that it is serially correlated and may be catastrophic. Even in the presence of Medicare and private health insurance, out-of-pocket expenditures for health care represent an important source of risk to the elderly wealth holdings. Out-of-pocket medical and nursing home expenses can be large, and thus generate significant net income risk for the elderly.

National Medical Care Expenditure Survey indicates that nearly 10% of elderly households spend a fifth or more of their incomes on out-of-pocket medical expenses. Furthermore, these figures neglect what perhaps is the most important contributor to health care costs—nursing home expenses. The likelihood that a typical sixty-five year old person enters a nursing home during her lifetime is 43%. Once admitted, the average stay in a long-term care facility exceeds one year. Because nursing home costs are virtually uninsured, admission to a long-term care facility can quickly deplete one's financial wealth. In their examination of IRS tax files, Slemrod (2007) find that 2 or 3% of elderly families incur medical expenses exceeding 40% of their adjusted gross incomes.

Consequently, uncertainty regarding future health care expenses, including those incurred during possible nursing home residences, effectively introduces random shocks to the pension incomes of retired households in the model. These random shocks provide the incentive for the elderly to engage in precautionary behavior with respect to current consumption. That is, households optimally maintain additional financial wealth to offset potentially large future out-of-pocket medical cost.

As stated above, out-of-pocket medical cost has a great impact on elder people's financial decisions. This paper adds medical expenditure to the constraint so as to take into account the effect of possible medical expenditure on the elder people's financial decisions. Additionally, the paper model health uncertainty by adding health status in to individual's utility function in an attempt to provide motives for precautionary saving, which helps to explain the under-spending puzzle for elderly Americans.

2.3 Health Status and Asset Holding

Since health tends to become riskier with age, the presence of health risk may explain why investors decrease their financial risk with age, continuing after retirement. A key element of the theory is the behavior of the mixed partial derivative of utility. If adverse health shocks increase the marginal utility of consumption (Lillard and Weiss, 1997), then investors respond to the risk of falling into poor health by holding safer financial assets. Aggregate and individual-level data suggest that health risk may explain 60% or more of the decline in financial risk taking after retirement.

Guiso, Jappelli and Terlizzese (1996) find that Italian households headed by individuals who spent more days sick tended to hold safer financial portfolios, even after controlling for many other variables. Rosen and Wu (2004) show a robust association between low health status and safe portfolios in the Health and Retirement.

Retirees face risks associated with their health status, a fact that previous theoretical models have not emphasized. By adding health uncertainty to the utility function, this paper take account the health risks faced by the elderly people. The paper then tries to estimate the model by using data from the Health and Retirement.

3. Modeling

3.1 The Utility Function

As discussed above, many former theoretical models treat health shocks simply as shocks to income and they find that health risk generates precautionary saving in the same way that income risk does. But this type of specification restricts the mixed partial derivative of utility to be zero: health shocks do not affect the marginal utility of consumption. Some other papers treat health shocks as a dummy variable: "1" represents good health while "0" means bad health. This paper builds primarily on previous research by Hubbard, Skinner and Zeldes (1994), in which aspects of health uncertainty are incorporated into life-cycle consumption models. Different from former models, I treat health status as a continuous random variable which ranges from "0" to "1".

Theoretically, the sign of the mixed partial is ambiguous. If adverse health shocks increase the marginal utility of consumption, then investors respond to the risk of falling into poor health by holding safer financial assets.

The empirical literature doesn't provide clear direction either. Viscusi and Evans (1990) find that chemical workers expect their marginal utilities of income to decline in bad health, as a result of job risks. They report that temporary health conditions like burns and poisonings seem not to affect the marginal utilities of surveyed adults. Lillard and Weiss (1997) report that among elderly households, adverse health shocks raise the marginal utility of consumption, induce transfers from the healthy to the sick partner, and provide an important precautionary motive for saving. These studies use completely different data and focus on individuals of different ages. It is unclear what may be driving the different results across these studies.

In this paper, I use a time-separable power utility since it is common in macroeconomics and finance and it encapsulates constant relative risk aversion (CRRA).

The agent tries to maximize the time zero expected value of the following form:

$$\sum_{t=0}^{\infty} \beta^t H_t^{\alpha} \frac{C_t^{1-\gamma}}{1-\gamma} \tag{1}$$

As shown above, in this model, we include the health status into the utility function. The effect of health status on marginal utility can be either positive or negative, depending on the sign of ð. Distinguished from the former papers; health status is a continuous random variable and is ranged between 0 and 1.

3.2 Expenditure Constraints

My analysis focuses on people who have already retired, which allows us to concentrate on savings and consumption decisions, and abstract from labor supply and retirement decisions. The retired people face the following budget constraint:

$$X_t = \theta X_{t-1} - C_t \tag{2}$$

($X_{-1} > 0$, $\theta_0 > 0$; X_{-1}, θ_0 are given. Here C_t is consumption at time t. X_{t-1} is invested wealth at time t-1. θ_t is a random variable whose value is realized at time t. The agent's choices of C_t and X_t can only depend upon $X_{-1} \ldots X_{t-1}$ and $\theta_0 \ldots \theta_t$.)

In the above formula, consumption includes both the consumption of regular goods and the consumption of medical services. The consumption of medical services for the retired includes both out-of-pocket health care expenditures and potential nursing home costs.

3.3 Expenditure Constraints

The nature of asset returns is an opaque topic. The benchmark assumption is that returns are independently and identically distributed (i.i.d). In this paper, I make the same assumption that $\{\theta_t\}_{t=0}^{\infty}$ is an i.i.d sequence with ln θ_t distributed according to a normal distribution with mean μ and variance σ^2. In my analysis, I assume $\sigma^2 = 0$. As a result, θ_t is a constant and can be represented as θ.

