Mathematics Education and Teaching Practice to Bring Up History of Mathematics Culture

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Abstract: This article, one of the documents about education of history of mathematics culture, discusses about sundials. There are three types of sundials, which are equatorial, horizontal, and vertical sundials. Students were taught to build an equatorial sundial and to convert the initial model into a horizontal sundial. Classes using sundials as teaching materials become the practicality of solid geometry and trigonometric function, the application of congruent triangles, and the summary of construction, and are also proved to be effective in teaching cultural history. In this paper is one part of the lecture I gave last year at the Onomichi City Municipal Junior High School Mathematics Seminar. In my lecture, I taught the teachers how to make a sundial.

Key words: mathematics education, history of mathematics culture, teaching practice

1. Introduction

Unlike in astronomy, the types or principles of sundials were converted to mathematics and organized into teaching materials that teach the solar movement and time from the solid geometrical view by me. Sundials have been used not only as arithmetic or mathematic teaching materials but also as incorporative learning materials. The writer conducted a middle school mathematics teachers’ study group last summer during which the teachers actually constructed sundials themselves as in Picture 1. The study group also summarized proof systems based on definition, axiom, and theorem through adapting logical geometry and taught teaching methods using the principles of sundial construction. The writer also lectured about a relationship between two specific sundials to mathematics teachers from elementary, middle, and high schools from a cultural history point of view: one was a design of a horizontal sundial by Hayashi Shihei, in Shiogama shrine museum in Shiogama, Miyagi Prefecture of Japan, where there recently has been a severe earthquake, and the other one was the sundial in the Netherlandish trading post in Dejima, Nagasaki Prefecture of Japan.

Based on these activities, this article reports that the topics on sundials are suggested to be effective textbooks which aid teachers to understand the history of mathematics culture.

2. Sundials and Cultural History

This section describes the reason sundials were chosen as cultural history. An explanation regarding how one will understand the history of mathematics culture is needed. This article acknowledges the mathematical culture
from the perspective of mathematics education. And as educational significance, this article explains the mathematical culture from four different viewpoints which are figure education, model conversion, cultural history, and relations among other textbooks.

First, Japan is not known for a well-built system of figure education. Especially the teaching system on space figure, also known as solid geometry, is not well-prepared compared to other countries’ systems. Such problem for students can be solved naturally by using sundials. Specifically, many theorems regarding figures are used in sundial textbooks, such as corresponding angles in parallel lines, perpendicularity of a radius of a circle and its tangent, theorem of three perpendiculars, spherical relation of position, sphere and tangential plane, parallel planes, and dihedral angle. Such sundial textbooks discuss space figures in a different view, a view from solid geometry. They also provide the proof of the solar movement by using analytic geometry of space for high school students as well as mathematics from certain levels. The next description is based on the viewpoint of model conversion. After an equatorial sundial, the most basic type of sundials, is built, it provides the foundation of a horizontal sundial construction. As the type of sundial changes from equatorial to horizontal, the method of drawing hour lines on the base changes. And thus, it is possible to develop mathematical models. This model conversion plays an important role in the comparative analysis of Hirado’s sundial and the sundial in the Shiogama shrine museum which is known as Hayashi Shihei’s Design that is discussed in the following paragraph.

Also, the analysis of the sundial known as Hayashi Shihei’s Design draws the conclusion that the sundial should be positioned in the same latitude of Hirado, not in Shiogama where it is currently located. It is possible to help understand and recognize the importance of explaining historical events scientifically using mathematics. Looking at equatorial sundials for example, the dihedral angles between the base and the ground are different in varying latitudes. As you can see, sundials are closely related to history or cultural assets, and at the same time, they can be important materials to develop global perspectives in a sense of mathematics culture. Sundials have been used as natural sciences teaching material in Japan until now. It is, however, possible to connect mathematics and natural sciences by using sundials. In order to do so, teachers need to study the textbooks thoroughly.

3. Sundials of Japan and the World

The basic parts of sundials are a shadow stick called gnomon which casts a shadow and a base which is made
by hour lines produced by the shadow. Depending on the position of the base relative to the ground, sundials may be one of the three types: equatorial, horizontal, or vertical. Sundials in different regions or countries show varying characteristics. Equatorial sundials are used in China or Korea, and horizontal sundials are used in Japan. And the higher latitude of Europe forced them to use vertical sundials. In my lecture, I taught the teachers how to make a sundial. I explained three types of sundial to the teachers. Those are the equatorial sundial, the horizontal dial, and the vertical sundial. This is an equatorial sundial that belongs to the Seiko Clock Museum as in Picture 2.

The following is the Seiko Museum’s description of this sundial.

This sundial is an object from China’s Qing Dynasty. You align the gnomon with Polaris, with the dial plate placed perpendicular to the gnomon. The sun rotates around the gnomon’s extension, and the resulting shadow can be used to read the time on the dial plate. Picture 3 is what we can see from the front. And Picture 4 is what we can see from the side.

With equatorial sundials, the vernal equinox and autumnal equinox make the border, and the sun’s shadow falls on either side of the dial plate (from vernal to autumnal), so that the time can be shown on either side. The time can be told on either side of the plate in Picture 5.
Picture 5  The Time Can be Told on Either Side of the Plate

Picture 6 is a horizontal sundial from the Shiogama Shrine Museum. The museum provides this description.

