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Applying Mathematical Tasks on Measures Strand and Promoting Classroom Interaction to Develop Students' Cognitive Skills and Mathematical Ability

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Abstract: How well students learn mathematics and what teachers' teaching strategies used to facilitate students' effective mathematical learning have been a concern in Hong Kong since the implementation of the new curriculum in 2002. Major curriculum reform and changes are the role of the teacher as a facilitator in the classroom, to help students develop cognitive skills and to foster students' learning interest and motivation in mathematics. The reform propels teachers towards a paradigm shift from a largely textbook-based teacher-centred approach to a more interactive and learner-centred approach. The purpose of this study is to examine the teaching and learning of mathematics through adopting high-level mathematical tasks in the Hong Kong primary classroom. Nowadays, teachers largely still use the textbook in a routine, chalk and talk mode as the main material for the introduction and consolidation of mathematical concepts by students. These teachers' classrooms are dominated by traditional teaching practices. In addition, researches and the mathematics curriculum reform documentation suggest that a learner-focused approach is a means to meet the best interests of learners. This paper deals with successes and difficulties six teachers go through in two schools as they move on to a student-centred approach by applying mathematical tasks and classroom discussion to develop students' cognitive skills and mathematical ability.

Key words: mathematics education, curriculum reform, cognitive skills, classroom interaction, mathematical task

1. Introduction

Discussion on mathematics curriculum reform at the primary level in Hong Kong and how to strengthen students' mathematical ability have been a concern of primary mathematics teachers since the implementation of the curriculum guide (Curriculum Development Council (CDC), 2000) in 2002. One of the major objectives of mathematics education systems around the world is to understand the processes in mathematics education that provide opportunities for all students to successfully learn mathematics. The purpose of this study is to examine mathematics learning and teaching in the Hong Kong primary schools at classroom level as the curriculum reform (CDC, 2001) has set out the general directions for curriculum development in Hong Kong for the next 10 years. Recent researches suggest that the majority of teachers in Hong Kong largely still use the textbook in a routine,

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"chalk & talk" mode as the main materials for the introduction and consolidation of mathematical concepts by students (Wong N. Y., Lam C. C., Leung F. K. S., Mok I. A. C. and Wong P. K. M., 1999; Leung K. M., 2006). These teachers' classrooms are dominated by traditional teaching practices such as direct instruction and exposition by teachers. Furthermore, researches (e.g., Boaler, 1998; Groves S. & Doig B., 2002) and the curriculum reform documentation (CDC, 2000) suggest that a learner-focused approach is likelier to meet the best interests of learners; that diversified learning and teaching strategies are likelier to suit the different needs of learners; and, that the teachers' role should be to facilitate students to learn how to learn through using the higher-order thinking skills in mathematics, especially for adopting the mathematical tasks (Stein M. K., Grover, B. W. & Henningsen M., 1996).

To achieve the above-mentioned targets, the investigation and exploration of students' interaction that takes place in classrooms continues to be an issue of great interest among mathematics educators. Towards this direction many researchers have developed theoretical constructs for the study of the relation between student's cognitive development and social interactions in the classroom. For example, interpretative constructs for this purpose are the social and sociomathematical norms (Yackel & Cobb, 1996) and the thematic patterns of interaction (Voigt, 1995). In this paper I present a case study of two primary mathematics teachers who participated in the research program (Leung K. M., 2008) and the teacher, Ms X of School A, located in a rural area of Hong Kong joined the study in the first year as well as Ms Y of School B, located in urban area, joined the study in its third year. The focus topics are on the strand of Measures. The purpose of the research was to investigate the way that students can be actively involved in mathematical activities as they cooperate in small-groups to solve mathematical problems. More specifically, I study the use by the two teachers of mathematical tasks designed to encourage student participation in small groups and to develop students' cognitive skills such as collaboration and problem solving skills, and to strengthen their mathematical ability as well.

2. Theoretical Framework

For constructing the mathematical tasks in this study I used Stein et al.'s framework (1996). Stein et al define a mathematical task as a classroom activity the purpose of which is to focus students' attention on a particular mathematical concept, idea or skill. I then implemented the tasks in the classroom with the collaboration of the two teachers. The tasks focus on developing certain aspects of mathematical understanding that mathematics education research sees as important (Ainley J., & Pratt D., 2002). When I consider learning as situated in a social and cultural context, the sociocultural perspective provides a useful lens for understanding how schools might function to provide opportunities for all students to learn mathematics the same situation was raised by mathematics experts of Eastern regions/countries (Fan L., Wong N. Y., Cai J. & Li S., 2004). In this paper, I aim to refer to the above guiding principles to discuss the present situation regarding primary teachers' mathematics teaching in Hong Kong; in particular the teachers' attempts at using mathematical tasks in the classroom so as to enhance students' learning and foster their higher-order abilities.