In order to make the model simple and analytically solvable, I make the assumption that $\{H_t\}_{t=0}^{\infty}$ is also an i.i.d sequence with H_t distributed uniformly between f and g.

In short, the theoretical model proposed in my paper assumes power utility, IID stock returns, infinite horizons, no labor income and no explicit bequest motive.

4. Analytical Solution I

As stated above, I want to maximize the time zero expected value of

$$\sum_{t=0}^{\infty} \beta^t H_t^{\alpha} \frac{C_t^{1-\gamma}}{1-\gamma} \tag{3}$$

By choice of sequences the utility function is subject to the following constraints:

$$X_t = \theta_t X_{t-1} - C_t$$
$$C_t \geq 0$$
$$X_t \geq 0$$
$$\theta_0 = \theta_t \succ 0$$

For all t where $X_{t-1} > 0$ and $\theta_0 > 0$ are given. Here C_t is consumption at time t, $X_{t-1} > 0$ is invested wealth at time t-1, β is a discount factor, and θ_t is a random variable whose value is realized at time t. θ_t gives the rate of return of wealth invested at time t-1. Let $\{\theta_t\}_{t=0}^{\infty}$ be an i.i.d. sequence with $\ln \theta_t$ distributed according to a normal distribution with mean μ and variance σ^2. Let $\{H_t\}_{t=0}^{\infty}$ be an i.i.d. sequence with H_t uniformly distributed between f and g. The agents choices of C_t and X_t can only depend upon $X_{-1} \ldots X_{t-1}$ and $\theta_0 \ldots \theta_t$

We guess that the value function if of the following form:

$$V(X, H) = Q(H)\frac{X^{1-\gamma}}{1-\gamma} \tag{4}$$

Where Q(H) is some function of H.

I assume that θ is constant, then, the bellman equation for the model is of the following form:

$$V(X,H) = \max_{\bar{x}}[H^{\alpha}\frac{(\theta X - \bar{X})}{1-\gamma} + \beta EV(\bar{X},\bar{H})]$$

$$= \max_{\bar{x}}[H^{\alpha}\frac{(\theta X - \bar{X})}{1-\gamma} + \beta\int V(\bar{X},\bar{H}\prod(\bar{H})d\bar{H})] \tag{5}$$

$$V_2 = \max_{\bar{x}}\left[H^{\alpha}\frac{(\theta X - \bar{X})^{1-\gamma}}{1-\gamma}\right] + \beta EQ(\bar{H})(\frac{\bar{X}^{1-\gamma}}{1-\gamma})] \tag{6}$$

Let $M = EQ(\bar{H})$ (M is a constant and H is i.i.d.), then the value function becomes:

$$V_2 = \max_{\bar{x}}\left[H^{\alpha}\frac{(\theta X - \bar{X})^{1-\gamma}}{1-\gamma}\right] + \beta M(\frac{\bar{X}^{1-\gamma}}{1-\gamma})] \tag{7}$$

Take first order condition with respect to $\bar{X}$, we get:

$$-H^{\alpha}(\theta X - \bar{X})^{-\gamma} + \beta M(\bar{X})^{-\gamma} = 0$$

Therefore,

$$\beta M(\bar{X})^{-\gamma} = H^{\alpha}(\theta X - \bar{X})^{-\gamma}$$

$$(\beta M)^{\frac{1}{\gamma}}\bar{X} = H^{\frac{\alpha}{\gamma}}(\theta X - \bar{X})$$

$$(\beta M)^{\frac{1}{\gamma}}\bar{X} = H^{\frac{\alpha}{\gamma}}\theta X - H^{-\frac{\alpha}{\gamma}}\bar{X}$$

$$(\beta M)^{-\frac{1}{\gamma}}\bar{X} + H^{-\frac{\alpha}{\gamma}}X = H^{-\frac{\alpha}{\gamma}}\theta X$$

As a result, we get

$$\bar{X}_{MAX} = \frac{H^{-\frac{\alpha}{\gamma}}\theta X}{(\beta M)^{\frac{1}{\gamma}} + H^{-\frac{\alpha}{\gamma}}} \tag{8}$$

$\bar{X}_{MAX}$ is the optimal decision rule for the choice variable, in order to find M, we *plug* $\bar{X}_{MAX}$ back to the value function and get:

$$V = H^{\alpha}\frac{\left(\theta X - \frac{H^{\frac{\alpha}{\gamma}}\theta X}{(\beta M)^{-\frac{1}{\gamma}} + H^{\frac{\alpha}{\gamma}}}\right)^{1-\gamma}}{1-\gamma} + \beta M\frac{\left[\frac{H^{-\frac{\alpha}{\gamma}}\theta X}{(\beta M)^{-\frac{1}{\gamma}} + H^{-\frac{\alpha}{\gamma}}}\right]^{1-\gamma}}{1-\gamma}$$

$$= H^{\alpha}\frac{\left(\frac{(\beta M)^{-\frac{1}{\gamma}}\theta X}{(\beta M)^{\frac{1}{\gamma}} + H^{\frac{\alpha}{\gamma}}}\right)^{1-\gamma}}{1-\gamma} + \beta M\frac{\left(\frac{H^{-\frac{\alpha}{\gamma}}\theta X}{(\beta M)^{-\frac{1}{\gamma}} + H^{-\frac{\alpha}{\gamma}}}\right)^{1-\gamma}}{1-\gamma}$$

$$= H^{\alpha}\frac{\left(\dfrac{\beta M)^{-\frac{1}{\gamma}}\theta}{(\beta M)^{\frac{1}{\gamma}}+H^{\frac{\alpha}{\gamma}}}\right)^{1-\gamma}X^{1-\gamma}}{1-\gamma}+\beta M\frac{\left(\dfrac{\beta M)^{-\frac{\alpha}{\gamma}}\theta}{(\beta M)^{\frac{1}{\gamma}}+H^{\frac{\alpha}{\gamma}}}\right)^{1-\gamma}X^{1-\gamma}}{1-\gamma}$$

