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Paradigm and Pan-Paradigm in Mathematics and Art History

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Abstract: Mathematics teaching is often more effective when teachers connect the contents of mathematics with history, culture, and social events. In the history of mathematics, the "paradigm" theory from Thomas Kuhn's scientific revolution is very effective to explain the revolutionary process of development in mathematics, and his theory has been widely quoted in the history of science and economics. However, it has not been appropriate to use his theory in the other fields. This is due to the fact that the scope of Kuhn's paradigm theory is limited to mathematics and science. This research seeks to explain styles in art history and the appearance of new theories in mathematics history by the concept of "pan-paradigm". Since this concept can be expanded to the entire history of culture, this researcher experienced effectiveness in teaching a mathematics course which is convergent in nature in a university setting.

Key words: paradigm, pan-paradigm, *Gampil* method, Differential·Integral calculus, Set theory, Topology, abstraction, Baroque art, Rococo art

1. Purpose of Study

In the field of mathematics education in which creativity is demanded through the use of an interdisciplinary approach, teachers are often effective in their teaching when they connect the contents of mathematics with historical backgrounds and cultural events (Kye, 2006a). Thus this researcher has presented the following research results by focusing on the mathematics and art history of the West: that is, the vanishing point of the 16th century Renaissance drawing, the point at infinity in Projective Geometry, differential·integral calculus and Baroque·Rococo art in the 17th and 18th centuries, set theory and the pointillism of art, and topology and the development of abstract painting in the impressionist paintings since the 19th century, have all demonstrated that they display the same spirit of the age (Kye, 2003; 2005; 2006b; 2009). However, Thomas S. Kuhn's theory of "paradigm" by scientific revolution has been proven to be not effective in explaining the revolutionary process of development in mathematics, nor appropriate to explain the same spirit of the age in the interdisciplinary studies between mathematics and art. Kuhn was not truly interested in the cultural and social backgrounds of mathematics and scientific revolution. And this is due to the fact that the scope of the paradigm theory was limited to mathematics and science (Kuhn, 1984).

Each time a new revolutionary theory in mathematics history appeared, there was also a parallel change in art style. The "vanishing point", which is a concept of the infinite that Renaissance painting had begun to introduce had influence on the birth of Projective Geometry as the mathematicians went beyond the traditional Euclidian

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geometry in order to actively utilize the concept of infinity. The concept of the infinite, the infinitesimal of Newton and Leibniz in the 17th century, played a leading role in the scientific revolution by creating differential·integral calculus; the concept of differential calculus found its expression in Baroque and Rococo art. The set theory of Cantor since the 19th century, which found its expression in the pointillism of Seurat and Signac, is a mode of thinking of infinity expressed in reductionism, which had been an object of interest since the time of ancient Greece. The paradigm theory of Thomas Kuhn is not adequate to explain the phenomenon of this kind of spirit of the age.

On the other hand, when we consider Oriental art (China, Korea, and Japan) in its 17th and 18th century period, in spite of the fact that they didn't have differential·integral calculus, they depicted momentary actions quite well in the style of Baroque. Also in the Orient, 700 years before the introduction of topology, painters painted using the *Gampil* method, which is a simplified pencil line similar to that found in the paintings of Matisse, the abstract painter of the 20th century. The mathematics and art of the Orient also cannot be explained away with the theory of Thomas Kuhn. This phenomenon shows that it is not that mathematics made a direct impact upon art, nor did painters make conscious impacts upon mathematics. But one can sense the significant birth of a new spirit of the age through the revolution of mathematics and art. This research seeks to explain the spirit of the age of mathematics and art in Western culture with the "pan-paradigm" of KIM Yong Woon who generalized the concept of paradigm; furthermore, this paper will attempt to explicate the comparison between the mathematics and art of the Orient and the West through "pan-paradigm" and "prototype".

