Journal of Modern Education Review, ISSN 2155-7993, USA
July 2013, Volume 3, No. 7, pp. 555–560

http://www.academicstar.us

The Principle of the Coherent Mathematics Teaching Materials for Teacher Education

Karasawa Toshimitsu
(University of Manchester, UK)

Abstract: This paper, as well as offering two- and three-dimensional figures as immediately usable educational materials to teachers of math from primary school through secondary school and sixth forms, also provides materials using four-dimensional through nth p-dimensional figures for university students hoping to become teachers and for teachers themselves taking part in in-service training. That is to say, the materials provided form, as a single group, a coherent system for primary schools through secondary schools, university students hoping to become teachers, and professional educators. In fact, I can report that, when presented in the course of a lecture at a teachers' in-service training, these materials received good marks from the teachers present.

Key words: task design, teacher education, mathematics teaching materials

1. Introduction

In general, in-service training for teachers utilizes either materials exclusively aimed at their primary or secondary school students, or materials focused on modern mathematics which are aimed only at teachers themselves. The former is unsatisfying for teachers who are actively interested in mathematics. The latter is considered an unhelpful training experience for teachers who are interested only in what they can put directly to use in the classroom. Materials which can be immediately used for primary or secondary students in the classroom, and which are at the same time problems which teachers must think seriously about in order to solve, fulfill the original intention of teacher training. The purpose of this paper is to present mathematically unified educational materials which can be used for primary students through secondary and sixth form students, for university students hoping to become teachers, and for teachers themselves. In fact, I can report that, when presented in the course of a lecture at a middle school teachers' in-service training, these materials received good marks from the teachers present.

2. The Issue of Matchstick of Tsubota

At a conference on primary school arithmetic education which was held in Hakone in Japan in December 2011, I heard Kozo Tsubota speak on matchstick problems. Tsubota stated that a class which asks children to figure out how many matchsticks are necessary to make the figure in Figure 1 encourages arithmetical activity.

Karasawa Toshimitsu, Ph.D., Local Assistant Researcher, University of Manchester; research areas: education, information science, history of science. E-mail: karasawa@sings.jp.

Tsubota offered four answers (6 × 6, 4× 9, 9 × 4, 4 + 12 + 20) as primary school students' typical answers. He also stated that he had made a three-dimensional figure out of matchsticks but that it was unstable. Even if educationally effective as a class which encourages arithmetical activity, this idea is mathematically closed and cannot be used for older students. This may be the difference between primary school arithmetic and mathematics.

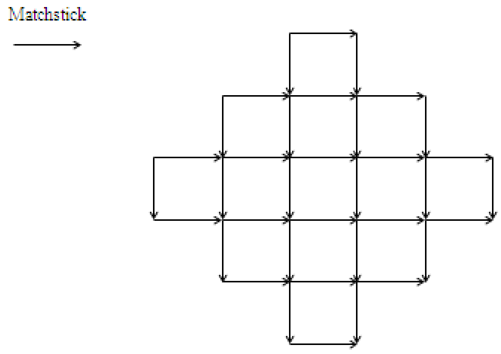

Figure1 The Issue of Matchstick of Tsubota

3. The Issue of Matchstick as the Problem of the School Mathematics

This paper's approach, then, is not to solve problems individually, but to reapproach problems in general dimensions to seek general answers. The solution method is to find the total number of matchsticks needed for nth 2-dimensional figures and 3rd 3-dimensional figures, applying this to nth 3-dimensional figures. The problem of finding the total number of matchsticks needed for 3rd 3-dimensional figures can be used for Key Stage 3 students, and that of finding the number of matchsticks which compose nth 2-dimensional and 3-dimensional figures for Key Stages 4 and 5 students. In the upper years, one hears that there are few good study tools which use mathematical induction, but this is an excellent example which is both specific and natural. As well, there are two ways to think about this solution method. One is to use the circumscribed least square or the circumscribed least cube as a supplementary figure. The other is to find the number of endohedral squares or cubes which compose the figure. First, we solve the problem using the first method. The number of matchsticks which compose the 1st 2-dimensional figure can be found with 1× 4. The number composing the 2nd figure is 1 × 4 + 3 × 4. Specifically, we surround the figure desired with the least square, as in Figure 2.

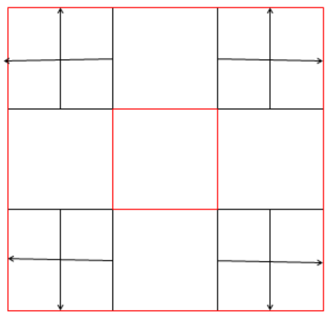

Figure 2 Request of the Number of the Matchstick of the Figure of the Two Dimensions Second

When we do this, a number of the matchsticks composing the figure can be moved, following the arrows, to make a square. Therefore, there are three matchsticks which compose each side of the square, and as it has four sides the total number of matchsticks is 3×4. However, the matchsticks composing the least square at the center of the figure remain on the original figure, so we add those. Thus it becomes 3×4+1×4. The number of matchsticks which compose the 3rd figure is made clear in Figure 3.