2.1 Adopting Mathematical Tasks in Classrooms

Teaching through mathematical tasks, teachers assume students can develop their creative abilities. Some educators (Cicero Ana Maria Lo, De La Cruz Yolanda, Fuson Karen C., 1999) put their focuses on the central activities of Children's Math Worlds (CMW) that is a research-based mathematics curriculum for kindergarten through grade 3. Basically, it uses the Vygotskian model as a springboard for formulating and solving problems; as

well as saying or drawing their own stories to develop thinking and creativity. The related activities of CMW focus on: (1) linking mathematical activities in the classroom to children's mathematical experiences outside school and (2) creating a rich and sustained environment for learning to write, solve, and explain ways of solving word problems (Cicero Ana Maria Lo, De La Cruz Yolanda, Fuson Karen C., 1999). Finally, it is noted that students' connections to mathematics with their daily life experiences will facilitate their learning. It is similar to the task-based approach discussed in the study that advocates using the local context of students' daily-life experience in the learning task.

2.2 Why Applying Mathematical Tasks so Important

Mathematical tasks can be examined from a variety of perspectives including the number and kinds of representations evoked, the variety of ways in which they can be solved, and their requirements for student communication (Stein M. K. et al., 2000). In the study, I examine mathematical tasks in terms of fostering students' higher-order thinking skills, the kind and level of thinking required in order to successfully solve the task. Why is fostering students' higher-order thinking skill through mathematical tasks so important? As stated in the Professional Standards for Teaching Mathematics (NCTM, 1991; 2000), opportunities for student learning are not created simply by putting students into groups, by placing manipulatives in front of them, or by handing them a calculator. Rather, it is the level and kind of thinking in which students are engaged that determines what they will learn. Tasks that require students to perform a memorized procedure in a routine manner lead to one type of opportunity for student thinking; which tasks that demand engagement with concepts and that stimulate students to make purposeful connections to meaning or relevant mathematical ideas lead to a different set of opportunities for student thinking. From students' daily experience, the cumulative effect of their experiences with mathematical tasks is students' implicit development of ideas about the nature of mathematics (Stein M. K. et al., 2000).

The above discussion is consistent with an emerging perspective in mathematics education that highlights both the social and mathematical norms in a mathematics classroom. In particular, students' interactive discussion in the classroom and active participation on the tasks will be the crucial factors to help students develop their cognitive skills and mathematical ability.

3. Methodology

The study I draw on in this paper is a part of the doctoral study that started in 2001. By using a case-study approach in a preliminary phase of the research, 8 task-based lessons were designed and implemented. Lessons were observed. Observation focused on how the teacher implemented the designed lesson and how students behaved during these lessons. In addition, in order to include the perspective of the students a group of 4–5 students involved in the lessons were invited to an interview once lesson implementation was complete. As for the characteristics and nature of the evidence and data collected, qualitative data analysis could provide a holistic picture of the phenomenon and triangulation. Preliminary analysis suggested that the new learning environment contributed significantly in the development of students' higher-order thinking skills; that by using the mathematical tasks, it created a new platform for students to learn in a collaborative mode; and that the increased opportunity for communication meant that students, even those who are usually reticent in the classroom, could freely put forward their ideas and suggestions.

This pilot study contributed to the conceptualization, methodological and substantive, of the main study. The pilot study was set up with the intention to encourage more student-student interaction in the classroom, to

enhance students' thinking and communication skills and to use diversified learning activities and tools (including mathematical tasks & information technology) for improving learning and teaching. These purposes remained the same for the main study conducted in 2003. In three collaboration meetings with Ms X and Ms Y I designed a series of lessons for students aged 9–11 on the following topics on the strand of Measures at Key Stage 2: Area of Triangle, Angles (degree), Volumes of Cubes & Cuboids and Area of Polygon. Twenty-three lessons were observed. Thirteen teacher and seven student (focus-group) interviews were conducted. All data was audio recorded except for the lessons that were video taped. I also collected observation fieldnotes, students' annotated work and other relevant documentation. The qualitative analysis was performed and findings are presented in the following section.

4. Findings and Discussion

In the following I exemplify from the above mentioned body of data. In particular I focus on the lessons designed on the topic of "Volumes of Cubes & Cuboids" in Grade 5 as there were four topics in Ms X's lessons and two in Ms Y's. This topic was common to both. Below I cite summarizing excerpts from the transcript of one of the collaborative meetings, the pre-lesson conference. The aims of the pre-lesson conference were: to discuss and identify the learning objectives of the lesson on the particular topic; to identify mistakes/difficulties students are expected to face with regard to the particular topic; and to prepare the tasks that will be used in the lessons. Ms X reflected: "*students have little difficulties to put numbers into the formula to calculate the volumes. They will soon remember that the formula of volume of cuboids is (length)×(breadth)×(height).*" and "*…..firstly divide the students into groups and give them task sheets to do. We will then require them to calculate the number of cubes in a single stack, …..*" in one of the pre-lesson conferences. Based on the discussion, five mathematical tasks were developed and adopted in the lessons. Different types of questions were designed so as to help students discuss the cases during the lessons.

