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Modeling of Mobile Sage and Graphing Calculator

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Abstract: Over the last 20 years, our learning environment for mathematics has changed dramatically, and mathematical tools now have an important role in our classrooms. Sage is the popular mathematical software which was released in 2005. This software has efficient features to adapt to the internet, and it can describe most mathematical problems: for example, problems within algebra, calculus, linear algebra, combinatorics and numerical mathematics. Nowadays, there are more mobile devices than personal computers all over the world. Furthermore, the most sophisticated Smartphones have almost the same processing power as personal computers and can connect to the internet easily. For example, we can connect from a mobile phone to any Sage server through the internet. We have developed our mobile infrastructure of Sage in the mobile-learning environment and have utilized it in our teaching of mathematics with Smartphones (Ko et al., 2009; Lee & Kim, 2009; Lee et al., 2001). In this paper, we introduce how to develop our Mobile Sage environment and features of Sage Graphers that have been developed using it (http://matrix.skku.ac.kr/mobile-sage-g/sage-grapher.html).

Key words: sage, mobile sage, mobile mathematics, graphing calculator, visualization

1. Introduction

After the “e-Campus Vision 2007 (2003–2007)” in Korea, students can review not only text based lectures but also real lectures with sound and movie clips; they can even enjoy these activities presented during class time from a remote location (MOE-Korea, 2002; Lee & Ham, 2005). All of our lectures are saved and can be reviewed right after. Also, students can use all lecture clips and web contents with CAS (Computer Algebra System) tools to study for themselves without teachers. We have realized all of these outputs can be used in a mobile environment.

The information and communications technology (ICT) has been the leading source of innovation in education. After we discovered some possibilities of this mobile tool for mathematics education, we started to develop CAS tools, create lecture clips, and design our lectures for the mobile environment in 2009. The Korean Minister of Education announced “Strategies for Advancement of Mathematics Education” in January, 2012. They had a declaration ceremony, “The Year of Mathematics Education: 2012” in April 17th, 2012. According to their report, using CAS tools at all levels of mathematics education in Korea will be visible for the first time.

For over 20 years, the issue of using adequate CAS tools in the instruction of mathematics has been raised continuously (Shiskowski & Frinkle, 2010; Butt, 2011). A variety of CAS tools have been introduced in several

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mathematics textbooks. However, due to problems in adopting those tools, none of them have been readily utilized in the classroom, and the theoretical aspects of mathematics have been focused on heavily in teaching and learning. We have, hence, tried to find or make a simple and comprehensive method for using ICT in our classes.

Due to the dramatic changes in the educational environment, we were aware that CAS tools and internet resources would be incorporated into classrooms sooner or later. Most countries with fairly advanced information technology have been interested in the applications of ICT in their respective educational environments, and Korea is no exception. Most Korean teachers wanted a comprehensive, uncomplicated and low cost, learning aid in their classes using CAS and ICT. We had to keep those issues in mind. Korean students are not well-trained with hand calculators or graphing tools, but they are now prepared for internet-based computational tools.

Students can get a lot of educational resources through the internet, and most instructors now understand this situation and make a constant effort to provide good educational content to them. Instructors try to develop content and tools that show a lot of relevant visualizations in class.

However, until recently there was no reasonable CAS tool for Korean. So, most secondary students depended on foreign softwares, such as Mathematica, MATLAB, Maple, GSP, etc. They also faced the language barrier by using these softwares. Additionally, the high cost of these softwares was also an obstacle.