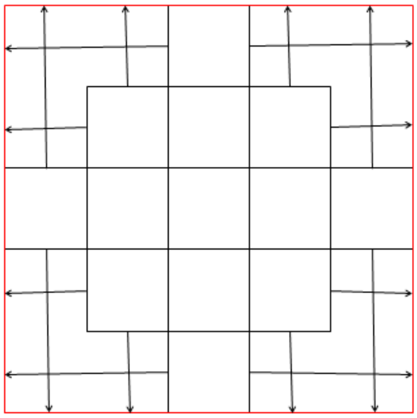

Figure 3 Request of the Number of the Matchstick of the Figure of the Two Dimensions Third

As when solving the 2nd figure above, we move a number of the matchsticks toward each side of the circumscribed least square. Thus, the matchsticks which cannot be moved exactly compose the 2nd figure, and so the total number of matchsticks we find is 5×4 + (3×4 + 1 × 4) = (5 + 3 + 1) × 4 = 36. The answer to 3-dimensional problems can be found in the same way, using the circumscribed least cube. The 1st 3-dimensional figure is itself the circumscribed least cube, so the total number becomes 1 × 4 × 3 = 12. The 2nd 3-dimensional figure, as we see in Figure 4, becomes 3 × 4 × 3 × 2 = 72.

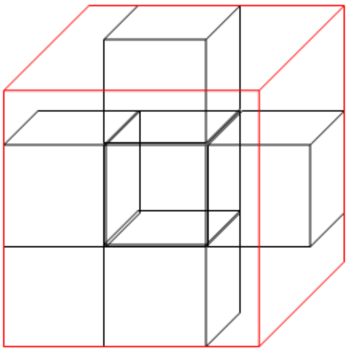

Figure 4 Request of the Number of the Matchstick of the Figure of the Three Dimensions Second

The 3rd figure, as we see in Figure 5, becomes {5 + (3 + 1) × 2} ×3×4 = 156. These are useful learning tools for Key Stage 3 students. Next, I present the latter method. The goal of this is to find the total number of matchsticks which compose nth 2- and 3-dimensional figures as well. The way to find the total number of matchsticks in 2- and 3-dimensional figures is demonstrated in Figures 6 and 7. We see in Fig. 6 that the total number of matchsticks composing each figure is, in the 1st figure where there is one endohedral square, 1 × 4 = 4, and in the 2nd figure, where there are four, 4 × 4 = 16. In the 3rd figure, there are nine, and it becomes 4 × 9 = 36. Therefore, the total number of matchsticks which compose an nth 2-dimensional cube is as follows.

$$\sum_{k=1}^{n}(2k-1)\times 4 = 4n^2$$

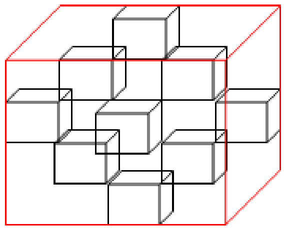

Figure 5 Request of the Number of the Matchstick of the Figure of the Three Dimensions Third

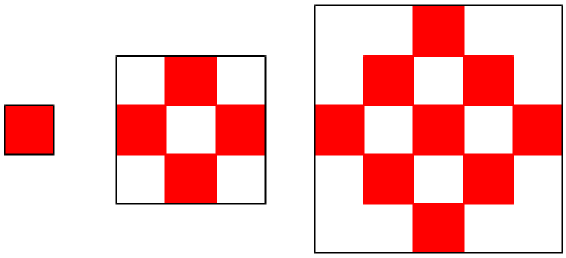

Figure 6 Request of the Total Number of the Matchstick Constituting A Figure to the Two Dimensions Third that Pointed to A General Solution

As well, we see in Fig. 7 that the total number of matchsticks composing each figure is, in the 1st figure where there is one endohedral cube, 1 × 4 × 3 = 12, in the 2nd figure, where there are six, 6 × 4 × 3 = 72, and in the 3rd figure, where there are thirteen, 13 × 4 × 3 = 156. Therefore, the total number of matchsticks which compose an nth 3-dimensional cube can be found by the following formula.

$$\left\{2\sum_{k=1}^{n}(2k-1)-(2n-1)\right\}\times 4\times 3=2\left[2\left\{2\times 1/2\times n(n+1)-n\right\}-2n+1\right]\times 12=12(2n^2-2n+1)$$

These materials are effective for use with Key Stages 4 and 5 students.

