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Application of SABR Model to Strategy of Accumulation of Reserves of Central Bank in México

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Abstract: The purpose of this paper is to review and to calibrate the SABR volatility model in order to estimate the volatility of foreign currency (US dollar) in the case of Mexican market and applied the model to the valuation of the option premium (O_c) specifically for the case of international reserves accumulation of the Central Bank on Mexico. It was found that the estimation of volatility and the premium O_c when the historical and implied volatilities are used have different estimation respect the SABR volatility case, principally when forward price is not the same from strike price.

Key words: SABR; volatility; international reserves; differential geometry
JEL codes: C61, G10, G12

1. Introduction

Since Black-Scholes original paper (1973) plain vanilla options have been valued using market parameters and financial assumptions. However later it was discovered that Black-Scholes model estimated the same volatility for different strike prices, a fact not observed in the real world, in other words at different strike prices produce different volatilities, this behavior is observed in the market and it is known as skew or volatility smile.

Market volatilities smiles and skews[1] are usually managed by using local volatility models. It was discovered that the dynamics of the market smile predicted by local volatility models is opposite of observed market behavior. When the price of the underlying decreases, local volatility models predict that the smile shifts to higher prices; when the price increases, these models predict that the smile shifts to lower prices. Due to this contradiction between model and market, hedges derived from the model can be unstable and may perform worse than Black-Scholes hedges.

The concept of local volatility was developed simultaneously by Dupire (1994) and Derman and Kani (1994). This idea represents the greatest advance in the understanding and calibration of the smile and volatility skew. In another direction, an important approximation of stochastic volatility was proposed by Hull and White (1987) and other model was the volatility model proposed by Heston and Loewentein W. (1993). Finally it is specially important for this work the contribution of Hagan, Kumar, Lesniewski and Woodward (2002) on SABR model and of course mention the version of SABR of Labordere (2009).

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[1] The smile of volatility of foreign currency and the skew of volatility of equities could be explained by the difference between real and lognormal theoretical distribution.

The local volatility model is consistent with smiles and volatility skews and also free arbitrage. However it has been observed in the market, according to Hagan Lesniewski and Woodward (2002), that dynamic behavior of volatility smiles and skews predicted by the local volatility model are opposite to the real observed behavior, SABR model volatility try to correct that fails of Hagan Model.

The most outstanding consequence of this result on the local volatility is that in the Hagan model (2002) the hedging often is worse than the Black-Scholes model hedge as they are actually inconsistent with the movement of the smile of the market, so that the SABR model is considered better model that the local volatility. In the original work of Hagan Lesniewski and Woodward (2002) and Lieke de Jong (2010) used perturbation theory to evaluate the price of SABR model and its associated implied volatilities.

The main themes discussed in the following sections are: In section 2 we present the introduction of volatility models and the most important results of the SABR model. In section 3, the paper made a review of the issue of international reserves in the case of Mexico. In section 4 this paper proposed a review of the purchase of foreign exchange model using put options (Banxico O_c option) and its adjustment in volatility with the model SABR, at the end there are the results and conclusions of this work.

2. SABR Model to Strategies of Accumulation of Reserves of Central Bank in México

2.1 Antecedents (Black-Scholes and Volatility)

One option is a financial contract that giving to the holder the right, but not the obligation to buy or sell a certain asset at an agreed price at some future time. In particular, a European (an option that could be exercised only one time) call option with strike price K and maturity T on an underlying S is a contract that giving the right to buy or sell the underlying asset at a price K at time T.

The Black-Scholes equation was published 1973 and was derived by Fischer Black and Myron Scholes (and simultaneously Merton). This equation assumes that the underlying behavior associated with the derivative term can be modeled by a stochastic brownian motion and its derivation makes use of at least two basic concepts of finance: the hedging and arbitrage.

For the derivation of the Black-Scholes model is considered the following assumptions: there is a risk-free rate and variance (both constants), the market is liquid for the asset underlying and the derivative, there are no transaction costs, it can be lent and borrowed at the same rate, no dividends are paid and there are no arbitrage opportunities.

Volatility is a measure of dispersion of information about the mean plus a measure of uncertainty in returns and typically is related to the standard deviation of an asset. The volatility varies over time and high volatility persists for long periods before reaching a long-run equilibrium, an effect known as clustering. Moreover, the volatility increases more than proportionally when yields increase when lower yields and this property is known as leverage. There are different ways of modeling the volatility as: parametric estimation, the historical moving average, using time series (ARMA, GARCH), the stochastic process with Brownian motions and implied and local volatilities.

For the parametric method volatility is a parameter that not changes over time and maintains the same value for the entire sample of size n corresponds to the sample variance or standard deviation. The main disadvantage of this model is that the volatility forecast is based in information of the past.

The moving average historical volatility opens a window of size n. Unlike the previous case, volatility is not a parameter but a process that evolves over time. By using the moving average of a sample of size n for each variance estimate is added a new observation at the end of the series and simultaneously eliminates the former. Some of the disadvantages of this method is its sensitivity to the number of observations moving average or the size of the window and on the other hand the weight to each observation receives is the same regardless of whether the information can be recent or distant.