2. Localization of Sage

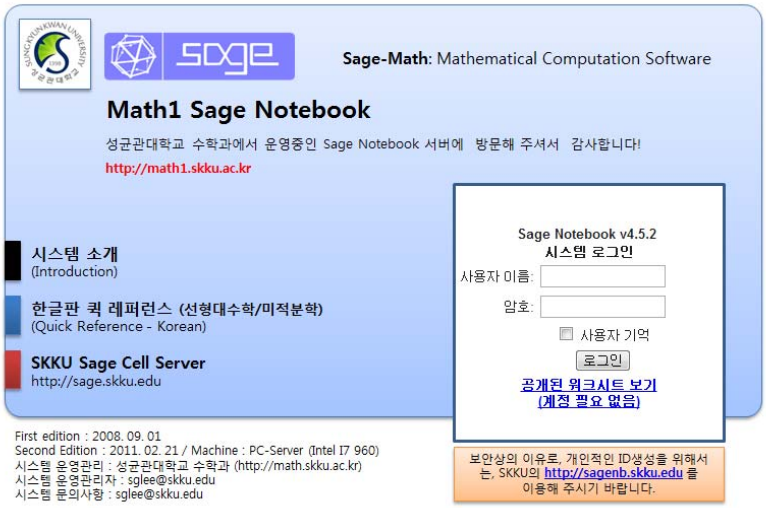
Now, Korea has a decent internet infrastructure, but there have been virtually no reasonable CAS tools for our mathematics classrooms. Sage (Software for Algebra and Geometry Experimentation: free open source software to do mathematical computations) was developed by William Stein with a development group at the University of Washington and mathematicians from around the world (Stein & Joyner, 2005). It has a similar grammar system to commercial softwares, and it follows the GPL (GNU Product License) and is therefore free. Sage has been revised by many developers after the National Science Foundation grants for William Stein. It has been released on its website: http://www.sagemath.org. Furthermore, Sage has a client-server model which is well-adapted to the internet.

Since its release, Sage has shown its versatility over other mathematical programming system. Sage has compatibility with Mathematica, Matlab, and Maple commands[1]. The reform of the educational environment and the development of educational tools and content has been a hot issue these days. In this reform, we had to develop useful tools which most students could utilize in their future careers.

We found that Sage could be an excellent solution for our needs, so a lot of effort has been made at our “Brain Korea 21 Mathematical Modelling Human Resources Development Division” regarding Sage. We found that Sage could be used within the framework of mathematics teaching to further students’ understanding.

Now, instructors can show and compute whatever they wish to show in class. Many students can see and manipulate visualized outputs. It also helps students to understand more complicated mathematical concepts. Students may install the Sage software on their laptop if they have an access to internet. A snapshot of the website is provided in Figure 1. In order to use the full features of Sage, Google Chrome or Safari browsers are recommended. The main hosting site[2] in Korea is the following:

[1]http://www.sagemath.org/doc/reference/sage/interfaces/mathematica.html.
[2]SKKU Sage Notebook Server (http://math1.skku.ac.kr, http://sagenb.skku.edu), SKKU Sage Cell Server (http://sage.skku.edu, http://sage1.skku.edu).



http://math1.skku.ac.kr (ID: skku, Password: math)

Figure 1 FrontPage of One Author's Sage Server

Sage is programmed in the Python language, and it can be modified freely. We can utilize many pre-published Sage worksheets for calculus and linear algebra at http://math1.skku.ac.kr/pub.

The aforementioned language barrier has been an obstacle for software popularity in Korea. Therefore, we had to develop a Korean version of Sage. Since Sage is open source software, we could modify the source codes for using Korean language. Because source codes of Sage are complex, there were some difficulties to apply our local language to this software. But we were able to make a localized version which can make many modules to treat Korean 2-byte characters (Kim et al., 2010). Now, Korean students can use Sage in their native language.

3. Modeling of Mobile Sage

Nowadays, cloud computing that provides network-based services has become more and more popular. As well, Smartphones are widely used and now play an important role in everyday life. Since 2007, Smartphone ownership has increased rapidly, as Figure 2 shows, all over the world. Smartphones have become the most utilized mobile phone device in the world.

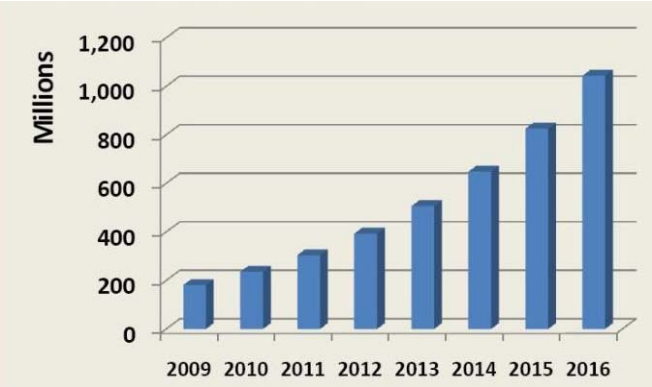


Figure 2 Worldwide Smartphone Shipments[3]

[3]Source: Telecom Trends International, Inc.