2. Paradigm and Pan-Paradigm

The idea of paradigm was first introduced by Thomas S. Kuhn, an American scientist and a philosopher, in his book, *The Structure of Scientific Revolutions*. Using the theory of paradigm one can properly analyze with the structure of development in the history of Western science. In the case of the history of mathematics, Kuhn's paradigm presents the categories of Oriental math, ancient Greek math, Medieval European math, Renaissance math, pre-modern math, and modern math, each of which shows its own unique characteristics. To add a remark on the categories, through these categories, mathematics has developed from practical addition and the theory of the equation, to logical geometry, to monastery-centered mathematics, to calculation-centered mathematics necessary for commerce, to the differential·integral calculus of Newton and Leibniz, and to abstractly structured mathematics by set theory (Kim Y. W. & Kim Y. K., 1997).

In each category of these revolutionary developments, the object of mathematics has changed. In the beginning the shapes of figures were the objects of mathematics, but later numbers, quantity, function, etc., which are abstract realities, became the object of mathematics. Now modern mathematics deals with the relation between more abstract objects such as arithmetic calculation, pattern and structure (Kim Y. W. & Kim Y. K., 1997).

Once there is a change of object in mathematics, a change in the methodology of mathematics is required as well. Europe in the 17th century, having undergone the Renaissance, was in an age in which they could not solve problems, meet the demands of the age and handle the social upheaval at all with the Greek deductive geometry which they traditionally believed as truth. Thus a new paradigm of analytic geometry, which combined Greek geometry and algebra, came forth by Descartes and Fermat. Based on the power of analytic geometry, the concept of the infinitesimal and the infinite created differential·integral calculus by Newton and Leibniz, which all promoted modern science. In the 19th century, Europe insisted on autonomy, and the set theory by Cantor, which is

the ancient Greek reductionism newly materialized, was brought forth into life. What becomes clear is that every time when the former culture faced crisis and when traditional knowledge collapsed, there always appeared a genius who came forth with a revolutionary theory in the history of mathematics (Kim Y. W. & Kim Y. K., 1997). This phenomenon is clearly a paradigm shift, and it occurs within the framework of scientific revolution.

In Kuhn's theory, paradigm is a concept about the structure of scholarship development, and it defines the reason for existence, structure, and trend of a scientist group. As an academic accomplishment it is a concept that implies the suggestions of scientific problems for significant period, format, model of solution, etc. On the other hand, Japanese scholar, Shigeru Nakayama's book *Scholarship as History* explains what a paradigm is, "sacred book style of ancient literature, which dictates the course of scholarship, justifies the professional and vocational activities, and regulates the future course of development of the standardized scholarship" (Kim, 1994).

Although mathematics and art in the West have displayed the same spirit of the age, upon a detailed look, there was nothing revolutionary in the field of art, even though the appearance of the infinite and infinitesimal in mathematics were significantly revolutionary within the scholarship. The appearance of the set theory also was quite a revolutionary upheaval, but the new direction toward abstract art was not considered inner revolution in the field of art. It is simply that photography skills developed, and through the use of the optical science, art has progressed toward abstract art. Therefore, although mathematics and art in the West have displayed the same spirit of the age, Thomas Kuhn's theory of paradigm is not appropriate to explain the spirit of the age of the two genres, and thus this researcher borrows the pan-paradigm of Kim Yong Woon. Especially, Oriental science never criticized the scope of the application of reason which Immanuel Kant did. Oriental science studied religion, mathematics, history, philosophy, and all other subjects from a single perspective. Kim (1994) compared and analyzed the history and mathematics of Korea and Japan, pointed out that it is not appropriate to widely apply the paradigm on the general trend of culture. Thus he provided the concept of pan-paradigm. The way he defined the pan-paradigm was this: it is a spirit that integrates the entire culture when the harmony and balance break between the cultural factors and elements in response to the demand from the society (spirit of the age). That is, Adam Smith's "Invisible God's Hand" or Hegel's "The World Spirit" works underneath all, so that it brings forth harmonious culture in all aspects of the culture. In other words, the pan-paradigm is this: At the root of various cultures there are always mutual interaction between "collective unconsciousness" and "demands of the times", and these two factors by interaction form values and controlling principles common to various parts of its culture, and this synergism leads the cultural activities. Furthermore, culture is always formed by pan-paradigm. Thus he suggested "Theory of National Prototype" between Korea and Japan since the national prototype penetrates within the pan-paradigm. He insisted that the 'trend of the times' that the European thinkers used at the end of the 19th century had ignored the relationship with collective unconsciousness. Yet since the pan-paradigm presupposes the collective unconsciousness, he argues for the differentiation from the trend of the times (Kim, 1994; Kim Y. W. & Kim Y. K., 1997; Kuhn, 1984).