The ARMA model (an autoregressive and Moving Average) considers that the average yield has a value of zero and models the variance of returns or squared returns using a linear regression with the square of the returns from previous periods. This model can be thought of as a generalization of the previous case where the coefficients or weights are determined through a process of regression. This methodology has the advantage of being able to make predictions of the intertemporal structure of volatility and reduce the clustering effect estimate and exploit the leverage effect.

The objective of the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) is to estimate the conditional variance of returns on financial assets. The simplest model GARCH (1.1) by linear regression assumes that the conditional variance as a function of a term independent of the prior period error and variance of the previous period corresponding to the autoregressive term. To be stationary in model requires that the estimators or regression coefficients are positive (including the constant term) and the sum of the first two is less than or equal to one. To estimate the variance of the mean is required which in turn is also estimated using a regression where the current period is explained by the performance of the previous period plus a random term. The main advantage of this model is to allow forecast volatility in any future period and thus build the term structure of volatility.

The stochastic volatility model proposed a Brownian geometric motion, the behavior of the variance consists of a drift term and a stochastic term. This model uses the Ito calculus in the derivation.

Several of the above models are based on historical information and in general in the predictions do not incorporate structural changes, extreme events o are far from reality. To correct this you must include the information in implied volatility in the price of options.

The model implied volatility takes prices of options contracts traded on the market and calculate the volatility on the market. This model uses the option pricing Black-Scholes method and numerical approaches (such as Newton or Newton-Raphson) to estimate the volatility considering the knowledge of the remaining parameters. According to this model from the price of European options contracts (call or put) the underlying asset price, the expiration of the option and interest rate risk-free estimated volatility for each point in time.

2.2 SABR Volatility Model

The SABR Model (by abbreviations of stochastic the parameters alpha beta, rho) proposes a forward rate (f) (which may be a forward swap rate, a forward exchange rate, forward actions price etc.) and its volatility described by a parameter α, both are described by two stochastic processes.

It is important to remember the following observations about the SABR model: (1) forward rate and their volatility are martingale (2) all the parameters of the model ν, β, ρ are constant (3) each forward rate lives in its own measure and does not know anything about the other forward rates. The forward rate and it volatility are described by the following stochastic equations:

$$df = \alpha f^{\beta} dW_1 \tag{1}$$

$$f(0) = f_0$$

$$d\alpha = \upsilon \alpha W_2 \tag{2}$$

Where dW_1, dW_2 are Brownian motions, f is the forward rate of some underlying, α is the volatility, and ν is the volatility of the volatility.

$$dW_1dW_2=\rho_{12}dt, \ -1 < \beta < 1$$

the value of European call and put options are given by the formula of Black, in the case that the underlying are futures assets:

$$C = e^{-r(T)}\big(fN(d_1) - KfN(d_2)\big) \tag{3}$$

$$P = C + e^{-r(T-t)}(K - f) \tag{4}$$

with

$$d_{1,2} = \frac{\log\left(\frac{f}{K}\right) \pm \frac{1}{2}\sigma_B^2 T}{\sigma_{B\sqrt{T}}} \tag{5}$$

Where T is the expiration date, K is the strike price, f is the forward rate, C and P are the call and put options premium. The SABR volatility $\sigma_B(f, K)$ is taken from Labordere (2009), it should be mentioned that the deduction from work is based in Heat Kernel methodology and differential geometry,

$$\sigma_B(f,K) = \alpha(fK)^{(\beta-1)/2}\left\{1 + \frac{(1-\beta)^2}{24}log^2\left(\frac{f}{K}\right) + \frac{(1-\beta)^4}{1920}log^4\left(\frac{f}{K}\right) + \cdots\right\}^{-1}\left(\frac{z}{x(z)}\right)$$

$$\left\{1 + \left[\frac{(1-\beta)^2}{24}\frac{\alpha^2}{(fK)^{1-\beta}} + \frac{1}{4}\frac{\rho\beta\alpha\upsilon}{(fK)^{1-\beta/2}} + \frac{2-3\rho^2}{24}\upsilon^2\right]T + \cdots\right\} \tag{6}$$

Where

$$z = \frac{\nu}{\alpha}(fK)^{(1-\beta)/2}log\left(\frac{f}{K}\right) \tag{7}$$

and x(z) is defined by

$$x(z) = log\left\{\frac{\sqrt{1-2\rho z+z^2}+z-\rho}{1-\rho}\right\} \tag{8}$$

For special case when the financial options are in money, i.e., if K = f the expression is reduced to

$$\sigma_{ATM} = \sigma_B(f,f) = \alpha/(f)^{(1-\beta)}\left\{1 + \left[\frac{(1-\beta)^2}{24}\frac{\alpha^2}{f^{2-2\beta}} + \frac{1}{4}\frac{\rho\beta\alpha\nu}{f^{1-\beta}} + \frac{2-3\rho^2}{24}\nu^2\right]T + \cdots\right\} \tag{9}$$

and for the gaussian case ($\beta = 0$) the volatility becomes:

$$\sigma(K) = \alpha\frac{lnf/K}{f-K}\left(\frac{z}{\chi(z)}\right)\left\{1 + \left[\frac{\alpha^2}{24fK} + \frac{2-3\rho^2}{24}\nu^2\right]T\right\} \tag{10}$$

$$z = \frac{\nu}{\alpha}\sqrt{fK}ln\frac{f}{K} \tag{11}$$

$$\sigma(K) = \alpha\left(\frac{z}{\chi(z)}\right)\ \{1 + [\frac{\rho\alpha\nu}{4} + \frac{2-3\rho^2}{24}]\nu^2\{1 + [\rho]T\} \quad (12)$$

$$z = \frac{\nu}{\alpha}ln\frac{f}{K} \tag{13}$$

Briefly it is possible to say about parameters and the qualitative behavior of the model, according Labordere (2009):