$$=\left(\frac{X^{1-\gamma}}{1-\gamma}\right)\left(\frac{1}{(\beta M)^{-\frac{1}{\gamma}}+H^{\frac{\alpha}{\gamma}}}\right)^{1-\gamma}\left\{\beta M(H^{-\frac{\alpha}{\gamma}}\theta)^{1-\gamma}+H^{\alpha}\left[\theta(\beta M)^{-\frac{1}{\gamma}}\right]^{1-\gamma}\right\}$$

$$=F(H)(\frac{X^{1-\gamma}}{1-\gamma})$$

Therefore, we get:

$$F(H)=\left(\frac{1}{(\beta M)^{\frac{1}{\gamma}}+H^{\frac{\alpha}{\gamma}}}\right)^{1-\gamma}\left\{\beta M(H^{\frac{\alpha}{\gamma}}\theta)^{1-\gamma}+H^{\alpha}\left[\theta(\beta M)^{-\frac{1}{\gamma}}\right]^{1-\gamma}\right\}$$

$$=\left(\frac{\theta}{(\beta M)^{-\frac{1}{\gamma}}+H^{-\frac{\alpha}{\gamma}}}\right)^{1-\gamma}\left\{\beta MH^{\frac{\alpha\gamma-\alpha}{\gamma}}+H^{\alpha}(\beta M)^{\frac{\gamma-1}{\gamma}}\right\}$$

$$=\left(\frac{\theta}{(\beta M)^{-\frac{1}{\gamma}}+H^{-\frac{\alpha}{\gamma}}}\right)^{1-\gamma}*\beta MH^{\alpha}*\left\{H^{-\frac{\alpha}{\gamma}}+(\beta M)^{\frac{-1}{\gamma}}\right\}$$

$$=\frac{\theta^{1-\gamma}\beta MH^{\alpha}}{\left[(\beta M)^{-\frac{1}{\gamma}}+H^{-\frac{\alpha}{\gamma}}\right]^{-\gamma}}$$

Since M is the expected value of $F(H)$, therefore, we get:

$$M=EF(H)=E\frac{\theta^{1-\gamma}\beta MH^{\alpha}}{\left[(\beta M)^{-\frac{1}{\gamma}}+H^{-\frac{\alpha}{\lambda}}\right]^{-\gamma}} \tag{9}$$

In order to estimate M numerically and obtain an analytical solution for the value function, I assume that H is uniformly distributed between f and g. Based on equation (8), I solved the following equation:

$$\int_f^g\left[\left(\frac{1}{g-f}\right)\frac{\theta^{1-\gamma}\beta MH^{\alpha}}{\left[(\beta M)^{\frac{1}{\gamma}}+H^{\frac{\alpha}{\gamma}}\right]^{-\gamma}}\right]dH-M=0 \tag{10}$$

I use Mat lab programming to numerically solve for *M*. First, I assign some original values for the four parameters *ϒ, α, θ* and *β*. *ϒ* is the risk-aversion, I assume that $ϒ = \beta = 1.1$; *α* reflects the effect of health status on people's optimal choice of consumption, I assume that $\alpha = C = 1$; *β* is the discount rate, I set the original value of *β* to be 0.8, $\beta = d = 0.8$. *θ* is the asset return, I set the original value of 0 to be 1.05, $\theta = \alpha = 1.05$. Table 1 describes how the value of *M* changes as I change the parameters of *ϒ, α, θ* and *β*.

Table 1 Change of Risk Aversion and Expected Health with Respect to Parameters

$\Theta = a$	1.05	1.05	1.05	1.20up	0.95down	1.05	1.05
$ϒ = b$	1.1	1.1	1.1	1.1	1.1	1.1	1.1
α = c	1	1	1	1	1	1.15up	0.85down
$\beta = d$	0.8	0.95up	0.65down	0.8	0.8	0.8	0.8
M	6.3163	27.1175	3.4198	5.8921	6.6704	6.3210	6.3116
$ϒ_{MAX}$	1.98	1.34down	2.44up	2.51up	1.74down	1.99up	1.98equal
M_{MAX}	56.2106	55.6428	57.0338	56.3898	56.4185	56.2527	56.1685

The first column of the form shows that if I fixed the value of $\theta = 1.05$, $ϒ = b$, α = 1, $\beta = 0.8$, then the numeric solution of *M* is 6.3163. The largest *ϒ* to get a numerical solution for *M* is 1.98 and the maximum value of *M* is 56.2106. The second column describes that if I increase *β* to be 0.95, other things unchanged, *M* is 27.1175. The largest *ϒ* is 1.34 to obtain a numerical solution for *M* and the maximum value for M is 55.6428. The third column describes that if *β* to is reduced to be 0.65, other things equal, then, *M* is 3.4198. The largest *ϒ* is 2.44 to obtain a numerical solution for *M* and the maximum value for M is 57.0338. The fourth column describes that if I increase *θ* to be 1.20, other things equal, then, *M* is 5.8921.In this case, the largest ϒ is 2.51 for us to get a numerical solution for *M,* M_{MAX} in this case is 56.3898. The fifth column describes that if I reduce *θ* to be 0.95, other things equal, then, M is calculated to be 6.6704, the largest *ϒ* is 1.74 to obtain a numerical solution for *M* and the maximum value for M is 56.4185. The sixth column shows that if α is increased to be 1.15, and other things equal, M is calculated to be 6.3210, the largest *ϒ* is 1.99 to obtain a numerical solution for M and the maximum value for M in this case is 56.2527. The seventh column describes that if I reduce α to be 0.85, and other things equal, then, M is 6.3116, the largest ϒ to get a numerical solution for M is 1.98, M_{MAX} in this case is 56.1685.