“Sundial”, a gift to Tomoaki Fujitsuka.
From 1792 (*seventeen ninety two*) — (The fourth year of the Kansei Era.).
This sundial, which was gifted to Tomoaki Fujitsuka, was made by his friend, Shihei Hayashi. The hour lines have been engraved with Roman letters, and though it is difficult to read because of weathering, there is an engraving of the words, “product of Holland named Daitounichi,” “Completed Kansei 52 (1852), presented to Tomoaki Fujitsuka,” and on the side, “signed by stonecutter Izumi Sanemon.” Though it appears to be a “Shihei Hayashi design”, if Shihei was involved in the production, then we should consider it, as it is written, an “imitation”.
The instructions are also written, and they say.
Picture 7 is what we can see from the front. And Figure 8 is what we can see from the side.
Pictures 9 & 10 are vertical sundials from Europe. It was taken for me by a staff of the university I work for.

Europe is at such a high altitude that outside of the east, early morning hours cannot be recorded on such sundials. For this reason, the eastern areas have also installed vertical sundial.

4. The Principles of Sundials

4.1 The Principle of Dial Construction from A Solid Geometry Point of View

The mathematical structure of horizontal sundials is shown in Figure 1. The ☆ in the picture indicates Polaris, which gnomons are made to face. At an arbitrary time, the shadow of the gnomon on the dial is on the segment OR. Thus, the segment PR is the hour line of the horizontal sundial at that arbitrary time.
Even if PQ=1, we do not lose the generality.
Let PQ=1, ∠QOR=T, ∠QPR=X, and the latitude be A.
From the right triangle POQ, OQ=sin A.
From the right triangle QOR, QR=OQ • tan T=sin A • tan T, and tan X=QR
Thus tan X=sin A • tan T.
Therefore A=sin⁻¹(tan X / tan A)
This is the formula used to calculate latitude in the previous page.

Figure 1  The Relationship between Latitude and Hour Lines

4.2 The Principle of Dial Construction from A Geometrical Construction Point of View

Mr. Yoich Tan taught the principle of dial construction to junior high school students using like that figures from Figure 2 to Figure 9. In this paper, these figures from Figure 2 to Figure 9 were drawn for me by Ms. Maruyama. Figure 2 shows the relationship between equatorial and horizontal sundials.

Refer to Figure 3 and find the location of point O using construction. Next, construct a right triangle with a hypotenuse QP and one side OQ with known lengths and measure ∠OPQ.
This angle measurement will be the degree of the latitude where the sundial would be located.

\[ \angle OQP = (90^\circ - \text{latitude}), \quad \angle OPQ = \text{latitude} \]

Since the latitude of Onomichi is about 34 degrees north latitude, we will construct a sundial that can be used here. A good teaching method would be to let the students think about how the space between hour lines, which is 15° on an equatorial sundial, would change on a horizontal sundial using an equatorial sundial which has already been made. However, let them think that the gnomon line of the sundial reaches the horizontal plane.

In Figure 5 the shadow of the gnomon AP becomes the face that includes \( \triangle ARO \) on the upside of the equatorial sundial.

Relative to the horizontal plane, the shadow of the gnomon AP becomes the line of intersection of the face which includes \( \triangle ARP \) and the horizontal plane. And so, the hour line on an equatorial sundial base which is divided into 24 sections, 15° apiece, is stretched to the line of intersection of the equatorial plane and the horizontal plane, in order to make a point of intersection. Decide the location of point P in Figure 2 by construction and connect point P to the point of intersection from before. In order to find the location of point P, first choose a length for OQ, and then construct \( \triangle OQP \) using \( \angle Q = 90^\circ - \text{latitude} \) and \( \angle O = 90^\circ \). The position of point P can be decided by measuring the length of PQ from the picture.
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Figure 5  The Shadow of Gnomon AP

Figure 6  The Shadow of Gnomon AO

Figure 7  The Right-Angled Triangle Which Is Similar for the Construction
If the hours are recorded around the horizontal plane in a rectangular shape as in Figure 8, a horizontal sundial base is created as in Figure 9.

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**Figure 8**  The Horizontal Plane in A Rectangular Shape Recorded Hours

5. Conclusion

The following are the teacher evaluations of these materials.

A male teacher with two years teaching experience said, “I’d like to try studying the equatorial sundial in the elective class. With the horizontal sundial, I think it is even more interesting to consider how the gnomon stands either perpendicular to the ground or at an angle.”

A female teacher with 2 years of teaching experience said, “I felt like I would like to try this in class. It
seemed like something the students could enjoy while building. I would have liked to learn more about the horizontal sundial’s dial face.”

A male teacher with 24 years teaching experience said, “I felt that it would be easier for junior high school students to make a horizontal sundial than to make an equatorial sundial. In fact, I think it would be possible to have students do readings of horizontal sundials.”

A male teacher with 29 years teaching experience said, “I thought it would be good to have students actually try making it in class, then go out to test the accuracy of their sundials.”

A male teacher with 26 years of teaching experience said, “I think the students will be interested in this. It will be good practice for them to draw diagrams with perfect parallel lines and tangent lines.”

Most of the teachers who attended the study group showed similar evaluations. In conclusion, sundials definitely could be effective teaching materials for middle schools.

Through delivering this lecture to the junior high school teachers, I felt that sundials make an excellent teaching tool for teaching math, history, and culture.

References