(1) If the initial volatility increases, σ_0^T moves the smile curve towards the top

(2)The β exponent has three effects on the smile curve: a) the first one is a progressive lifting of the smile curve as β ranges from 1 to 0 b) the second one is a reduction in the level of the smile as β increases c) the third is the introduction of curvature to the smile as β goes from 1 to 0

(3)If ρ moves from 0 to -0.5 the smile is becoming more negative in its slope

(4)The increase of υ increases the curvature of the smile

2.3 International Reserves in México

International reserves are biggest assets of general balance from the Central Bank of México (Banco de

México or Banxico[2]. The objective of accumulation of reserves is to contribute to the stability of the national currency purchasing power through the compensation of imbalances of the balance of payments. The reserves are used as a kind of insurance to deal with contingencies that could damage trade flows or the balance of payments generated by macroeconomic and financial imbalances. The reserves are financial assets that the central bank invests abroad from Mexico and that can be easily converted into means of payment in other words whose main characteristic is the liquidity. A situation of imbalance in the balance of payments requires that assets are easily convertible in means of payment to pay off the obligations in foreign currency.

International reserves are invested in financial instruments that have with certain attributes in the case of Mexico[3] which stipulates that reserves are made up of foreign currency, gold and must be free of any tax, have total availability, it is in addition to assets that represent obligations of entities not residing in Mexico, as deposits in financial institutions from abroad and they must be called in freely convertible currencies.

The same law of Banxico establishes that international reserves in Mexico are only: banknotes and foreign coins, deposits, titles and values payable outside the national territory, considered first-order in the international markets, with a relatively high credit rating agencies and liquidity, credit of central bank not more than six months which this service current and finally special drawing rights of the International Monetary Fund (IMF).

Mexican Oil (Petróleos mexicanos or PEMEX) is the main provider of foreign currency. Pemex sold dollars directly to Banxico proceeds from the sale of oil and it is the main source of foreign currency, which makes up the international reserve. Also occasionally as result of the external debt, the federal Government also sells dollars to the Bank of Mexico and finally another important source of foreign currency comes from the policy of accumulation of international reserves, which in his case, determines the Commission of changes when instructed the central bank to buy foreign exchange in the market.

The appropriate level of international reserves is not a fixed value in time but it depends on the situation and the time for each country and the amount depends is a tradeoff between cost and benefit.

In the case of high levels of reserves, the benefits are among others: the perception of the strength of the currency of a country, the strength of its economy for its obligations with the rest of the world that improves the perception of the risk country, lowers the cost of external financing and reduces the vulnerability to exogenous shocks.

The costs associated with maintaining international reserves, is the maintain a reserve inverted in extremely liquid financial instruments (such as notes of treasury of United States notes) instead of investing in instruments afforded greater performance as long-term instruments.

It is necessary to mention that an excessive accumulation of reserves is funded with liabilities issued by the Bank of Mexico. If the local rate that the Central Bank must pay for these liabilities is greater than the foreign rate that receives of average performance of the reserve assets, generates a cost for the central bank. Reserves represent a long position in foreign currency leading to major losses or gains of capital.

It is necessary to say that almost all central banks of the world, including of Mexico do not follow any theoretical model for determining reserves, and do not have any particular indicator in particular. If do not primarily based on the analysis cost benefit.

Since the crisis of the nineties (1994-1995) the balance of reserves in the Bank of Mexico has been increased considerably and its evolution is linked to the exchange-rate regime. With a fixed exchange rate, the Central Bank

[2] According to article 18 of the law on the same central Institute.

[3] Reserves must comply with the provisions of article 19 of the law of the Bank of Mexico.

intervened in the market to keep the pegs of the exchange rate and with the flexible exchange rate the international reserve attenuates the economy's vulnerability to external shocks.

The flexible exchange rate currency regime has coexisted in different stages in the policy of accumulation of international reserves, which have detonated several mechanisms of operation of the Mexico Bank in the foreign exchange market.

The experience of Mexico with a floating exchange rate regime has been success and this international reserves accumulation policy has been designed to influence the regime of free float as little as possible. The central bank has implemented various measures to lessen the impact on the Mexican economy, in particular in October 2008 began to sell foreign currency with the purpose of providing liquidity to the market and to cope with the volatility of the exchange rate. The evolution of international reserves of Banxico appear in Figure 1 at beginning of 1996 international reserves was less 20,000 millions of US dollars and sixteen years after the accumulation of reserves are close to160,000 millions of US dollars. It is a notorious increment in international reserves.