Therefore, from the perspective of pan-paradigm, one can see that there is a unique national prototype that penetrates through the culture of the Western people. Also one can analyze that the reason that mathematics and art have demonstrated the same spirit of the age in each period, whether it is ancient Greece, medieval Europe, the Renaissance, or the pre-modern and modern period, is because the pan-paradigm is different in each period. That is, my research papers, "Projective Geometry and Renaissance Art", "The 17th, 18th century Differential·integral calculus and Baroque·Rococo Art", "Modern Mathematics and the Abstractness of Modern Art", are not about paradigm but all about pan-paradigm. This concept can be applied to the entire history of culture, and has been an

interesting subject in complex mathematic courses at the university level.

3. 17th and 18th Century Western Art and Chosun Dynasty Drawing

Figure 1 CHO Young Suk (Putting Horseshoe)

Figure 2 KIM Hong Do (Roof-tiling)

Figure 3 KIM Deuk Shin (Killing Time)

The unique characteristic of 17th century painting in the West is that it was good at capturing the action of a moment and showing the contrast between light and darkness. Thus the painting of this period has been quite dynamic in comparison to the Classicism of the 16th century. The first lady painter, Gentileschi; the best religious painters, Rubens and Van Dyke; the painter of light, Rembrandt; the painter of a moment, Hals; are all leading painters of the Baroque style who produced dynamic paintings. In the 18th century, although Rococo art was decorative, splandid and decadent, when one looks at the paintings of Fragonard from a mathematician's perspective, they exhibit the spirit of the age in which differential calculus came forth as they expressed each action of every moment (Kye, 2005; 2006b).

Now I seek to focus on Oriental art, especially the Korean paintings of the Joseon dynasty in the 18^{th} century. In CHO Young Suk's Painting 1, the apex of the Joseon dynasty, a man is attempting to put a horseshoe on a horse that is jumping off due to pain and fear, while the other man is trying to pacify the horse, as the horseshoer endeavors to put the horseshoe on quickly. Painting 2 is the work of KIM Hong Do and describes a vivid portrayal of the workers working with roof tiles. Some of them are binding tiles, some others throwing them, some others planning, while the noblemen are supervising. All of this captures well the actions of the moment, as well as those of the Baroque and Rococo arts in the West. Painting 3, the art work of KIM Deuk Shin, illustrates the mother hen trying to chase a cat who was attempting to catch chicks, while the owner, having been smoking, was jumping off from the deck to chasten the cat, as his wife was running out to the yard. Here again all of this excellently depicts the reality by capturing the actions of the moment (Lee, 2000). Meanwhile, the mathematics of the Joseon dynasty in Korea, including China, did not see the appearance of the concept of the infinite and the infinitesimal. However, just as we have seen, the spirit of Baroque is exactly expressed in the Oriental paintings. How can we explain this phenomenon?

4. Western Mathematics and Western Abstract Art

In the 19th century, the presentation of set theory by Cantor in 1883 started the process of abstraction in the entire field of mathematics. The background for it was due to the demand of Europe for "autonomy", and

"reductionism". At the same time, the "pointillism" of Seurat and Signac appeared. On the other hand, the one-point perspective style of the Renaissance, in which painters had to work in one single space under the strict ratio, began to disappear and welcome the abstract painting style. Just in time, Louis Daguerre invented photography, which replaced the detailed paintings in their function as the painters turned their attention to the inner world and pursued abstract painting. In mathematics Cantor announced set theory; in art, the giant of cubism, Picasso, denied the perspective technique and destroyed the traditional stereotypes at the same time. In the 1930s, geometry did not find problematic the traditional problems of figuring out angles of figures, parallel lines and areas. The focus was only on the continuity of a figure that corresponds from a point to point, and with it a new geometry and topology appeared (Kline, 1972; Kye, 2005; 2006b).