The above results show that as discount rate *β* goes up from 0.8 to 0.95, other things equal, $ϒ_{MAX}$ decreases from 1.98 to 1.34 and if *β* goes down from 0.8 to 0.95, ceteris paribus, $ϒ_{MAX}$ decreases from 1.98 to 1.34. If rate of asset return *θ* goes up from 1.05 to 1.20, ceteris paribus, $ϒ_{MAX}$ increases from 1.98 to 2.51, similarly, if rate of asset return *θ* goes down from 1.05 to 1.20, ceteris paribus, $ϒ_{MAX}$ increases from 1.98 to 2.51. If the health factor α goes up from 1 to 1.15, ceteris paribus, $ϒ_{MAX}$ will increase from 1.98 to 1.99; if α goes down from 1 to 0.85, ceteris paribus $ϒ_{MAX}$ will be equal to 1.98. The maximum expected value of health M_{MAX} does not change much as the parameters change, it stays around 56.

5. Analytical Solution II

In order to prove the robustness of the analytical solution for the first model, I try to solve the model using another utility function. I maximize the time zero expected value of the following equation:

$$\sum_{t=0}^{\infty}\frac{(H_t^p C_t)^{1-\gamma}-1}{1-\gamma}=\sum_{t=0}^{\infty}\left[\frac{(H_t^p C_t)^{1-\gamma}}{1-\gamma}-\frac{1}{1-\gamma}\right] \tag{11}$$

By chince of sequences $\{C_t, X_t\}_{t=0}^{\infty}$ subject to the constraints

$$X_t = \theta_t X_{t-1} - C_t$$
$$C_t \geq 0$$
$$X_t \geq 0$$
$$\theta_t = \theta_0 \succ 0$$

For all t where $X_{-1} > 0$ and $\theta_0 > 0$ are given. Here C_t is consumption at time t, $X_{t-1} > 0$ is invested wealth at time $t-1$, β is a discount factor, and θ_t is a random variable whose value is realized at time t. θ_t gives the rate of return of wealth invested at time $t-1$. Let $\{\theta_t\}_{t=0}^{\infty}$ be an i.i.d. sequence with $\ln\theta_t$ distributed according to a normal distribution with mean μ and variance σ^2. Let $\{H_t\}_{t=0}^{\infty}$ be an i.i.d. sequence with H_t uniformly distributed between f and g. The agents choices of C_t and X_t can only depend upon $X_{-1}...X_{t-1}$ and $\theta_0...\theta_t$.

We guess that the value function is of the following form:

$$V(X,H) = Q(H)\frac{X^{1-\gamma}}{1-\gamma} - \frac{1}{(1-\gamma)(1-\beta)} \tag{12}$$

Let $\alpha = p(1-\gamma)$, the bellman equation for the model is of the following form

$$V(X,H) = \max_{\bar{x}}\left[H^{\alpha}\frac{(\theta X - \overline{X})}{1-\gamma} - \frac{1}{1-\gamma} + \beta EV(\overline{X},\overline{H})\right] \tag{13}$$

$$V_2 = \max_{\bar{x}}\left[H^{\alpha}\frac{(\theta X - \overline{X})^{1-\gamma}}{1-\gamma} - \frac{1}{1-\gamma} + \beta EQ(H)(\frac{\overline{X}^{1-\gamma}}{1-\gamma}) - \frac{\beta}{(1-\gamma)(1-\beta)}\right] \tag{14}$$

Let $M = EQ(H)$, then we have:

$$V_2 = \max_{\bar{x}}\left[H^{\alpha}\frac{(\theta X - \overline{X})^{1-\gamma}}{1-\gamma} - \frac{1}{1-\gamma} + \beta M(\frac{\overline{X}^{1-\gamma}}{1-\gamma}) - \frac{\beta}{(1-\gamma)(1-\beta)}\right] \tag{15}$$

Then, after taking the first order condition with respect to choice variable *X*, we get:

$$-H^{\alpha}(\theta X - \overline{X})^{-\gamma} + \beta M(\overline{X})^{-\gamma} = 0$$
$$\beta M(X)^{-\gamma} = H^{\alpha}(\theta X - \overline{X})^{-\gamma}$$
$$(\beta M)^{\frac{1}{\gamma}}\overline{X} = H^{\frac{\alpha}{\gamma}}(\theta X - \vec{X})$$
$$(\beta M)^{\frac{1}{\gamma}}\overline{X} = H^{\frac{\alpha}{\gamma}}\theta X - H^{-\frac{\alpha}{\gamma}}\overline{X}$$

Then, we get

$$(\beta M)^{\frac{1}{\gamma}}\overline{X} = H^{\frac{\alpha}{\gamma}}\theta X - H^{-\frac{\alpha}{\gamma}}\overline{X}$$

Therefore, we get the optimal choice variable:

$$\overline{X}_{MAX} = \frac{H^{-\frac{\alpha}{\gamma}}\theta X}{(\beta M)^{-\frac{1}{\gamma}} + H^{-\frac{\alpha}{\gamma}}} \tag{16}$$