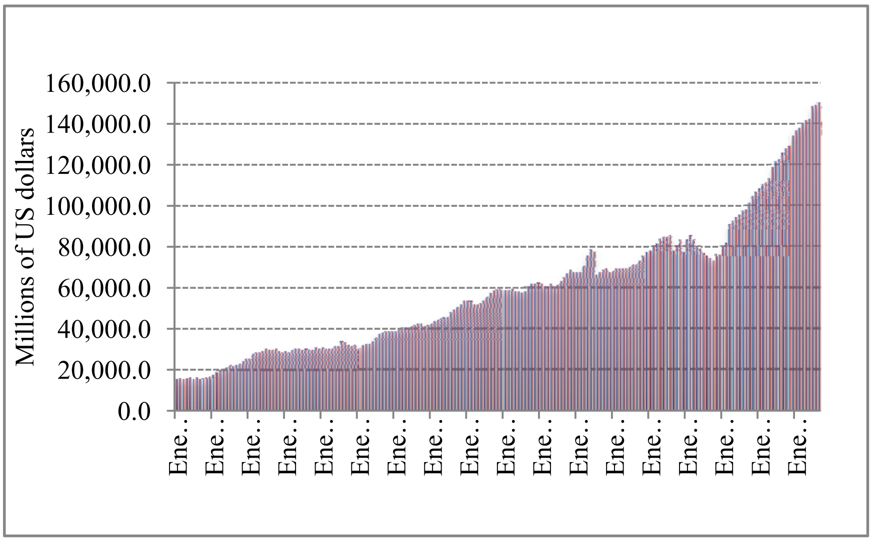

Figure 1 Accumulation of Reserves of Central Bank

2.4 Valuation of O_c Options in the Strategy of Accumulation of Reserves of the Bank of México

In 1996 Bank of México proposed the possibility of buying dollars on the market, taking care to not send wrong signals to the market. Although it is advisable to have an enough amount of reserves, the central bank wanted a scheme that encourages the purchase of dollars when the market is offered and inhibits in the opposite case in order to affect the free float regime as little as possible. In August of the same year, the Banxico issued a statement to financial institutions and credit of Mexico where invited through to the payment of a premium (by means of a put option) to sell dollars to the central Institute. it should be noted that these options have characteristics somewhat different from the plain vanilla put options.

The option proposed by the Bank (O_c) can be considered at the same time like a portfolio of some options with a day of maturity, but which may be exercised only once. However there is a risk to accumulate reserves in this way is when the exchange rate shows a trend of depreciation if foreign currency from one day to another, it would be optimal for the holders of the put options exercise at that time then the Bank accumulate reserves through the purchase of currency on the market and at the end would have an effect of devaluation pressure. In order to avoid this behavior the central bank propose that the exercise of the option has made one conditional to the type of this change at the bottom of a certain level, in such a way that the single put option can be exercised if and only the strike price of the exchange is not higher than the arithmetic average of the price of 20 working days before of fix exchange rates.

In Figure 2 the graph shows the contrast between the spot of foreign US dollar/peso[4] and by the other hand the amount of put options exercised, in almost all cases when the exchange rate had an appreciation tendency.

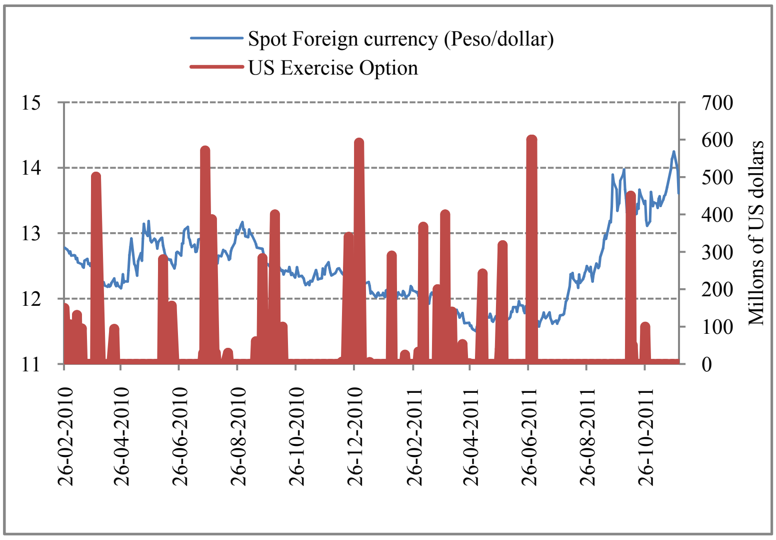

Figure 2 Spot Exchange Rate Peso/US dollar and Exercise of Options O_c

Galan, Duclaud and García (1996) proposed an approximation to the value of the put option for the sale of dollars to the Bank of Mexico considering the history of the fix peso/dollar exchange rate in order to estimate the volatility. As mentioned previously the choice of Bank Mexico can be seen as a portfolio of European put options "at the money", maturing to a day with exercise price determined by the survey of Bank of Mexico from the previous day, once option is exercised the remaining options are lost. In addition it should be remembered that the option only may be exercised when the strike is equal to or less than the arithmetic average of the survey of the average of the 20 days prior to the date of exercise. In summary, to determine the value of the portfolio of options put in it is necessary shall consider the following factors:

(1) put Option value "at the money"

(2) probability to comply with the restriction of the average of the "n = 20" days;

(3) probability to exercise the put option only on a particular day

$$O_C = \sum_{t=1}^{n} desc * Put(At\ money)^{BS} * Prob(NoRestricción) * W(ejercicio\ in\ t) \tag{14a}$$

or

$$O_C = \sum_{t=1}^{n} e^{-r(T-1)} * Put(At\ money)^{BS} * N(d_1) * N(C)(1-N(C))^{t-1} \tag{14b}$$

Where each one of the options are joined in the portfolio and O_c is a product of (1) a discount factor, (2) the value of the modified Black-Scholes formula "at the money" that mix up the plain vanilla put option with version Garman M. and S. Kohlhagen for currencies and forwards, (3) the term of the probability of exercise in a given day and finally (4) the probability of meet the restriction of the option.