Many types of operating systems of Smartphones already exist. Since most Smartphones use Google Android as an operating system, we have started our study in the Google Android operating system. Android provides the proper environment for Mobile Sage. In the mobile learning for mathematics, the development of Mobile Sage can play an important role as a CAS tool. Android is applied not only to mobile phones but also to many kinds of consumer electronics. So, the structure of the environment implemented with Android makes it possible to create many kinds of framework for education and research. But currently, there are some shortcomings in the mathematical content and tools based on the Android operating system. However, due to the popularity explosion of mobile devices using Android, it is reasonable to develop mathematical tools with Sage for Android Smartphones. We found there are two ways:

(1) Use JAVA and XML language.

(2) Show the web pages directly.

As Sage is optimized for the web application, we have chosen the second way. That means it has already been optimized for the mobile environment on the web, so there is no difficulty in applying Sage on Smartphones. However considering the user interface, we should focus a bit more on research. The method of using Sage on the Smartphone is the same as it on the web (showed in Figure 3). Simply put the codes in the given cell, press the "Evaluate" button, and the result will be shown.

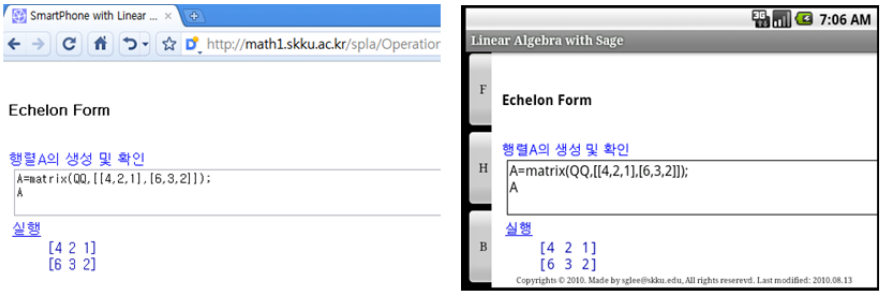


Figure 3 Using Sage on Web and Android Phone

Also, we have used Sage Cell Server to model various aspects of Mobile Sage content. The Sage Cell Server is an open-source, scalable, and easy-to-use web interface for Sage, a comprehensive open-source math software system. Additionally, the Sage Cell Server can be used to embed Sage computations into any webpage. The way to embed a Sage cell into a webpage is straightforward.

(1) Include the following HTML code in the head of the webpage (optionally replacing "sage.skku.edu" with the name of your server).

```
<script src="http://sage.skku.edu/static/jquery.min.js"></script>
<script src="http://sage.skku.edu/embedded_sagecell.js"></script>
<script> $(function () { sagecell.makeSagecell({"inputLocation": ".sage"}); }); </script>
```

(2) Include this code in the body of the page. The code is wrapped in <script> tags so that it is not treated as HTML.

```
<div id="sage">
<script type="text/code"> 1+2 # sage code </script>
</div>
```

4. Application of Mobile Sage: Sage Grapher[4]

As a visual expression of mathematics, graphs make it easy visualize mathematical principles or rules so that they can be understood more easily. In classes, teachers always use graphs to explain problems in an intuitive way. For mathematical education, graphs are often used to help students understand problems more clearly.

In this section, we will introduce the modeling process of Mobile Sage and the result of the developed graphing tool which can be used for Calculus, one of the basic courses of college mathematics. We call it Sage Grapher. Since it is based on Sage, it can be used on the PC and mobile devices through the internet to do cloud computing. Here, we will mainly consider the mobile environment for Sage Grapher.

Sage Grapher can be used as a tool to teach students, and graphs teachers make can be saved in the server so that students can review them after class. Also, students can use Sage Grapher on their computer or mobiles to graph anytime, anywhere through the internet.