In order to get the solution for *M,* we plug $\overline{X}_{MAX}$ back to the value function, we get:

$$V = H^{\alpha}\frac{\left(\theta X - \frac{H^{\frac{\alpha}{\gamma}}\theta X}{(\beta M)^{-\frac{1}{\gamma}} + H^{\frac{\alpha}{\gamma}}}\right)^{1-\gamma}}{1-\gamma} - \frac{1}{1-\gamma} + \beta M\frac{\left(\frac{H^{\frac{\alpha}{\gamma}}\theta X}{(\beta M)^{-\frac{1}{\gamma}} + H^{\frac{\alpha}{\gamma}}}\right)^{1-\gamma}}{1-\gamma} - \frac{\beta}{(1-\gamma)(1-\beta)}$$

$$= H^{\alpha}\frac{\left(\frac{(\beta M)^{-\frac{1}{\gamma}}\theta X}{(\beta M)^{-\frac{1}{\gamma}} + H^{-\frac{\alpha}{\gamma}}}\right)^{1-\gamma}}{1-\gamma} + \beta M\frac{\left(\frac{H^{-\frac{\alpha}{\gamma}}\theta X}{(\beta M)^{-\frac{1}{\gamma}} + H^{-\frac{\alpha}{\gamma}}}\right)^{1-\gamma}}{1-\gamma} - \frac{1}{(1-\gamma)(1-\beta)}$$

$$= H^{\alpha}\frac{\left(\frac{(\beta \mathrm{M})^{-\frac{1}{\gamma}}\theta}{(\beta \mathrm{M})^{\frac{1}{\gamma}} + H^{\frac{\alpha}{\gamma}}}\right)^{1-\gamma} X^{1-\gamma}}{1-\gamma} + \beta M\frac{\left(\frac{H^{-\frac{\alpha}{\gamma}}\theta}{(\beta M)^{-\frac{1}{\gamma}} + H^{\frac{\alpha}{\gamma}}}\right)^{1-\gamma} X^{1-\gamma}}{1-\gamma} - \frac{1}{(1-\gamma)(1-\beta)}$$

$$= F(H)\left(\frac{X^{1-\gamma}}{1-\gamma}\right) - \frac{1}{(1-\gamma)(1-\beta)}$$

As a result, we get *F*(*H*) as the following:

$$F(H) = \left(\frac{1}{(\beta M)^{-\frac{1}{\gamma}} + H^{-\frac{\alpha}{\gamma}}}\right)^{1-\gamma} + \left\{\beta M(H^{-\frac{\alpha}{\gamma}}\theta)^{1-\gamma} + H^{\alpha}\left[\theta(\beta M)^{-\frac{1}{\gamma}}\right]^{1-\gamma}\right\}$$

$$= \left(\frac{\theta}{(\beta M)^{-\frac{1}{\gamma}} + H^{-\frac{\alpha}{\gamma}}}\right)^{1-\gamma} + \left\{\beta M H^{\frac{\alpha\gamma-\alpha}{\gamma}} + H^{\alpha}(\beta M)^{\frac{\gamma-1}{\gamma}}\right\}$$

$$= \left(\frac{\theta}{(\beta M)^{-\frac{1}{\gamma}} + H^{-\frac{\alpha}{\gamma}}}\right)^{1-\gamma} + \beta M H^{\alpha}\left\{H^{-\frac{\alpha}{\gamma}} + (\beta M)^{\frac{-1}{\gamma}}\right\}$$

$$= \frac{\theta^{1-\lambda}\beta M H^{\alpha}}{\left[(\beta M)^{-\frac{1}{\gamma}} + H^{\frac{\alpha}{\gamma}}\right]^{-\gamma}}$$

Since *M* is the expected value of *F*(*H*), therefore, we get:

$$M = EF(H) = E\frac{\theta^{1-\gamma}\beta MH^{\alpha}}{\left[(\beta M)^{-\frac{1}{\gamma}} + H^{-\frac{\alpha}{\gamma}}\right]^{-\gamma}} \tag{17}$$

In order to estimate M numerically and get an analytical solution for the value function, I assume that H is uniformly distributed between f and g. Based on equation 16, I need to solve the following equation:

$$\int_f^{\varepsilon}\left[\left(\frac{1}{g-f}\right)\frac{\theta^{1-\gamma}\beta MH^{\alpha}}{\left[(\beta M)^{\frac{1}{\gamma}} + H^{\frac{\alpha}{\gamma}}\right]^{-\gamma}}\right]dH - M = 0 \tag{18}$$

I can see that equation 17 is the same as equation 9. Therefore, I will get the same M using Mat lab programming.

In this case, the value function will be:

$$V(X,H) = Q(H)\frac{X^{1-\gamma}}{1-\gamma} - \frac{1}{(1-\gamma)(1-\beta)}$$

6. Analytical Solution III

Based on the analytical solution II, as γ approaches to 1, I have the following results:

$$\lim_{\gamma\to 1}\frac{(H^pC)^{1-\gamma}-1}{1-\gamma} = \lim_{\gamma\to 1}\frac{-(H^pC)^{1-\gamma}\log(H^pC)}{-1}$$
$$= \log(H^pC)$$
$$= p\log H + \log C$$

We want to maximize the time zero expected value of the following equation:

$$\sum_{t=0}^{\infty}\log(H_t^pC_t) = \sum_{t=0}^{\infty}(p\log H_t + \log C_t) \tag{19}$$

By choice of sequences $\{C_t, X_t\}_{t=0}^{\infty}$ subject to the constraints

$$X_t = \theta_t X_{t-1} - C_t$$
$$C_t \geq 0$$
$$X_t \geq 0$$
$$\theta_t = \theta_0 \geq 0$$

For all t where $X_{-1} > 0$ and $\theta_0 > 0$ are given. Here C_t is consumption at time t, $X_{t-1} > 0$ is invested wealth at time $t-1$, β is a discount factor, and θ_t is a random variable whose value is realized at time t. θ_t gives the rate of return of wealth invested at time $t-1$. Let $\{\theta_t\}_{t=0}^{\infty}$ be an i.i.d. sequence with $\ln\theta_t$ distributed according to a normal distribution with mean μ and variance σ^2. Let $\{H_t\}_{t=0}^{\infty}$ be an i.i.d. sequence with H_t uniformly distributed between f and g. The agents choices of C_t and X_t can only depend upon $X_{-1}...X_{t-1}$ and $\theta_0...\theta_t$.