With the formula Galan, Duclaud and García (1996) on the condition that the value of the fix exchange rate is lower than the average of the 20 days prior.

[4] In all paper pesos refer to Mexican pesos.

$$Prob(S_t \le Y_t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d_1} e^{-\frac{x^2}{2}} d_2 = N(d_1) \tag{15a}$$

with

$$d_1 = \frac{Y_1^* + \left(\frac{t-1}{n} - 1\right)S_0^* - \frac{1}{n}\sum_{i=1}^{t-1} S_{-n+i}^* + \mu t\left(\frac{t-1}{2n} - 1\right)}{\sigma_{z(t)}} \tag{15b}$$

Where

e_t : the exchange rate (fix) determined by the Bank of Mexico day t

S_t : the natural logarithm of e_t with $S_t = \mu + S_{t-1} + \varepsilon_t$ y $\varepsilon \sim N(0, \sigma_\varepsilon^2)$

Y_t : the moving average of n observations prior to S_t (n =20 days)

And with Z_t one auxiliary random variable with the following feature

$$Z_t = \sum_{i=1}^{t}\left(1 - \frac{t-i}{n}\right)\varepsilon_i \tag{16a}$$

$$Z_t \simeq N(0, \sigma_{z(t)}^2) \tag{16b}$$

and

$$Var(Z_t) = \sigma_\varepsilon^2 \frac{t}{n^2}\left(\frac{(t+1)(2t+1)}{6} + (n-t)(n+1)\right) = \sigma_{z(t)}^2 \tag{17}$$

On the other hand, also of Galan, Duclaud and García (1996), W(t) is the probability of exercise in one day t, once it has been exercised the option eliminates the possibility of subsequent options. It is noted that there is incentive to exercise the options as quickly as possible, once satisfied the above mentioned restriction and utility exceeds the value of the option premium, this will be exercised.

$$W(t) = prob(ejercicio = t) = N(C)(1 - N(c))^{t-1} \tag{18a}$$

$$N(C) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{C} e^{-\frac{x^2}{2}} d_2 = N(d_1) \tag{18b}$$

$$C = -\frac{(\mu + O_c)}{\sigma_\varepsilon} \tag{18c}$$

For equations (14a and 14b) it assumed that the portfolio of the options are of type put "at the money", with maturity of one day with the parameters of the exchange rate.

An adjustment is proposed in the previous model to the premium O_c of the portfolio of the Bank Mexico O_c, instead considering the historical volatility or implied volatility of Black-Scholes model.

We used in the model SABR volatility.

3. Results

In the first part of this section there are the results of the calibration model and which is the best estimation of the parameters, in the second part there is a comparative for the premium option O_c using historical, implied and SABR volatilities, their sensibilities (delta and vega Greeks)[5] and finally the differences for the total premium are presented in the cases when strike prices and forward prices are equal and the case when are not the same.

The data for the estimation of SABR volatility are taken from derivatives Mexican market (Mercado Mexicano de Derivados or Mexder[6]) and with complementary information of the other financial variables on the mexican market on one day (August 26, 2011). The parameters of volatility, rho, and niu to calibrate the model and the volatility obtained is compared with the implied volatility that is taken of real information (see in the Figures 3a, 3b and 3c), the best parameters of the calibration are shown in Table 1 and Figure 4:

5 Sensibility of option O_c to underlying and the volatility respectively.

6 http://www.mexder.com.mx/MEX/Boletin_Diario.html.