By using mobile Sage mentioned in section 3, we can model a dynamic visualization tool (Sage Grapher). Using Sage Grapher, there are a total of eight different versions developed: general function grapher, parameter function grapher, implicit function grapher, and so on. So, manipulating these tools with simulations can help us find the unique properties of various functions.

Sage Grapher is divided into two parts to do general modeling. In the first part, one inputs the functions and regulates the coefficients of the given functions, as Figure 4 shows. In Figure 5, the graphing result of the functions is shown. You can use Sage Grapher through the web page below:

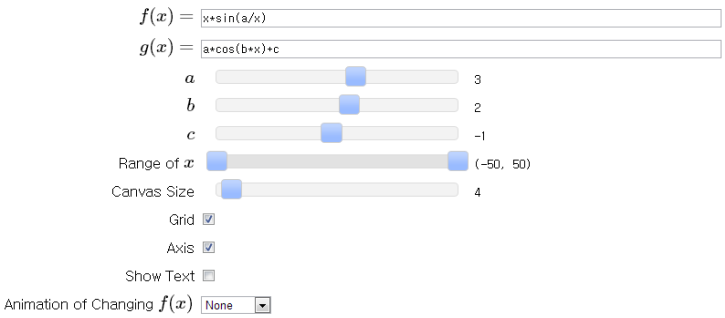


Figure 4 Simulation Part of Sage Grapher

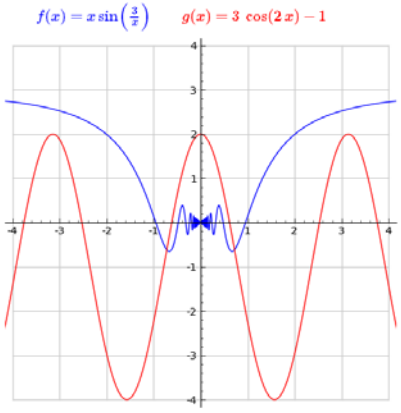


Figure 5 Simulation Result of Sage Grapher

[4]http://matrix.skku.ac.kr/mobile-sage-g/sage-grapher.html.

There are the main codes and algorithms of modeling within the Sage function grapher as listed below. It is possible to add more functions and new features.

Step 1. Declaration of variables and generating mathematical functions and slides of the coefficients.

```
var('a, b, c')
@interact
def graph(f=input_box('x*sin(a/x)', label="$f(x)=$"), ..., text_a_b_c=('- Text',False)):
  g=f.subs(a=Constant_a, b=Constant_b, c=Constant_c) # Defintion of the function
  ...
xm, xma = x_range # Generate a range of the slider(range of x)
```

Step 2. Whether or not to create x,y axises.

```
if axis:
axis_TF=1 # Show x, y-axis
  else:
axis_TF=0 # Hide x, y-axis
```

Step 3. Graphing f(x), g(x)

```
p=plot(g, (x, xm, xma), ... , plot_points=3000, figsize=7.2) # Graph of f(x)
p1=plot(0, (x,0,0.001), axes=axis_TF, figsize=7.2) # Prepare for creating grid
p3=plot(h, (x, xm, xma), ... , plot_points=3000, figsize=7.2) # Graph of g(x)
v=[ ]
p2=Graphics()
p2=p2+p1+p3 # Show the three graphs one time
```

Step 4. Graphing the grid

```
if grid:
xlabels=[-size+i for i in range(0,2*size+1)]
ylabels=[-size+i for i in range(0,2*size+1)]
  for x0 in xlabels:
  p1+=line([[x0,-size], [x0,size]], ... ) # Graphing the grid parallels to x-axis
  for y0 in ylabels:
  p1+=line([[-size,y0], [size,y0]], ... ) # Graphing the grid parallels to y-axis
  else:
xlabels=[ ]
ylabels=[ ]
```

Step 5. Add the information about functions f(x) and g(x).

```
if a_b_c=='None':
  html('$f(x)=%s$, $g(x)=%s$'%(latex(g), latex(h))) # Add informations about
  # functions by using Latex
  show(p1+p+p3)
```

