Journal of Modern Education Review, ISSN 2155-7993, USA
January 2013, Volume 3, No. 1, pp. 48-54

http://www.academicstar.us

Students' Understanding of Elementary Algebra in Indian School

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Abstract: This paper describes and presents the findings of a study which aimed to trace the pupil's understanding of algebraic concepts, i.e., linear equation with one variable and related cognitive difficulties in solving linear equations. Forty (40) pupils of seventh grade were involved as the participants of this study. The data collected through a set of problems (24 linear equations) with individualized in-depth interview reveals that the children who come across algebra for the first time often have problems in understanding the unknown/variable, with equal sign and algebraic linear equations. The research also shows that though the children have procedural knowledge to solve the equations, they face difficulties in solving the equations having unknown as a subtrahend or divisor and having unknown at different positions.

Key words: Algebra, understanding, linear equation, difficulties

1. Introduction

According to Locke, mathematics is a way to settle in the mind a habit of reasoning. Piaget suggested that the origin of the structures of knowledge is in the internalization of actions. The study of mathematics results in the development of power rather than the acquisition of knowledge. The chief characteristics of mathematics are abstractness, precision, generality, logical, analytical, systematic etc. It is self contained mental discipline with its own language, symbolism and structure.

Learning Mathematics is not just about acquiring and mastering computational and problem-solving techniques, or solely about understanding definitions , arguments and proofs, or even simply about how to work with examples and counter-examples. In addition to all these things, it also involves an individual in reconstructing his/her own thinking and work of other mathematicians, so that it becomes a part of their own thinking. In doing so, he/she undertakes his/her mathematical activity and exploration. Learning mathematics requires one to develop ways of thinking mathematically while doing mathematics, for many, the most exciting and creative element of all. Learning mathematics means understanding a system that goes beyond the examples one learns from. For this reason, we say that mathematics learning is generative, i.e., learning the system allows one to generate new facts that one was ever taught about. In order to understand children's learning of mathematics, we have to analysis their productions-the way they count, write numbers, solve equations etc. and try to figure out how they think (Victor Lee & Gupta P. J., 1995).

Every branch of mathematics furnishes ideas and techniques for representation of reasoning about, structural properties or patterns in observed or imaginary situations. Most of today's mathematics education world presents the position that algebra should be learned by all students if they are to be functional contributors to the world of

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the future. Traditionally we justify that the algebra should be studied because it is essential for learning advanced mathematics.

Algebra is a branch of mathematics which studies structure, relation and quantity. Pimm (1995) describes algebra as form and transformation. It seems to be dynamic, operating on or transforming forms. The concepts, principles and methods of algebra constitute powerful intellectual tools for representing quantitative information and then reasoning about that information.

Elementary Algebra is the most basic form which is taught to students who are presumed to have no knowledge of mathematics beyond the basic principles of arithmetic. The central concepts of algebra include variables, functions, relations, equations, and inequalities, and graphs. The central principles are the structural properties of the real- number system and its important subsets. Those concepts and principles combined together give a system of symbols for describing and drawing inferences from relationships among quantitative variables. Students must represent and handle quantitative information throughout their study of arithmetic at the level of elementary school.

Linear equations are not only a central part of any mathematics course, especially at elementary level, but they are also an integral part of a wide variety of algebraic, geometric, and trigonometric problems. Nickerson (1985) mentioned that an area of confusion in mathematics that deserves special mention involves the understanding of simple algebraic equations. A basic understanding of what an equation is would seem to be fundamental to success in any mathematically oriented endeavor. The algebraic representation has the advantage of conciseness and of making available the power of mathematics to develop the implications of sets of relationships among several variables. There are several features of linear equations which are not immediately apparent to the pupil and may even not be appreciated by novice teachers. Kieran (1992) found that the solution methods for linear equations are of procedural in nature as the students carried out arithmetic operations on numbers to yield numbers. For example, in solving the equation $2x + 1 = 5$, students tried to put various numerical values for "x" until a solution is found. As Kieran notes, "the objects that are operated on are not the algebraic expressions, but their numerical instantiations". She further suggests that the introduction of algebra, at this stage, requires a teacher-directed move away from procedural approach to a structural approach. Here the term structural refers to a different set of operations that are carried out, not on numbers, but on algebraic expressions. For example, the solution of $3x + 2 = 15$ may involve the step $3x + 2 – 2 = 15 – 2$ which has nothing to do with either final solution or any numerical instantiation.