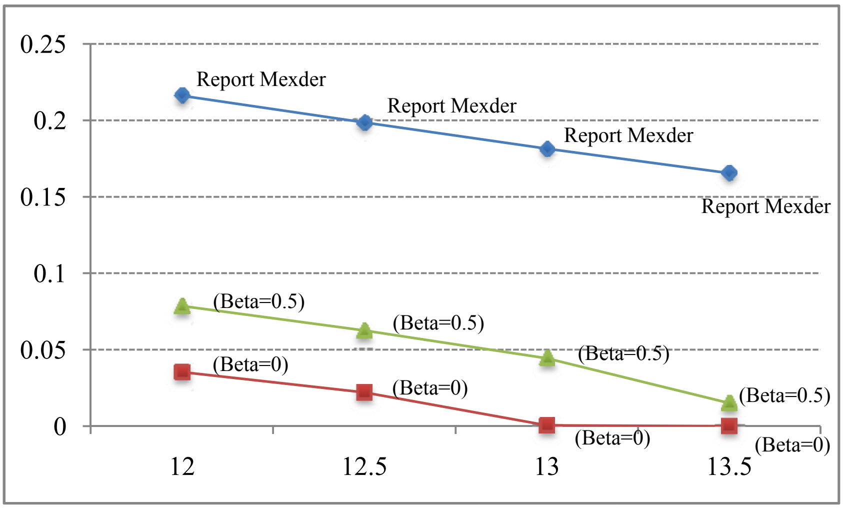

Figure 3a Calibration of SABR Volatility Different Beta Parameter

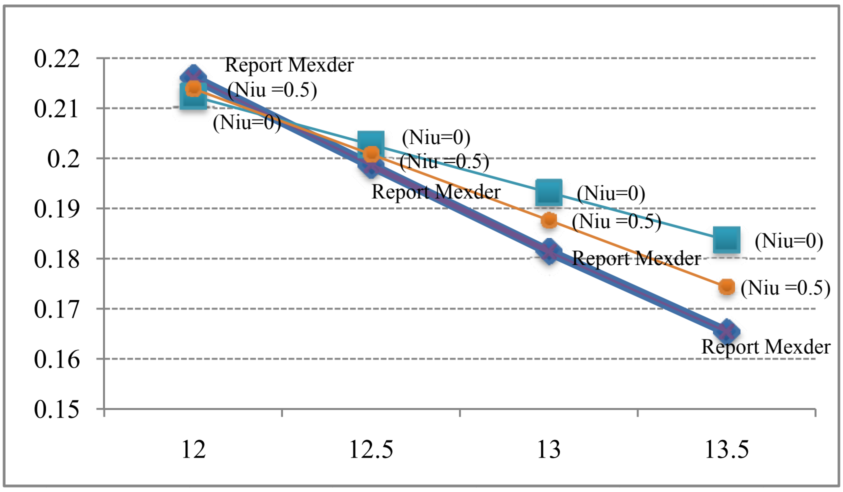

Figure 3b Calibration of SABR Volatility Different Niu Parameter

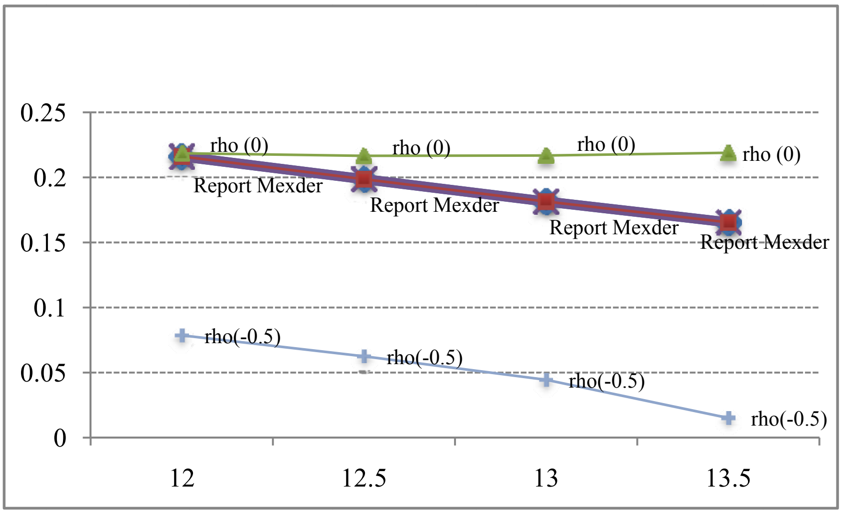

Figure 3c Calibration of SABR Volatility Different Rho Parameter

Table 1 Information of Parameters for Calibration

Parameters				
K (Strike Price)	12	12.5	13	13.5
F(Forward rate)	12.7	12.7	12.7	12.7
Beta	1	1	1	1
Sigma(I. Vol.)	0.205	0.205	0.205	0.205
Rho	-1	-1	-1	-1
Niu	0.9	0.9	0.9	0.9
T(Maturity)	0.8194	0.8194	0.8194	0.8194

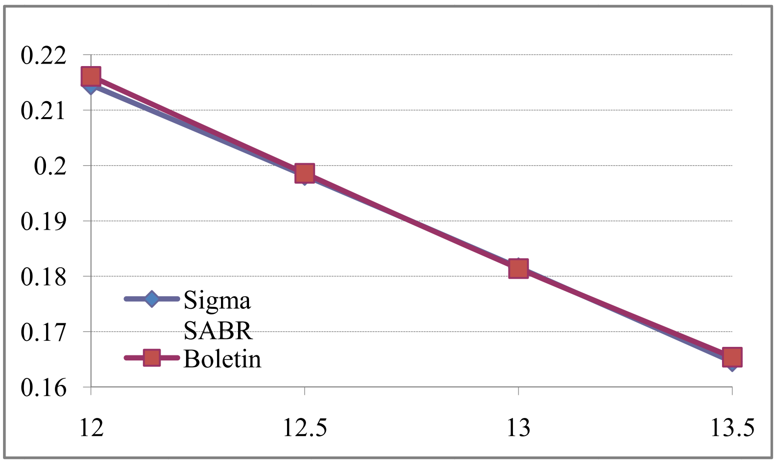

Figure 4 Implied Volatility vs SABR Volatility the Best Calibration

Now applying the SABR model volatility with information of the Mexican market of the day (26 August 2011) to estimate O_c that is one portfolio of options put to 20 consecutive days with maturities of one day for each of them. Equations (18a, 18b and 18c) estimate the probability of exercise an particular day and the condition to exercise the exchange rate peso/dollar is lower than the average of 20 days before to perform (15a and 15b). In order to solve solution equation and find O_c (14b, 15a, 15b, 16a, 16b, 17a, 18a, 18b, 18c) it is necessary to use a numerical methods because of the final equation is implied. Now it is propose that the O_c premium, rather than deal with the historical volatility dealt with the estimated SABR model and comparative of the results obtained is shown Figure 5.