Matisse, one of the two greatest master painters in the 20th century in addition to Picasso, getting the compliments of "genius of simplifying painting", eliminated all unnecessary elements, leaving only the most basic to the object of his painting. His painting is a work of simplified topological style which leaves only the most basic elements while ignoring the height, size and width of object. In Matisse' painting, there is no distinguishing line between a room, a wall or a floor. It signifies that the Euclidian three-dimensional space has been destroyed. As such, the appearance of both worlds of abstract art and abstract mathematics in the same period yet in different fields is a realization of the spirit of the age of the 20th century and pan-paradigm (Kye, 2005; 2006b).

5. Appearance of the Pointillism even in Oriental Art

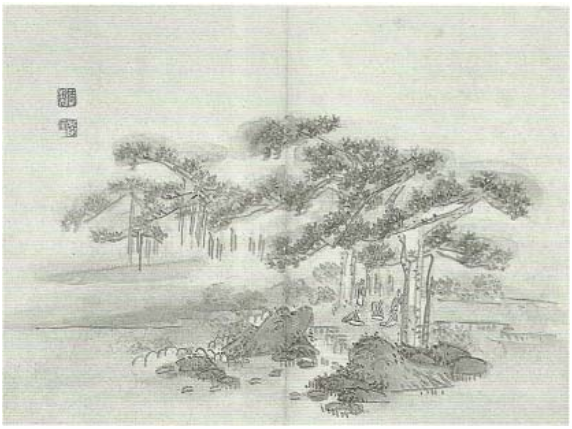

Fig. 4 KIM Soo Cheol <Pine Tree and Stream Gossip>

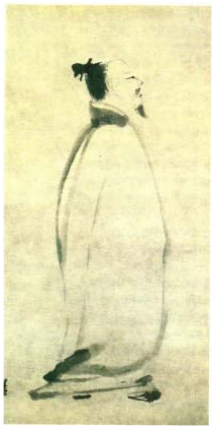

Fig. 5 Liang Kai <Li Bai>

Fig. 6 Matisse <Nude>

Seurat, a contemporary of Cantor who was known for his set theory, expressed his painting with dots in 1883, and in order to express brighter color, took advantage of the principle in which two colors are combined in our retina when we look at the small dots of solid color and primary color. He borrowed the power of optical science that was well developed in his days. To take a dot as an element of a painting and to take an element as an object of mathematics have everything to do with reductionism. On the other hand, KIM Soo Cheol, a painter during the Joseon dynasty in the 19th century, utilized a radically different style of technique than the traditional Oriental painting technique (see for example, *Pimajoon* or *Bubyukjoon*). He analyzed the object of his painting into a certain unit which he expressed by putting dots sparsely on paper. He was the great leading figure of Neoplasticism (Cahill, 2010; Lee, 2000). This too expresses the same spirit of the age as that of the pointillism of the West. However, there are no known traces of the Greek reductionism in the Oriental thoughts, nor the set theory of the West found in Chinese mathematics or in the Joseon mathematics called *Sanhak*. As such, while the

mathematics and art of the West display the same spirit of the age, how can we explain this phenomenon in Oriental art? (Kim Y. W. & Kim Y. K., 1997).