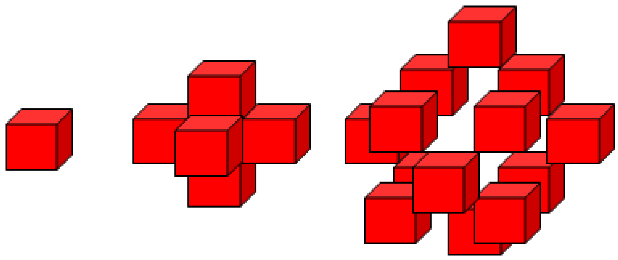

Figure 7 Request of the Total Number of the Matchstick Constituting A Figure to the Three Dimensions Third that Pointed to A General Solution

4. The Issue of Matchstick for A University Student and the Incumbent Teacher

Finally, we look for the total number of matchsticks which compose an nth p-dimensional figure. First, we solve generally for the situation of a p-dimensional least cube. We begin by thinking of a 4-dimensional least cube. A 4-dimensional least cube has eight vertices in total. From each vertex, four mutually perpendicular matchsticks emerge. Therefore, their total number is 4 × 8 = 32. Next, we think of a p-dimensional least cube. If we call the number of p-dimensional vertices V (p), then V (p) = 2^p. If we call the number of the side with E(p+1), we are as follows.

$$\begin{cases} E(1) = 1 \\ E(p+1) = 2E(p) + 2^{P} \end{cases}$$

Therefore, the number of matchsticks which compose the 1st figure, the p-dimensional least cube, is $p \cdot 2^{p-1}$. Further, the total number of matchsticks which compose an nth p-dimensional figure is, when we call the vertices V(p, n) and the number of matchsticks E(p, n), shown by the following recurrence formula.

$$\begin{cases} \mathrm{V}(p+1, n) = 2\{\mathrm{V}(p,1) + \mathrm{V}(p,2) + ... + \mathrm{V}(p, n)\} \\ \mathrm{E}(p+1, n) = 2\{\mathrm{E}(p,1) + \mathrm{E}(p,2)\} + 2\{\mathrm{V}(p,1) + ... + \mathrm{V}(p, n-1)\} + \mathrm{V}(p, n) \end{cases}$$

This must be made into a single formula using only p and n, but that is considerably difficult. These materials are effective for the mathematics study of university students hoping to become teachers and for teachers themselves.

5. Conclusion

When these materials were presented at an in-service training for middle school teachers, the following opinions were independently presented.

Teacher, male, 25 years' teaching experience: "I understood that these materials are basically an example showing the point of view of the maths teacher looking constructively at educational materials … (omission). When actually making them into teaching materials, I imagine they were adjusted to meet various conditions: for instance, could they be made into a graph for consideration (countable), could one change the conditions and investigate them with an analogous method and so on. The problem used this time is an interesting problem mathematically, but the way the difficulty level increases when the dimensions are increased is a point which, within the scope of middle school mathematics, won't go too well even if approached analogously, and I felt it was difficult as a study tool. However, the experience or the feeling of investigating this kind of problem and finding it interesting is one which we maths teachers must have in order to transmit the merits of mathematics. First we have to find maths interesting, to get caught up in investigating it, the importance of this…(omission). I want to pass this on at the mathematics subject meetings. Thank you for this opportunity."

Teacher, male, 15 years' teaching experience: "I feel I've learned a lot from hearing this talk, and I was also made aware of my own need for further study. I have a renewed desire to make the most of myself as an educator. I look forward to studying with you again."

Headmaster, 35 years' teaching experience: "Hearing yesterday's talk, the development of a problem from primary through secondary level and then its expansion to a general level, I felt again the interest and wonder of mathematics. The audience's reaction during the latter half of the lecture suggested that perhaps we should have requested a little more on the actual practice in the classroom. With this opportunity,… (omission)… I hope for your continued guidance and support of mathematics teachers in municipal middle schools. Thank you very much."

As stated above, Tsubota's lecture is probably effective as guidance to encourage mathematical activity (see Ministry of Education, Culture, Sports, Science and Technology [MEXT], 2008a). As well, it is thought that 3rd 3-dimensional figures in middle schools and nth 2- and 3-dimensional figures in senior high schools are effective for guidance of mathematical activity (MEXT, 2008b; MEXT, 2009).

It is also thought that figures from 4-dimensional least cubes to nth p-dimensional figures are effective for university students hoping to become teachers and for teachers themselves. In this way, it is not usually an easy proposition to find a single topic on which to build effective educational materials for primary schools, middle schools, senior high schools, universities, and teachers. However, this is thought to be a rare example of a good-quality study tool. The next task is to build up classroom practice in middle and senior high schools.

References

Ministry of Education, Culture, Sports, Science and Technology (2008a). *An Elementary School Course of Study Commentary Arithmetic*, Toyokan Publishing .

Ministry of Education, Culture, Sports, Science and Technology (2008b). *A Junior High School Course of Study Commentary Arithmetic*, KYOIKU・SHUPPAN. Co., Ltd.

Ministry of Education, Culture, Sports, Science and Technology (2009). *A High School Course of Study Commentary Arithmetic*, Jikkyo Shuppan Co., Ltd.