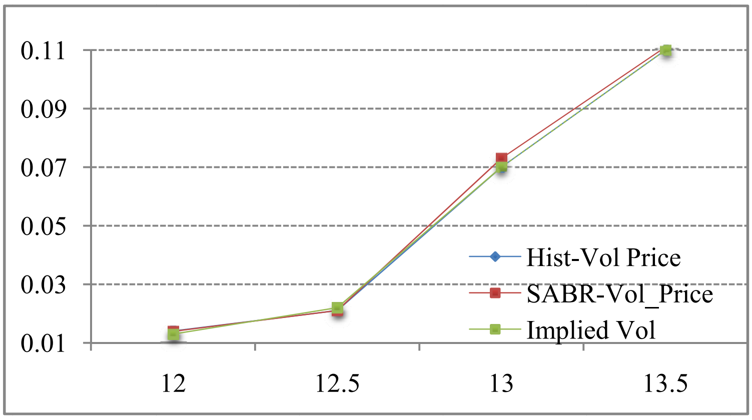

Figure 5 Comparative of Premium O_c Banxico Option

The sensibility of option O_c respect to change of underlying (foreign currency) and respect to volatility are shown on 6a and 6b. The maximum sensibility of the value O_c option respect to change of the dollar (Delta greek[7]) is when ST = K = 12.5 peso/US dollar and the sensibility of O_c respect to change volatility (vega greek) is decrease as forward rate increase.

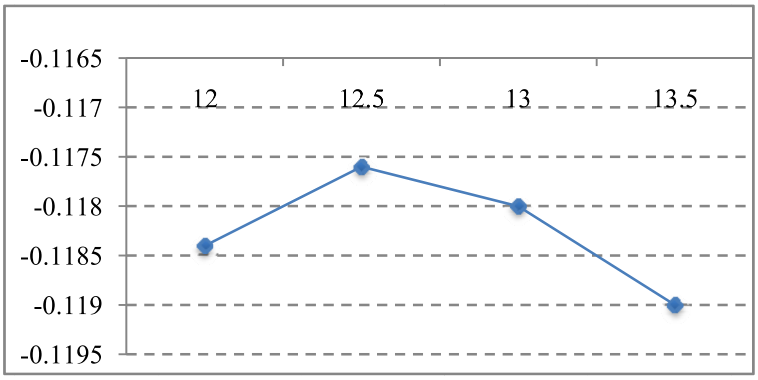

Figure 6a Delta Sensibility of O_c Option

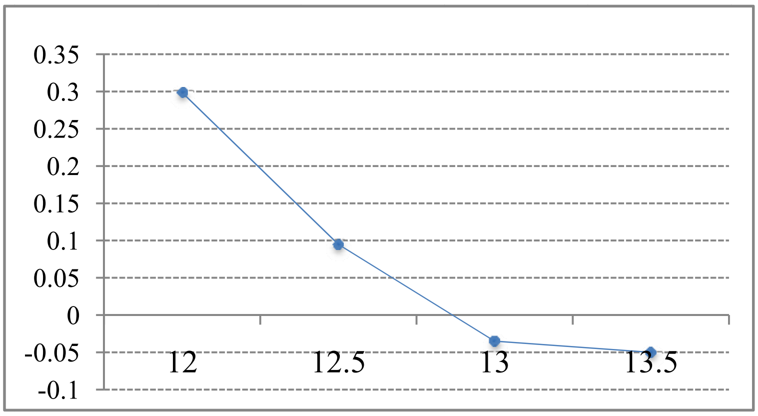

Figure 6b Vega Sensibility of O_c Option

By another way when the forward price of foreign currency coincides with exercise price on 12.5 pesos/US dollar the difference is minimal 0.001 pesos for the premium O_c option, but another case when the exercise price of 13 pesos/US dollar, the difference between Black-Scholes and SABR option value is from 0.003. For an offer auction for an amount of $600 million (the nearest day was July 1 of 2011), would represent a difference in income from the premium option by 600,000 or 1,8000,000 pesos respectively.

However if the difference between the strike price and the forward price of the US dollar increase (or decrease) every day one percent until the last one day (day 20) the difference among in the valuation for the O_c premium become more important and particularly for high values of the forward rate. In Figures 7a and 7b the graphs show the difference in the estimation of O_c premium value and in amount for one dollars auction of the option in the case of increase the forward rate and in the Figures 8a and 8b the correspond case for decrease the forward rate. In both cases the premium O_c almost stay constant while the premium O_c obtained from the estimation historical, implied and SABR (case F = K) increase with strike price. Moreover the Figures 7b and 8b show that the total amount paid for the holder's option and received as premium for the Banxico could become

[7] Sensibilities in financial option world are known like Greeks.

important in the case where strike and forward price are different.