$$V_0(X,H)=0$$

$$V_1(X,H)=\log H^p\theta X = p\log H+\log\theta+\log X = F_1+G_1\log X+S_1\log H$$

$$F_1=\log\theta, G_1=1, S_1=p$$

As a result, I guess the value function to be the following form:

$$V(X,H)=F+G\log X+S\log H$$

The bellman equation for the model is of the following form:

$$V(X,H)=\max_{\bar{x}}\left[\log\left(H^p(\theta X-\bar{X})\right)+\beta F+\beta G\log\bar{X}+\beta SE\log H\right] \tag{20}$$

Take the first order condition with respect to $\bar{X}$, we get:

$$\frac{-H^p}{H^p(\theta X-\bar{X})}+\frac{\beta G}{\bar{X}}=0$$

$$\frac{\beta G}{\bar{X}}=\frac{1}{(\theta X-\bar{X})}$$

$$\beta G(\theta X-\bar{X})=\bar{X}$$

$$\beta G\theta X=\bar{X}+\beta G\bar{X}$$

As a result, we get:

$$\bar{X}_{MAX}=\frac{\beta G\theta X}{1+\beta G} \tag{21}$$

We plug $\bar{X}_{MAX}$ back to the value function and we get:

$$\theta X-\bar{X}_{MAX}=\theta X-\frac{\beta G\theta X}{1+\beta G}=\frac{\theta X}{1+\beta G}$$

Maximize:

$$\log H^p\left(\frac{\theta X}{1+\beta G}\right)+\beta F+\beta G\log\frac{\beta G\theta X}{1+\beta G}+\beta SE\log H$$

$$=\log\theta+\beta G\log\theta$$

$$+\log X+\beta G\log X$$

$$+p\log H-\log(1+\beta G)+\beta F+\beta G\log\beta G-\beta G\log(1+\beta G)+\beta SE\log H$$

$$=(1+\beta G)\log\theta+\beta G\log\beta G-\beta G\log(1+\beta G)-\log(1+\beta G)+\beta SE\log H+\beta F$$

$$+(1+\beta G)\log X$$

$$+p\log H$$

As a result, we get:

$$G=1+\beta G$$

So we get:

$$G=\frac{1}{1-\beta} \tag{22}$$

Also, we get:

$$S=p \tag{23}$$

Since $G = \frac{1}{1-\beta}$, $S = p$, and we get:

$$F=(1+\frac{\beta}{1-\beta})\log\theta+\frac{\beta}{1-\beta}\log\frac{\beta}{1-\beta}-\frac{\beta}{1-\beta}\log\frac{1}{1-\beta}+\log(1-\beta)+\beta pE\log H+\beta F$$

$$F=(\frac{1}{1-\beta})\log\theta+\frac{\beta}{1-\beta}\log\beta+\log(1-\beta)+\beta pE\log H+\beta F \tag{24}$$

$$F=\frac{1}{(1-\beta)^2}\log\theta+\frac{\beta}{(1-\beta)^2}\log\beta+\frac{1}{1-\beta}\log(1-\beta)+\frac{\beta}{1-\beta}pE\log H$$

Since H is uniformly distributed between f and g, we get the following result:

$$E\log H=\int_f^g\frac{1}{g-f}\log HdH=\frac{1}{g-f}(H\log H-H)\Big|_f^g$$

$$=\frac{1}{g-f}(g\log g-g-f\log f+f)=\psi$$

Therefore:

$$F=\frac{1}{(1-\beta)^2}\log\theta+\frac{\beta}{(1-\beta)^2}\log\beta+\frac{1}{1-\beta}\log(1-\beta)+\frac{\beta}{1-\beta}pE\log H$$

$$=\frac{1}{(1-\beta)^2}\log\theta+\frac{\beta}{(1-\beta)^2}\log\beta+\frac{1}{1-\beta}\log(1-\beta)+\frac{\beta}{1-\beta}p\psi$$

As a result, the value function takes the following form:

$$V(X,H)=F+G\log X+S\log H$$

$$=\frac{1}{(1-\beta)^2}\log\theta+\frac{\beta}{(1-\beta)^2}\log\beta+\frac{1}{1-\beta}\log(1-\beta)+\frac{\beta}{1-\beta}p\psi+\frac{1}{1-\beta}\log X+p\log H$$

In order to get the optimal decision rule for X, we maximize:

$$\log\left(H^p(\theta X-\overline{X})\right)+\beta F+\beta G\log\overline{X}+\beta SE\log H$$

Take the first order condition with respect to $\overline{X}$, we get the following result:

$$\frac{-1}{\theta X-\overline{X}}+(\frac{\beta}{1-\beta})\frac{1}{\overline{X}}=0$$

Therefore,

$$(\frac{\beta}{1-\beta})\frac{1}{\overline{X}}=\frac{1}{\theta X-\overline{X}}$$

$$\overline{X}_{MAX}=\beta\theta X \tag{25}$$

In order to check the answer, we plug $\overline{X}_{MAX}$, G, K, F back to the value function, we get:

$$V=\log\left[H^p(\theta X-\beta\theta X)\right]+\beta F+\frac{\beta}{1-\beta}\log\beta\theta X+\beta p\psi$$

$$V = \log\left[H^{p}\theta X(1-\beta)\right] + \beta F + \frac{\beta}{1-\beta}\log\beta\theta X + \beta p\psi$$

$$= p\log H + \log\theta + \log X + \log(1-\beta) + \beta F + \frac{\beta}{1-\beta}\log\beta + \frac{\beta}{1-\beta}\log\theta + \frac{\beta}{1-\beta}\log X + \beta p\psi$$

$$= p\log H + \frac{1}{1-\beta}\log X + \frac{1}{1-\beta}\log\theta + \log(1-\beta) + \frac{\beta}{1-\beta}\log\beta + \beta p\psi$$

$$+\beta\left[\frac{1}{(1-\beta)^2}\log\theta + \frac{\beta}{(1-\beta)^2}\log\beta + \frac{1}{1-\beta}\log(1-\beta) + \frac{\beta}{1-\beta}p\psi\right]$$

$$= p\log H + \frac{1}{1-\beta}\log X + \frac{1}{(1-\beta)^2}\log\theta + \frac{\beta}{(1-\beta)^2}\log\beta + \frac{1}{1-\beta}\log(1-\beta) + \frac{\beta}{1-\beta}p\psi$$

$$= S\log H + G\log X + F$$

This result proves that our guessed value function is correct.