Step 6. Create the animation of changing the coefficient a of f(x) (for non-recording).

```
if a_b_c=='a':
  html('$f(x)=%s$, $g(x)=%s$'%(latex(g_a), latex(h)))
  for i in srange(-10,11,1):
  p2=p1+plot(f.subs(a=i, b=Constant_b, ...), (x,xm,xma))+p3
v.append(p2)
nh= animate(v, ... , figsize=[5.5,5.5]) # Make the animation of f(x)
nh.show(delay=round(100/speed,0))
```

Step 7. Create the animation of changing the coefficient a of f(x) (for recording).

```
if a_b_c=='a_record':
  html('$f(x)=%s$, $g(x)=%s$'%(latex(g_a), latex(h)))
  for i in srange(-10,11,1):
  p2=p2+plot(f.subs(a=i, b=Constant_b, ...), (x,xm,xma)) # Record
v.append(p2)
nh= animate(v, ... , figsize=[5.5,5.5]) # Make the animation of  f(x)
nh.show(delay=round(100/speed,0))
```

It is possible to graph curves with special shapes by Sage Grapher, change the range of x and y, and hide the grid and x, y-axises. Thus, we can draw many special shapes, as Figure 6 shows. In particular, by using the scroll bars and the features of the animation, we can change the values of coefficients to draw a “nice” graph, such as the moving graphs in Figure 7.

$$(x(t), y(t)) = (12\sin(t)^3, -5\cos(2t) - 2\cos(3t) - \cos(4t) + 8\cos(t)) \quad (-\pi \le t \le \pi)$$

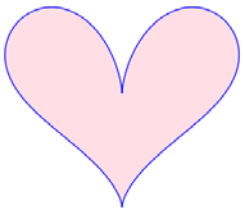


Figure 6 Graph of Heart Curve by Using the Parameter Function

$$(x(t), y(t)) = (a\sin(t)^3, -5\cos(2t) - 2\cos(3t) - \cos(4t) + 8\cos(t)) \quad (-10 \le a \le 10)$$

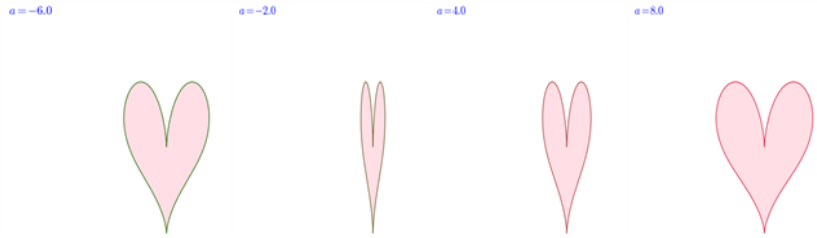


Figure 7 Graphs of Heart Curve by Changing the Coefficient

5. Conclusion

In this paper, we introduced the free public mathematical program, Sage, as a CAS tool which is convenient to use within the framework of mathematics education in Korea. Also, we showed the Korean version of the Sage server and how we utilized it in our teaching of college mathematics.

In particular, it is necessary to use CAS tools in some mathematical courses that require a lot of complex calculations, such as Calculus[5] and Linear Algebra. However, it is not easy to make CAS tools to adapt to math classes because of expensive costs, language barriers, challenges of learning new programming languages, etc.

We have developed a Korean version of Sage to solve all the above problems. Korean students have been able to do mobile computing and mathematical modeling freely. We expect that it will save several millions of dollars in Korea and improve our innovative competitiveness globally.

[5]http://matrix.skku.ac.kr/Cal-Book.

Through this study, we show that one of the main features of the Sage Grapher and mobile mathematics with Smartphone is that it is more convenient than other tools utilized in the problem-solving process. We can simply click instead of logging in to get results anywhere, anytime through the internet. Also, just by changing coefficients of functions in Sage Grapher, students can visualize most of the functions and apply what has been practiced for other subjects. What is more, students have no need to learn a new programing language to use the tools.

The mobile computing can supply an environment to do mathematical modeling anywhere, anytime, to improve mathematical research within Korea. In the future, while the change from a mobile society to a ubiquitous and smart society, the development of mobile mathematics will become increasingly important. So, it will continue to play a pivotal role in the upcoming decade.




Determinant[6] Characteristic polynomial[7] Eigen system[8] LU-decomposition[9]

Figure 8 Some Mathematical Models of Mobile Computation

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