6. Oriental *Gampil* Method

Figure 7 Japanese 8th century <Mapo Bosalsang>

Figure 8 KIM Myung Gook <Dalmado>

Picture 5 is the work that the Chinese Zen painter Liang Kai produced between the 12th and early 13th century. It portrays a walking Li Bai who loved to drink reciting a poem. In comparison to *Nude* that Matisse drew in 1940 in France, the concept of abstract painting in Oriental art came 700 years earlier. In China, from the ancient period it was known as *Gampilbup*, which is a method of simple drawing while eliminating unnecessary details. This method has been mainly used for portraits, and KIM Myung Gook in the Chosun dynasty also painted *Dalmado* using this *Gampilbup* (Cahill, 2010; Lee, 2000; Lee, 2005). This researcher argued that simplified drawing of the Western painting by eliminating the details was under the influence of the topology of the mathematics that appeared in 1930s, and that it is the expression of the same spirit of the age. If so, then why has this *Gampilbup* preceded the Western art for 700 years even though no concept such as topology in the Oriental mathematics has been found? This researcher argues that it is due to the influence of Chinese Zen Buddhism, a Taoist philosophy, which emphasized non-possession, self-control and a simple life. Picture 7 is *Mapo Bosalsang* from the middle of 8th century, and is Japanese Buddhist art that portrays a statue of a Bodhisattva sitting on a cloud painted with Chinese ink stick on a hemp cloth. The pose of fingers, long sleeves, clouds below all of these help us to feel strongly a three-dimensional effect. In Oriental art, they simply used brush strokes without shadow effect in order to create effectively a three-dimensional perspective. This, too, is a form of art in which brush strokes are simplified and thus considered a forerunner of *Gampilbup*. Since Liang Kai's art work in China is of 12th and early 13th century, the Japanese *Mapo Bosalsang* precedes the works of Liang Kai by 300–400 years. In sum, analyzing the paintings of China, Japan and Korea, we cannot explain with Thomas Kuhn's paradigm this abstract concept which was used in paintings in spite of the absence of the concept of topology in the Oriental mathematics. Therefore, we can explain it with pan-paradigm.

7. Conclusion

The term paradigm, that Thomas Kuhn first used in his book, *Structure of Scientific Revolution*, has turned

out to be a common term in recent years to refer to a community's mode of thinking and to cultural phenomena that have become popular for a certain period of time. But in this study, this researcher introduced KIM Yong Woon's pan-paradigm as a general concept that encompasses all, since through a strict analysis in the field of mathematics and art, Thomas Kuhn's theory of paradigm does not explain the phenomena. That is, at the root of various cultures there exist always a "collective unconsciousness" and "demands of the times", and these two factors by synergism form values and controlling principles common to various parts of the culture, and this synergism leads the cultural activities, the process of which is a phenomenon called pan-paradigm.

In mathematics, the invention of differential·integral calculus by Newton and Leibniz and Descartes' analytic geometry applied by analysis and synthesis were scientific revolutions, and each pursued its own independent academic discipline. However, although there was not scientific revolution in the contemporary Baroque and Rococo arts, the fact that they demonstrated the same spirit of the age can be explained by pan-paradigm. Furthermore, comparing Oriental (Korea, China and Japan) art and mathematics, the Western paradigm indeed does not fit to explain these phenomena. The reason why the Oriental art developed the simplified painting method of *Gampilbup* early in history, in spite of the fact that Oriental art did not experience the making of mathematics to be abstract, is because of the influence of Chinese Zen Buddhism, which is a Taoist philosophy, that emphasized a life of non-possession, self-control and simplicity.

KIM Yong Woon (1994) argues that the reason why Korean culture and Japanese culture have shown different aspects is due to the difference of pan-paradigm, yet the collective unconsciousness that flows through the two cultures is the same, which he labels as the theory of National Prototype. This researcher ends this abstract by quoting the contents of the study of a cognitive psychologist of the United States, Richard Nesbit (2004) in order to argue that mathematics and art in the Western culture are of pan-paradigm; furthermore, the reason why the pan-paradigms of the Oriental culture and the Western culture are different is because the National Prototypes of the Oriental people and the Western people are different. That is, Professor Nisbett argues that the difference in the national prototype of the Oriental and Western people comes from the difference in ecological environments and economy. Greece had a commerce and industry-centered city-state and republican government in contrast to China which was a family-centered agricultural society and this resulted in the differences of social structure. He insists that this social structure difference caused different social norms and different methods of child rearing, which in turn gave birth to different understandings about the essence of the universe, and all of which caused them to have different perceptions and theories of epistemology (Nisbett, 2004).

Therefore, if we consider all the cultural phenomena including both mathematics and art in the Western culture, it is explained by pan-paradigm. And when we compare the West and the East in the fields of the history of mathematics and art, we see the difference of the prototypes of collective unconsciousness, and accordingly, they are due to the differences of pan-paradigms between the two. Furthermore, this researcher concludes the abstract by arguing that this pan-paradigm theory, which goes beyond the history of mathematics and arts between the East and the West, is a concept worthy of further research and analysis by expanding its application over the entire history of culture.

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