Learning to solve equation should not be just memorizing a set of rules. Perso (1996) expressed concern that students who solve equations only by a set of memorized rules tend to have misconceptions about solving equations. For example, when the idea of inverse operations is eclipsed by the memorized rule "change the side, change the sign", students may not see that inverse operations can be performed without changing the equality of the equation. Solving equations is not limited to finding a solution. Wagner and Parker (1993) claimed, that having an understanding that a variable in the original equation can be replaced by the value determined by the solution, demonstrates knowledge of the nature of variable and the purpose of solving equations.

When solving equations with a single unknown, children can fall back on their knowledge of arithmetic and find a solution using a variety of strategies. Generally they do not fully understand the structural rules that govern the equal sign and its use. Therefore, they do something to the equations without a contextual understanding as how to proceed. This indicates that something is done to the equations and not with them. Gallardo and Rojano (1987) studied the pre-algebraic behavior and the phenomenon of transition from arithmetical thinking to

algebraic and found the areas of difficulties in algebra learning as follows: operations, the nature of numbers, primitive methods(The strategy of trial and error), the interaction between the semantics and syntax of elementary algebra and the didactic cut in the study of linear equations.

It is generally accepted that the concept of a variable is difficult for beginning algebra students to comprehend. They may have been exposed to the idea of a variable in a multitude of settings prior to coming to algebra class. Booth (1984) found that the students used letters as labels or as specific values, rather than as variables. One of the findings of Kuchemann (1981) revealed that few students were able to interpret letters as variables and a very small percentage of 13–15 year old pupils were able to consider the letter as a generalized number. Kieran (1992) contented that students do not view the equal sign as a symbol of equivalence but rather as an announcement of the result or answer of an arithmetic operation. They think that the right side should indicate the answer, that is, 4 + 3 = 7. Knuth et al. (2006) highlighted the students inability in understanding the concept of equality by stating that studies on students' understanding contended that the notion of "equal" is complex and difficult for students to comprehend.

Apart from the above difficulties, there is a general perception that mathematics and particularly algebra is a tough subject which requires some particular kind of ability and skill and thus cannot be mastered by each and everyone-but only by a few, possessing these special abilities or skill. The students' inability to understand the algebraic concepts even when given appropriate instructional guidance is a serious concern to all of us. In India, a good number of studies have been done to find out the achievement in mathematics education as well as the causes responsible for low achievement of students in mathematics education. But hardly a few studies have been done to investigate the understanding of mathematics among students and the difficulties (what is the error and what is the source of error?) of students in mathematics In general and algebra in particular. This may give insight into the main obstacles in learning mathematics with specific reference to algebra. Thus the difficulties in mathematics/algebra have been an area of immense interest to researchers.

2. Methodology

The sample for the present study consists of 40 students of class VII selected from a school that was selected on the basis of the convenience as well as the basis of fulfilling the need of the data collection of the study i.e. the students were available for the long duration interview whenever needed. A set of problems (24 linear equations adopted from Herscovics & Linchevski, 1994) was administered on the students to assess the student's understanding of linear algebraic equations and to investigate the difficulties faced by the students. The investigator conducted interview on the students individually for an in-depth analysis of the same.

3. Results and Discussion

The data gathered through the set of problems and an in-depth interview, was analyzed in terms of success and difficulties faced by the students in solving linear equations with one variable. On the basis of the responses given by the students, the equations were categorized such as:

(1) Equations involving addition and subtraction

(2) Equations involving multiplication and division

(3) Equations involving the grouping of the numerical terms

(4) Equations involving both additive and multiplicative operations

(5) Equations involving double occurrence of unknown on the same side

(6) Equations involving double occurrence of unknown on the both sides

These categorization of the equations have been done on the position of unknown/variable and the type of operation (e.g., addition, subtraction, multiplication and division).