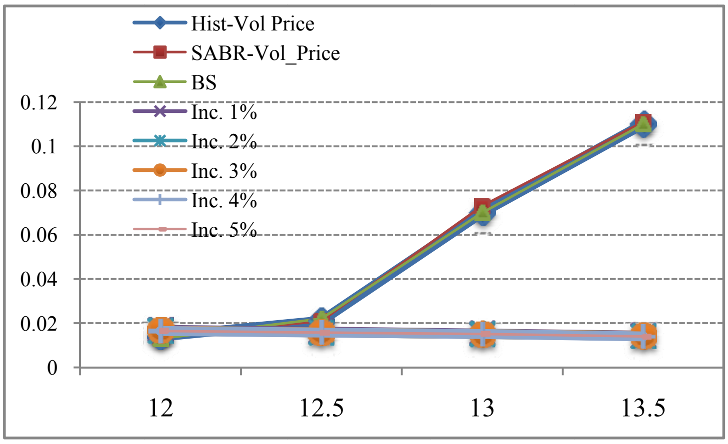

Figure 7a Premium Option O_c When the Difference of Strike Price and Forward Strike Increase 1% Each Day

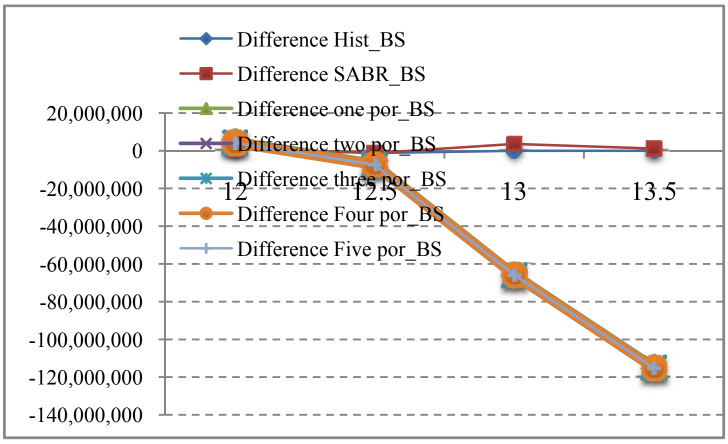

Figure 7b Considering the Total Amount Received by Premium O_c and One Auction for US 1,200 Millons (Increase 1%)

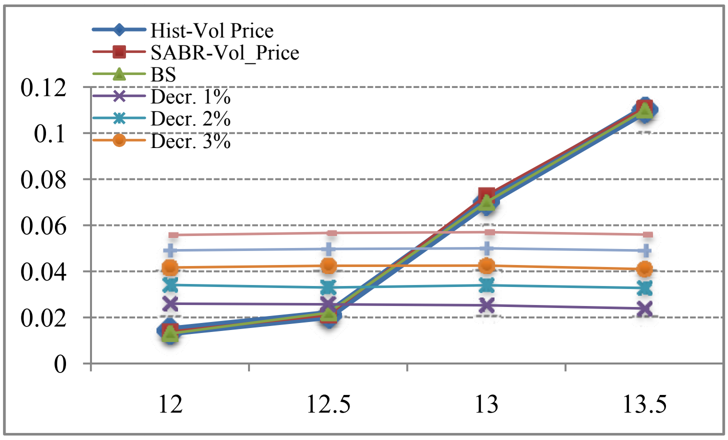

Figure 8a Premium Option O_c when the Difference of Strike Price and Forward Strike Decrease 1% Each Day

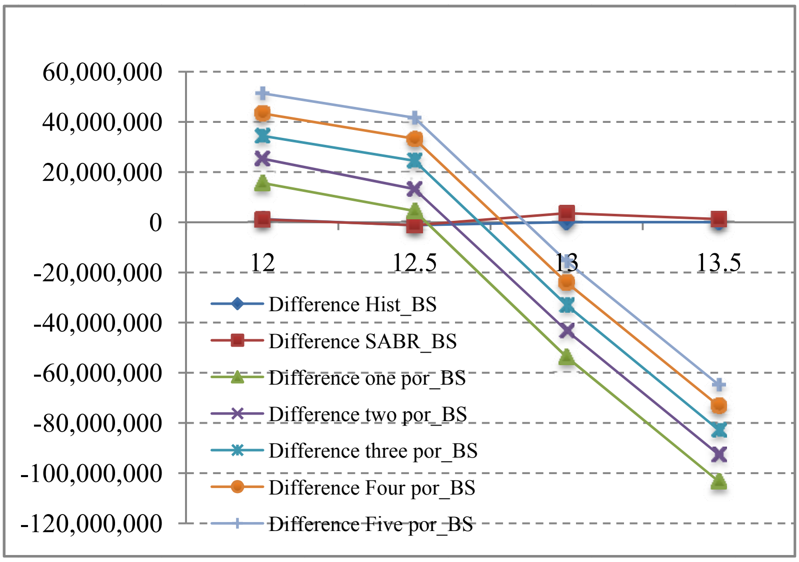

Figure 8b Considering the Total Amount Received by Premium O_c and One Auction for US 1,200 Millons (Decrease 1%)

4. Conclusions

It is important to mention that the volatility function and premium of option of to the SABR model obtained with Differential Geometry is the same that original proposed of Hagan using perturbation theory.

In the present work is presented the calibration of the parameters of volatility SABR for the case of foreign currency (US dollar) for the SABR volatility model.

There almost is not difference of the premium of Banxico Option O_c "At the money" when the volatility used is Implied, Black-Scholes or SABR.

However the difference O_c in is important in all cases different of "at the money" especially for high values of futures of foreign currency dollar.

The total premium received for the issue of put options should be much less if the future of the foreign currency is different from the strike price.

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