7. Comparing Analytical Solution II and III

Analytical Solution II and III are closely related as γ approaches 1,

$$\lim_{\gamma\to 1}\frac{(H_t^{p}C_t)^{1-\gamma}-1}{1-\gamma} = \log(H_t^{p}C_t)$$

As proved through analytical solution II, the value function for $\sum_{t=0}^{\infty}\frac{(H_t^{p}C_t)^{1-\gamma}-1}{1-\gamma}$ takes the following form:

$$V(X,H) = Q(H)\frac{X^{1-\lambda}}{1-\gamma} - \frac{1}{(1-\gamma)(1-\beta)} \tag{26}$$

As proved through analytical solution III, the value function for $\sum_{t=0}^{\infty}\log(H_t^{p}C_t)$ takes the following form:

$$V(X,H) = F + G\log X + S\log H \tag{27}$$

As γ approaches 1, we want to prove that the expected value of the two value functions equation 26 and equation 27 are equal to each other. We want to prove:

$$\lim_{\gamma\to 1}\left\{E\left[Q(H)\frac{X^{1-\gamma}}{1-\gamma} - \frac{1}{(1-\gamma)(1-\beta)}\right]\right\} = E(S\log H + G\log X + F)$$

We want to prove:

$$\lim_{\gamma\to 1}\left\{E\left[Q(H)\frac{X^{1-\gamma}}{1-\gamma} - \frac{1}{(1-\gamma)(1-\beta)}\right]\right\} = E(p\log H + \frac{1}{1-\beta}\log X + F)$$

For the purpose of simplicity, it is assumed that X is 1, therefore, we want to prove the following equation:

$$\lim_{\gamma\to 1}\left\{EQ(H)\frac{1}{1-\gamma} - \frac{1}{(1-\gamma)(1-\beta)}\right\} = F + pE\log H$$

Since $EQ(H) = M$, suppose H is uniformly distributed f and g, we get:

$$E\log H=\int_f^g \frac{1}{g-f}\log H dH=\frac{1}{g-f}(H\log H-H)\Big|_f^g$$
$$=\frac{1}{g-f}(g\log g-g-f\log f+f)=\psi$$

Since g = 1.2, f = 1,

$$\psi=\frac{1}{g-f}(g\log g-g-f\log f+f)$$
$$=\frac{1}{2}(1.2\log 1.2-1.2-1\log 1+1)$$
$$=5(1.2*0.1823-0.2)$$
$$=.0983$$

As a result, we want to prove:

$$\lim_{\gamma\to 1}\left\{M\frac{1}{1-\gamma}-\frac{1}{(1-\gamma)(1-\beta)}\right\}=F+p\psi \tag{28}$$

In the Mat lab programming, we give the original value for the parameters *θ, γ, p, β, g, f.*

We assume $\theta = a = 1.05$, $\gamma = b = 1.1$, $p = c = 1$, $\beta = d = 1/1.05 = 0.9523$, $g = 1.2$, f = 1. We will prove that equation 28 establishes as γ approaches 1.

For the right hand side, we get:

$$F+p\psi=\frac{1}{(1-\beta)^2}\log\theta+\frac{\beta}{(1-\beta)^2}\log\beta+\frac{1}{1-\beta}\log(1-\beta)+\frac{\beta}{1-\chi}p\psi+p\psi$$
$$=\frac{1}{(1-\beta)^2}\log\theta+\frac{\beta}{(1-\beta)^2}\log\beta+\frac{1}{1-\beta}\log(1-\beta)+\frac{1}{1-\beta}p\psi$$
$$\beta=0.9523,\psi=0.0938,p=1$$

Therefore, the right hand side becomes:

$$F+p\psi=\frac{1}{(1-\beta)^2}\log\theta+\frac{\beta}{(1-\beta)^2}\log\beta+\frac{1}{1-\beta}\log(1-\beta)+\frac{1}{1-\beta}p\psi$$
$$=\frac{1}{(1-0.9523)^2}\log 1.05+\frac{0.9523}{(1-0.9523)^2}\log 0.9523$$
$$+\frac{1}{1-0.9523}\log(1-0.9523)+\frac{1}{1-0.9523}*1*\psi$$
$$=\frac{1}{0.00275}(0.0488)+\frac{0.9523}{0.00275}(-0.0489)+\frac{1}{0.0477}(-3.04)+\frac{1}{0.0477}*1*(0.0938)$$
$$=17.745-16.93-63.73+1.966$$
$$=-60.949$$

For the left hand side, the results are shown by Table 2 and Table 3.