Table1 Equations Involving Addition and Subtraction

Eq. No.	Equations	Success rate of students (in %)		
		Solved	Partially Solved	Not Solved
1	14 + n = 43	92.5	7.5	0
2	35 = n + 16	90	5	5
3	n – 13 = 24	90	5	5
4	37 – n =18	70	20	10
5	17 = n – 15	65	22.5	12.5
6	23 = 37 – n	67.5	22.5	10

Most of the students solved the linear equations having addition operation. The students could not completely solve equations involving subtraction. A considerable number of students have attempted to solve the equations but failed to complete the solving process and could not get the right answer. The number of students who solved the equations partially was higher in the case of equations having subtraction operation with unknown at the right side of the equal sign. It seems that the students have a procedural thinking such as they have to subtract the number by shifting from one side of the equal sign to the other and unknown should be always on the left side of the equal sign.

Students were facing the difficulties in thinking as to whether sign of a number changes on shifting it from one side of equal sign to other side as sign of the unknown/variable does not change. They were not able to think if a number can be subtracted from an unknown number and vice versa. They had a question in their minds as to how a number can be subtracted from an unknown number and vice versa? Students were facing the difficulty of Sign detachment.

Table 2 Equations Involving Multiplication and Division

S. No.	Equations	Success rate of students (in %)		
		Solved	Partially Solved	Not Solved
7	16n = 64	67.5	0	32.5
8	2088 =174n	65	0	35
9	n ÷ 6 = 13	67.5	2.5	27.5
10	84 ÷ n = 4	52.5	22.5	25
11	15 = n ÷ 7	62.5	2.5	35

A large number of the students have solved the linear equations having multiplication. In equations, having division operation, less number of students could solve in comparison to the equations having multiplication operation. The equation, having variable as divisor, seemed to be more difficult to the students as only half of the students could solve it. Some of the students tried to solve the above said equation (e.g., 84 ÷ n = 4) but failed to complete the solving process and could not succeed to get the right answer.

Similar to the problem with the subtraction from an unknown number, the students had a question in their minds as to how a number can be divided from an unknown number and vice versa? Students were thinking that

only variable or unknown number comes on left side of the equal sign and number should be on the right side. They had a reason for that — "we have to find the value of variable". Some of the students were not understanding division operation. They were facing difficulty in operating on unknown numbers and having Concatenation problem, e.g., some of the students were not considering 16 as a multiple of "n" in 16n. They were considering 16n as a three digit number and were not able to find out even a wrong value of "n".

Table 3 Equations Involving the Grouping of Numerical Terms

S. No.	Equations	Success rate of students (in %)		
		Solved	Partially Solved	Not Solved
12	n + 34 = 29 + 38	75	0	25
13	23 + n +18 = 44 +16	72.5	0	27.5
14	4+ n - 2 + 5=11 + 3 -5	52.5	15	32.5

Majority of the students solved the equations involving the grouping of numerical term. A comparison between equations 12 and 13 involving an unknown between two numerical terms indicates that the presence of an unknown in the grouping of number is an indicator of difficulty for the students. Among these three equations, equation 14 gave the most startling results. It is found that 19 of 40 students failed in solving the equation and could not do grouping the numbers on the left side correctly. Some of them tried to solve the equation but could not succeed completely as they were trying to add 2 with 5 first and then 4 to get the total number at the left side.

Students were facing difficulties as discussed earlier, i.e., sign change problem, fixing left side for variable/unknown and sign detachment problem e.g. students were ignoring the minus sign preceding the number 2 in equation 4 + n – 2 + 5 = 11 + 3 – 5.

Table 4 Equations Involving Additive and Multiplicative Operations

S. No.	Equations	Success rate of students (in %)		
		Solved	Partially Solved	Not Solved
15	13n +196 = 391	55	0	45
16	420 = 13n + 147	47.5	2.5	50
17	16n – 215 = 265	50	0	50
18	63 – 5n = 28	40	12.5	47.5
19	188 = 15n – 67	50	0	50

Equations involving additive and multiplicative operations with unknown on left hand side to the equal symbol are solved by more number of the students than equations involving additive and multiplicative operations with unknown on right hand side to the equal symbol. Similarly, the equations involving subtractive and multiplicative operations on the left side are solved by more number of students than the equations involving subtractive and multiplicative operations on right hand side to the equal symbol. The equation 18, which has variable as subtrahend is seemed to be more difficult to the students. Students could not solve the equations because they only knew that they had to either subtract/add or and divide the numbers.