4.1 Strengthening Students' Mathematical Ability through Doing the Task

In the trial lessons, mathematical tasks, worksheets and group activities are adopted in the class. Ms X raised the daily-life investigative problems in the blackboard for students' discussion and developing their cognitive skills such as problem-solving skill. For example, the volume of a box, 18 cubic centimetres is given. If students need to place cubes of 2 cm^3 in it, how many cubes can students put in at most? So 18 cm^3 is divided by the number (2 cm^3). Thus, a simple answer is 18 ÷ 2 = 9 provided. Students easily solved the case by applying their mathematical knowledge as the problem was drawn on the blackboard and illustrated for students' understanding. In the last two trial lessons held in the main study, Ms X started lessons with revision on the volumes of a cube and a cuboid. She highlighted that the length, breadth and height of a cube were the same. The volume of a cube was then calculated. The following daily-life and open-ended problem was then discussed in order to help students develop their higher-order abilities such as communication, critical-thinking and problem-solving skills. The daily-life and open-ended problem can be summarized below:

Given: Dimensions of Container
length = 7 m, breadth = height = 2.8 m
Find the maximum number of cubic boxes with the side of 0.9 m that the truck can be stored.

During discussion Ms X's questions played a guiding role. Students worked on the problem in groups in a similar problem. The simplified case is the parcel with dimension 2 cm × 2 cm × 2 cm that need to place into the

Box A with dimension 18cm x 6cm x 4cm and Box B with dimension 17 cm × 7 cm × 4 cm. Students needed to find the maximum number of parcels placed into the Box A and Box B as students could not simply compute 476 ÷ 8 = 59 (parcels)…4 for Box B. Ms X's guidance and student-teacher interaction in the lessons are observed and increased through exploring the strategies to solve the problems.

4.2 Developing Students' Cognitive Skills

For data collected in the topic, "Area of Triangle", the teachers agreed that this approach helped developed skills such as collaboration and problem solving skills. Ms X reflected at her first lesson that "*The topic is given to students and let them explore. Students cut the papers in group basis and match them on their own to see the results. By letting them explore on their own, the abilities of exploration will be strengthened*" Another case-study teacher, Ms Y, in her lessons, emphasizes students' thinking ability can be improved through providing them more chance and space. The following captures her views on the above: "*They think more in their discussions. They may not understand if they think on their own, but the weaker students can be led by the other classmates during discussions and thus can think more*". Thus, through the communicative process in the lesson, students will also expand their capacity from working individually to solving problems in a collaborative way. Besides, to provide mathematical tasks during the lessons, classroom interaction is a crucial indicator. The following describes and analyses the data which reveal of the key features of using mathematical tasks in the lessons, i.e., encouragement of active participation and development of students' cognitive skills. Data from the transcription of the observed lessons provided some basic information on the use of individual work, pair work and group work. These data show the extent to which students are encouraged to explore and to ask questions. Taking Ms X and Ms Y's trial lessons in the topic of "Volume of Cubes and Cuboids" as an example, students' involvement in the tasks are highlighted so as to show how successful the lessons are in this respect. With regard to students' learning in the class, I observed the significant amount of time they spent in the lesson involved in activities. During lesson observation, students actively participated in the mathematical tasks and the tasks really aroused their interest. Since the group discussion on the worksheet is performed first, most students know how to solve the questions. For a more interactive classroom, the teacher is expected to select the most appropriate teaching approach and method to assist students to make progress towards the learning targets. Decisions about the techniques and strategies which are most effective for particular students and for particular purposes are based on the mathematical tasks for the students as well as the practical activities designed for them. My analysis of the observed lessons suggests that for the major part of the lesson students are on tasks and interact with teachers by answering teachers' questions. Thus, time for students' involvement in the mathematical tasks is significantly a crucial factor for a success lesson and an indicator for students to developing their cognitive skills.