Table 2 The Change of Value As Risk Aversion Approaches One from the Right Side

ϒ	M	lim
1.2	42.0458	-105.2291
1.18	39.6014	-103.3410
1.16	37.2991	-101.8692
1.12	33.0883	-100.7358
1.08	29.3530	-104.4122
1.04	26.0394	-125.9854
1.02	24.5257	-176.2850
1.01	23.8022	-280.2174
1.001	23.1693	-2.1693e+003
1.0001	23.1069	-2.1069e+004
1.00001	23.1007	-2.1007e+005
1.000001	23.1001	-2.1001e+006

γ is the risk averse, we set it to be close to 1. M is solved by mat lab using the following equation:

$$\int_f^g \left[\left(\frac{1}{g-f} \right) \frac{\theta^{1-\gamma} \beta M H^{\alpha}}{\left[(\beta M)^{\frac{1}{\gamma}} + H^{\frac{\alpha}{\gamma}} \right]^{-\gamma}} \right] dH - M = 0 \tag{29}$$

“lim” means the limit for the following equation as γ approaches 1:

$$\lim_{\gamma \to 1} \left\{ M \frac{1}{1-\gamma} - \frac{1}{(1-\gamma)(1-\beta)} \right\}$$

In the program, we set $\theta = a = 1.05$, $\gamma = b = 1.1$, $p = c = 1$, $\beta = d = 1/1.05 = 0.9523$.

Table 2 shows that as γ approaches 1 from the right hand side, (γ become, 1.2, 1.18. 1.16 … 1.0 + 1e-6), *M* approaches 23. The left hand side (lim) of equation 28 is around -100 for $\gamma > 1.01$, however, as γ gets closer to 1 *(γ* < 1.01*),* a numerical problem occurs and the left hand side (lim) approaches -∞.

Table 3 The Change of Value as Risk Aversion Approaches One from the Right Side

ϒ	M	lim
0.8	12.6925	-41.5373
0.83	13.8852	-41.8515
0.86	15.1901	-41.4993
0.90	17.1227	-38.7730
0.92	18.1794	-35.2577
0.94	19.3013	-28.3117
0.96	20.4925	-12.6880
0.98	21.7572	37.8600
0.99	22.4185	141.8546
0.999	23.0309	2.0309e+003
0.9999	23.0931	2.0931e+004
0.99999	23.0993	2.0993e+005
0.999999	23.0999	2.0999e+006

Table 3 shows that as ϒ approaches 1 from the left hand side, (ϒ become, 0.8, 0.83, 0.86 …0.9-1e6), *M* approaches 23. The left hand side (lim) of equation 27 is around -41 for ϒ< 0.98, however, as ϒ gets closer to 1 (ϒ> 0.999), a numerical problem occurs and the left hand side (lim) approaches + œ.

8. Maximum Consumption/Investment and Health Status

I set heath status (h) between 1 (g = 1) and 1.2 (f = 1.2) and consider invested wealth (x) to be 1 and 2. Graph 1 show the relationship between maximum consumption and health when we set the invested wealth (x) to be 1. Maximum consumption is computed according to the following equation:

$$C_{MAX} = \frac{(\beta M)^{\frac{1}{\gamma}} \theta X}{(\beta M)^{-\frac{1}{\gamma}} + H^{-\frac{\alpha}{\gamma}}} \tag{30}$$

$\theta = a = 1.05$, $\gamma = b = 1.1$, $a = c = 1$, $\beta = d = 1/1.05 = 0.9523$. M is calculated by mat lab to be 31.1647.

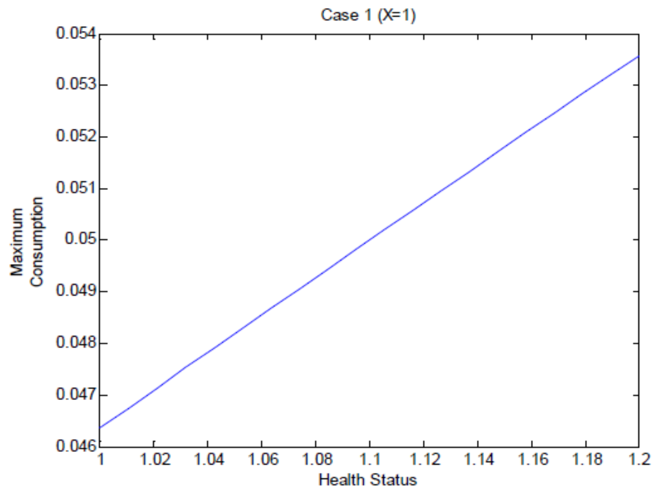

Figure 1 The Relationship between Consumption and Health Status

Figure 1 shows that that the maximum consumption and health status are linearly and positively related.

9. Conclusions

The paper obtained the analytical and numerical solutions for three health-consumption models with different utility functions. The first theoretical model proposed in my paper assumes power utility, IID stock returns, infinite horizons, no labor income and no explicit bequest motive. By numerically solving the first model, the results indicate that the discount rate and the risk-aversion are inversely related; the discount rate and people's risk-aversion are negatively related; the asset return and the risk aversion are positively related; and the health status and the risk aversion are positively related. The maximum expected value of health status doesn't change much with the parameters: it stays around 56. I also analytically solved another model with different utility function and proves that the results of the first model to prove the robustness of the result.

In addition, as risk aversion approaches one, the value function takes a third form and the paper shows that as risk aversion approaches 1 from the right hand side (1.2, 1.18, 1.16, ….), the expected value of health will approach 23. However, as risk aversion get closer to 1 (less than1.01), a numerical problem occurs and there is no numerical solution for the value function. Similarly, as risk aversion approaches 1 from the left hand side (0.8, 0.86, 0.9….), the expected value of health will approach 27. However, as risk aversion get closer to 1 (greater than 0.999), a numerical problem occurs and there is no numerical solution for the value function. Finally, the paper shows the relationship between the maximum consumption and health status to be linearly and positively related as suggested by the empirical literature.

The model is the first attempt to explain the elders' investment and consumption behavior by setting up a theoretical model that includes the health condition into the utility function and the health expenditure in the constraint to analyze the elder people's portfolio choice in a rigorous way. The author analytically and numerically solved the model and the results complies with the empirical literature by suggesting the elders' maximum consumption/ investment behavior is positively and linearly related with their health conditions.

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