Students were facing the problem of concatenation, fixing left side for variable and sign detachment They were also facing difficulty in operating on unknown numbers ignoring the minus sign preceding the term 5n in the equation 63 – 5n = 28. Students were facing difficulty in operating on unknown numbers. They were adding or subtracting only numbers, e.g., some students were adding 13 and 147 first and then subtracting the sum from 420

in equation 420 = 13n + 147, some students were adding 215 and 265 first and then subtracting 16 from the sum in the equation 16n – 215 = 265. They were also facing the concatenation problem, e.g., students were thinking 13n as three digit number and were putting value of n as 1, 2, 3 up to 10 to find the sum of 13n and 196 as 391.

Table 5 Equations Involving Double Occurrence of Unknown on the Same Side

Eq. No.	Equations	Success rate of students (in %)		
		Solved	Partially Solved	Not Solved
20	n + 5 + n = 55	50	12.5	37.5
21	11n + 14n = 175	52.5	7.5	40
22	9n – 4n = 35	60	0	40
23	7n + 5n + 7 = 55	55	5	40

Equations involving double occurrence of unknown on the same side without a coefficient with unknown is seemed more difficult to the students in comparison to the equations involving double occurrence of unknown on the same side having a coefficient with unknown. Of the students who could not solve the equations or could solve the equations partially, only knew that they had to either subtract/add or and divide the numbers.

Students were facing difficulty in operating on unknown numbers as they were adding or subtracting only numbers, e.g., some students were adding 11 and 14 first and then dividing 175 by the sum in equation 11n + 14n = 175, and some students were adding 7.5 and 7 (all the numbers only) first and then subtracting sum from 55 in the equation 7n + 5n + 7 = 55. They were facing concatenation problem, e.g., students were thinking 9n and 4n as two digit numbers and were putting value of n as 1, 2, 3 up to 9 to solve the equation 9n – 4n =35. Students were thinking that if an unknown number without any number is added with other unknown number, will give result again one unknown number only, e.g., n + n = n.

Table 6 Equations Involving Double Occurrence of Unknown on Both Sides

S. No.	Equations	Success rate of students (in %)		
		Solved	Partially Solved	Not Solved
24	5n + 12 = 3n + 24	50	2.5	47.5

Only half of the students solved the equation involving double occurrence of unknown on the both side. One student tried to solve the equation by subtracting 12 from 24 after shifting 12 to the right side of the equal sign but could not proceed further as he complained to the investigator by saying "'3n' should not be on the right side so it can't be solved". Students were facing difficulties almost similar to the equations involving double occurrence of unknown on the same side. Students were facing difficulty in operating on unknown numbers with concatenation problem, e.g., students were thinking 5n and 3n as two digit number and were putting value of n as 1, 2, 3 up to 9 to solve the equation 5n + 12 =3n + 24.

4. Conclusion

The findings reveal that the children had procedural knowledge to solve the equations as they had to add or subtract the number by shifting from one side of the equal sign to the other and unknown should be always on the left side of the equal sign. That is why the students faced the difficulties in solving the equations having unknown either on right side or as an addend or subtrahend. Most students chose a simple strategy that was sufficient for solving a particular equation, instead of a sophisticated strategy. It is important to note that using an inverse

operation to solve an equation involving division was very difficult for students. Students were facing different kinds of difficulties in solving the equations. They were facing problems of problem of sign detachment, thinking that sign of unknown does not change as it is shifted from one side of the equal sign to the other side, concatenation problem, difficulty in operating on unknown/variables and so on. Most of the findings of the research are consistent with the other researches in this area (Herscovics & Linchevski, 1994), but the significance of the study is that it affirms that Indian students' level of understanding of linear equations and their problems are similar to the students of other parts of the world.

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