4.3 Encouraging Student-student Interaction in the Lesson

To decide a lesson whether it is successful or not, students' views are a crucial indicator that includes "what a lesson looks like", "what activities are held in the lessons", "how teachers implement the lessons" and etc. While asking students what kind of a lesson they expect, they state they like to participate into the learning activities. The following phases are captured from their interviews, "having chances to come out to do calculation" and "I think the lesson can foster cooperation among ourselves, and we can know how to calculate the volume of cubes and cuboids". In addition, students like more group activities and working as a group. As compared students' views on the two trial topics held in School B, firstly, students thought that using PowerPoint to present the teaching contents was interesting as the lesson would be more dynamic, interactive and attractive. They also

emphasized that if the blackboard was used, the knowledge would be disseminated to them directly. Same result is collected from the next interview and they reflect that when the teacher writes and draws teaching notes on the blackboard that seems boring. However, when teaching notes are shown by using the PowerPoint, they are more interesting. More pictures and less words are presented. Students can read clearly by using the computer. Students feel the mathematics lessons interesting as there are more activities in the lessons and learning is like a game. There are more group discussions. Some of them even state that they do not need to do all the things alone and another echoed he will not be afraid anymore. It truly reflects students enjoy the lessons and the change of learning atmosphere and teaching approach makes them easily construct and understand the mathematical knowledge. Students are positive for task-based teaching and they express activities make learning more interesting. They like having group activities during the lessons and they can communicate with classmates easily. And of course, they can understand the mathematical knowledge easily and quickly. In both schools students felt more discussions in the class could help them share the idea and understand what teachers had taught more easily. Furthermore, they appreciated that they did not need to ask teachers constantly and directly. The following quotations from School B students are indicative:

Table 1 Interview with School B Students

44	S2:	If I don't know how to calculate, I'll ask teacher.
45	T:	If you don't know how to calculate now, you will not ask teacher. Which parts do you like the most in these few lessons?
46	Ss:	Group discussion.
47	T:	Why?
48	S3:	It's because we can discuss with others.
49	S2:	It's not as boring as before.
50	T:	Any more?
51	S5:	We can do (the task) in groups.

In particular, the students from School B said that they like group activities as they find them very exciting; that they dealt better with difficulty by doing the task in groups and lessons are not as boring as before due to more interaction and more opportunities to contribute in the discussion.

4.4 Applying Mathematical Task to Help Students Achieve Learning Outcomes

In this part, I am going to talk about students' learning outcomes that truly reflect what students have learnt or their performance in the lessons. Nowadays, students learn by having more interactions between their peers, having more activities to do actually, and they are not just told by their teachers what to learn but by finding the results when doing the activities. All these contribute to the understanding of the concepts and the long-term memory. They can understand more easily. This teaching strategy is a good approach for the trial topics. In the following, I show students' performance through data collected from teachers' interviews on the trial topic, "Area of Polygon" conducted in the same level of School A.

The following quotations extracted from the interviews are the evidence to show that by applying the tasks during the lesson, students' learning motivation can be increased and positive attitude is expected. Teachers also agree that students can easily get hold of the concept taught and their interests in mathematics have been raised. In addition, it is noted that students' academic achievement is a very good indicator to describe or measure the effectiveness of the teaching approach.

Table 2 Students' Performance

Class of teacher taught	Students' Learning Outcomes/Achievement
4A	"….. As they had responded, they had a greater sense of success. They achieved something……"
4B	"The majority of the students could master what they were expected to learn. Also, they learnt happily because they could participate in activities and had discussions and they did benefit from that."
4C	"They were very devoted when they use the 1 cm^2 squares to create the rectangles. It was because they actually participated in it. They could also find out the length, width and area easily. Besides, when I asked them to come out and see how and how many students could stand in the 1 m^2 area, they participated eagerly and answered my questions eagerly."
4D	"We always hope the topics to be more related to daily lives, as it is more interesting…. For example, I played a game of collecting objects with them. I asked the students to put their books into the 1 m^2 area to see how many books could be put inside. Therefore they would have a better concept of how big 1 m^2 is."

5. Concluding Remarks and Limitations

In the study, the following dilemmas faced by teachers are raised below for further consideration:

- teachers find this approach to be time-consuming. They are consequently worried about content coverage; and
- while students are on task, teachers have to contend with the fact that students still depend on the teacher to explain the text or the situation.

Suggestions for further research may be to replicate the present study, to include a larger sample of students from different social backgrounds and to cover different topics as to produce more reliable results.

The main purpose of this study is to explore the implementation of task-based lessons that increase student interaction and participation in two typical primary schools in Hong Kong. The rationale for the study is grounded on the popularity and acclaim, in some western countries, of this approach as numerous studies attest (e.g., Anghileri J., 2002; Edwards J., 2002). Locally, this study was partly conducted in response to HKSAR Government's concern for mathematics education as indicated in the report (CDC, 2001). In this students' learning with respect to "offering them essential learning experiences" and "providing them a learner-focused approach" was encouraged as was a shift away from a teacher-centred pedagogy (from teacher based to task based) and towards student interaction and active participation. This approach helps fulfil three major learning goals: promoting students' involvement and engagement in the lesson by allowing students to voice their own ideas; helping them develop better understanding by allowing them to think things and verbalize their thinking; and finally, helping students develop cognitive skills such as the self-confidence to voice their own opinions in public, solving problems and the ability to do so in a clear and concise way.

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